# DESIGN

OF A

# HIGH-SPEED STEAM ENGINE.



## Notes, Diagrams, Formulas and Tables.

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SECOND EDITION, REVISED AND ENLARGED.

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# PREFACE.

When the first edition of this book was published the intention was to make it Part I of a treatise on the Dynamics and Design of a High-Speed Steam Engine. Part II was to treat of the Dynamics and Design of Shaft Governors. This second part is now nearly finished, but proves to be so difficult and voluminous as to make it unsuitable for most undergraduate work in technical schools; it will, therefore, be issued as a separate and independent work.

The present volume drops its old designation as Part I, corrects the typographical errors of the first edition, and adds much new material in the form of appendices.

J. F. KLEIN.

*Bethlehem, Pa., February, 1903.*

NOTES, DIAGRAMS, FORMULAS AND TABLES

FOR THE DESIGN OF A

# HIGH-SPEED STEAM ENGINE.

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## DATA.

Determine the principal dimensions of an engine fulfilling the following requirements:

The work ordinarily required, = 100 *H. P.* net = 110 *I. H. P.*, but the load may vary from 50 *I. H. P.* to 170 *I. H. P.*

The engine is to run steadily under all these loads, the total variation ($N_1 - N_2$) of the number of revolutions of engine shaft being less than $\frac{1}{50}$ of the normal number of revolutions ($N$)

$$i.\,e.,\ N_1 - N_2 = \frac{N}{50} \text{ when } N = \frac{N_1 + N_2}{2}. \qquad (1)$$

The engine is to be of the non-condensing, high (rotative) speed type, durable, of compact and rigid form, and the steam consumption of the 100 *H. P.* per hour is to be a minimum. Moreover, this high-speed engine must be provided with generous wearing surfaces which are continuously lubricated, the reciprocating parts must be balanced and their weights so chosen that the tangential pressures on the crank pin will be as nearly constant as possible.

It is assumed that the workmanship will be excellent, the parallelism of crank pin and shaft, as well as the alignment of crank-shaft bearings, perfect, and that they will be maintained so by good foundations.

## ORDER OF CALCULATION.

I. Determination of diagram of effective steam pressures on piston of required engine.

*a.* Determination of boiler pressure.

*b.* Determination of clearance, real cut-off and apparent cut-off.

*c.* Determination of back pressure and amount of compression.

*d.* Method of constructing equilateral hyperbola and of determining points on the curves $pv^n$ = constant by tabulated coordinates.

*e.* Methods of determinining the mean effective pressures of indicator diagrams.

*f.* Determination of the cut-off corresponding to a given horse-power $H_1$ when the mean effective pressure $p_m$ and the total pressure $p'_m$ is known for another horse-power $H$ of the same engine running at the same speed, with same initial pressure $p_1$ and same counterpressures.

*g.* Diagrams of effective-steam-pressure-on-piston for forward and return strokes.

II. Determination of ratio of length of connecting rod to that of crank.

III. Determination of that mean accelerating force (per □″ of piston) necessary to start reciprocrating parts and corresponding to the most uniform tangential pressure on crank pin.

*a.* Determination of acceleration and construction of diagrams of accelerating forces.

*b*. Construction of horizontal or resultant pressures on cross-head pin (or on crank pin when friction is neglected) by combining diagram of effective steam pressures on piston with diagram of acceleration.

*c*. Discussion of effects on this diagram of valve setting, high speed and long and short cut-offs.

*d*. Conversion of horizontal pressures on crank pin into tangential pressures.

*e*. Construction of diagrams of tangential pressures on crank pin.

IV. Determination of diameter of cylinder, length of stroke, revolutions per minute and weight of reciprocating parts.

V. Preliminary estimate of dimensions of reciprocating parts, to see whether minimum weight of latter does not exceed that necessary to produce steadiness of running under the conditions assumed.

VI. Determination of weight of fly-wheel rim from tangential pressure diagram.

VII. Determination of dimensions of crank disk and distribution of its material so that both crank arm and reciprocating parts will be balanced.

VIII. Method of determining the influence, on the foregoing results, of the frictional resistances, of the weight of the rod and of the exact values of the forces of inertia.

IX. Calculation of width and diameter of belt pulley.

X. Graphical determination of diameter of crank shaft.

XI. Calculation of the length of crank shaft journal and graphical determination of the plane of division of its brasses.

XII. Calculation of dimensions of steam ports and pipes, also thickness of cylinder walls.

XIII. Valve diagrams and dimensions of valve and gear.

XIV. Drawing of details of engine.

XV. Drawing of plan and elevation of complete engine.

### Plates to be Drawn.

I. Six Indicator Diagrams.
II. Six Diagrams of effective steam pressures on piston.
III. Diagrams of accelerating and retarding pressures by approximate and exact methods.
IV. Six Diagrams of horizontal pressures on cross-head pin.
V. Diagram for converting horizontal pressures on cross-head pin into tangential pressures on crank pin.
VI. Six Diagrams of tangential pressures on crank pin.
VII. Four Diagrams of the forces shaking engine bed, for as many different degrees of counter-weighting.
VIII. Distribution of material on crank disk, and determination of its center of gravity.
IX. Determination of diameter of engine shaft by the methods of Graphical Statics.
X. Graphical determination of the direction of the resultant pressure on crank-shaft bearing.
XI. Valve Diagrams.
Plates containing details, plan and elevations of complete engine.

## I A.

## Determination of Boiler Pressure.

If we consider the cylinders as perfect non-conductors of heat, and consider economy of steam only, we must, in accordance with the deductions of thermodynamics, make our initial steam pressures as high as possible, for by so doing we increase the range of temperature of the steam in the engine, and thereby increase the efficiency of the engine. This is clearly shown by the expression for the thermal efficiency $\eta$ of the engine

$$\eta = \frac{T_1 - T_2}{T_1} \tag{2}$$

$T_1$ being the absolute temperature of the steam at the initial absolute pressure $p_1$, and $T_2$ the lowest absolute temperature

practicably possible for the engine, being in non-condensing engines $T_2 = 212° + 459.°4 = 671.°4$ Fahr. $T_1$ increases very slowly with the higher pressures of steam, see Roentgen's Thermodynamics, pp. 672 and 674.

Even for ordinary (cast iron) conducting cylinders it is true (up to not very well defined limits) that economy in the use of steam increases slowly with the initial pressure. Thus according to Prof. Thurston the weight of steam per hour per *H. P.* should in good engines be not more than that given by the empirical formula

$$\frac{200}{\sqrt{p_1}} \text{ pounds} \tag{3}$$

and for the best practice with large engines, dry steam, high piston speed and good design, construction and management the consumption of steam should be not more than

$$\frac{150}{\sqrt{p_1}}. \tag{4}$$

In (3) and (4) $p_1$ is the absolute pressure in pounds per sq. inch.

Sometimes the highest pressure which can be employed in an engine is prescribed by the strength of the boilers which are to furnish it with steam; at other times (with engines running under conditions in which the power occasionally sinks considerably below the average) a lower pressure (accompanied by a greater cut-off) than economy of fuel and steam would prescribe, must be employed in order that the engine may run at high speed with great steadiness and without reversing the direction of the horizontal pressures on crank pin. See limits to high speed, III, c.

The employment of very high pressures is also limited by the greater first cost of the boilers owing to the greater strength required, and the greater care which must be exercised in their construction. The engine also becomes somewhat more costly for the same reason. A practical limit is thus set to the general use of extremely high pressures.

In the present case we will start the calculation by assuming the initial boiler pressure employed by many makers of high-speed engines when designing and rating their engines, namely, 85 pounds gauge pressure.

## I B.

## Determination of Clearance, Real Cut-off and Apparent Cut-off.

Clearance is the volume or space included between the piston (when the latter is at the end of its stroke) and the cylinder head, and also includes the steam passages. It is usually expressed as a fraction of the volume swept through by the piston, *i.e.*, if $i$ represents the clearance, $A$ the area of piston and $S$ the stroke we have

$$i = \frac{\text{clearance volume}}{AS} \tag{5}$$

The clearance should be as small as possible, for when it is at all large it involves considerable loss by presenting a large space to be filled by fresh steam, which steam is not engaged in pushing back the piston when entering the cylinder, but in giving whirling motions to its own particles, though during expansion a portion of the energy of the steam which has filled the clearance spaces is utilized. On account of the loss occasioned by it, the clearance is sometimes called the hurtful space. Its hurtful influence can, however, be entirely removed by allowing the exhaust steam to be compressed till it reaches a pressure equal to the initial pressure. When this is done no fresh steam is needed to fill the clearance spaces at the beginning of the stroke; but as it is not always practicable (as will appear later) to compress the exhaust steam to so great an extent, it behooves the designer to diminish the clearance as much as possible.

The total amount of engine clearance employed in practice varies greatly with the type and size of engine employed. For example, in Corliss Engines the four separate valves employed

are placed close to the cylinder and the distance between end of piston and cylinder head left for variation of length of connecting rod (when the wear is taken up) is frequently not more than ¼″, so that in these engines the clearance often does not exceed ½ of one per cent. In high-speed engines, however, the steam passages are usually very large, as it is very important to admit the steam promptly, and at full pressure. In these engines the clearance sometimes amounts to 14 per cent., but this is very large. We will for the present assume that the clearance in our high-speed engine is as great as 7 per cent.

## Cut-off.

See Fig. 3, p. 16.

The volume of steam in the cylinder (on the live steam side of the piston) when the admission valve closes, divided by the volume of this steam (after it has expanded) when the exhaust valve opens, is called the ***real*** or ***virtual cut-off*** $= \frac{1}{r} = \epsilon$. It is always a proper fraction. The reciprocal of this fraction is the ***expansion***, and is of course a whole or mixed number.

The distance of the piston from the end of its stroke when the admission valve closes, divided by the whole stroke, is the ***apparent cut-off*** $= \frac{1}{r^1} = \epsilon^1$. The relation between the real and apparent cut-off is represented by the following equation:

$$\text{real cut-off} = \frac{1}{r} = \frac{\frac{SA}{r^1} + iSA}{SA + iSA} = \frac{\frac{1}{r^1} + i}{1 + i} = \epsilon. \qquad (6)$$

The following tables, pp. 8 and 9, give this relation, and were taken from Du Bois' translation of Weisbach, Vol. II. If our cylinders were non-conductors, and economy of steam were the only element of economy to be considered, we know from thermodynamics that it would be wise to employ both high pressures and great expansions. Unfortunately our cylinders are conductors of heat, and as a consequence there is a very marked

# TABLE I.

REAL CUT-OFF CORRESPONDING TO APPARENT CUT-OFF FOR DIFFERENT FRACTIONS OF CLEARANCE.

| Apparent Cut-off. | Clearance. | | | | | | | | | | Apparent Cut-off. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 | .10 | |
| .05 | .059 | .069 | .078 | .087 | .095 | .104 | .112 | .120 | .128 | .136 | .05 |
| .06 | .069 | .078 | .087 | .096 | .105 | .113 | .122 | .130 | .138 | .145 | .06 |
| .07 | .079 | .088 | .097 | .106 | .114 | .123 | .131 | .139 | .147 | .155 | .07 |
| .08 | .089 | .098 | .107 | .115 | .124 | .132 | .140 | .148 | .156 | .164 | .08 |
| .09 | .099 | .108 | .117 | .125 | .133 | .142 | .150 | .157 | .165 | .173 | .09 |
| .10 | .109 | .118 | .126 | .135 | .143 | .151 | .159 | .167 | .174 | .182 | .10 |
| .11 | .119 | .127 | .136 | .144 | .152 | .160 | .168 | .176 | .183 | .191 | .11 |
| .12 | .129 | .137 | .146 | .154 | .162 | .170 | .178 | .185 | .193 | .200 | .12 |
| .13 | .139 | .147 | .155 | .163 | .171 | .179 | .187 | .194 | .202 | .209 | .13 |
| .14 | .149 | .157 | .165 | .173 | .181 | .189 | .196 | .204 | .211 | .218 | .14 |
| .15 | .158 | .167 | .175 | .183 | .190 | .198 | .206 | .214 | .220 | .227 | .15 |
| .16 | .168 | .176 | .184 | .192 | .200 | .208 | .215 | .222 | .229 | .236 | .16 |
| .17 | .178 | .186 | .194 | .202 | .210 | .217 | .224 | .231 | .239 | .245 | .17 |
| .18 | .188 | .196 | .204 | .212 | .219 | .226 | .234 | .241 | .248 | .255 | .18 |
| .19 | .198 | .206 | .214 | .221 | .229 | .236 | .243 | .250 | .257 | .264 | .19 |
| .20 | .208 | .216 | .223 | .231 | .238 | .245 | .252 | .259 | .266 | .273 | .20 |
| .21 | .218 | .225 | .233 | .240 | .248 | .255 | .262 | .269 | .275 | .282 | .21 |
| .22 | .228 | .235 | .243 | .250 | .257 | .264 | .271 | .278 | .284 | .291 | .22 |
| .23 | .238 | .245 | .252 | .260 | .267 | .274 | .280 | .287 | .294 | .300 | .23 |
| .24 | .248 | .255 | .262 | .269 | .276 | .283 | .290 | .296 | .303 | .309 | .24 |
| .25 | .257 | .265 | .272 | .279 | .286 | .292 | .299 | .306 | .312 | .318 | .25 |
| .26 | .267 | .275 | .282 | .288 | .295 | .302 | .308 | .315 | .321 | .327 | .26 |
| .27 | .277 | .284 | .291 | .298 | .305 | .311 | .318 | .324 | .330 | .336 | .27 |
| .28 | .287 | .294 | .301 | .308 | .314 | .321 | .327 | .333 | .339 | .345 | .28 |
| .29 | .297 | .304 | .311 | .317 | .324 | .330 | .336 | .343 | .349 | .355 | .29 |
| .30 | .307 | .314 | .320 | .327 | .333 | .340 | .346 | .352 | .358 | .364 | .30 |
| .31 | .317 | .324 | .330 | .337 | .343 | .349 | .355 | .361 | .367 | .373 | .31 |
| .32 | .327 | .334 | .340 | .346 | .352 | .358 | .364 | .370 | .376 | .382 | .32 |
| .33 | .337 | .343 | .350 | .356 | .362 | .368 | .374 | .380 | .385 | .391 | .33 |
| .34 | .347 | .353 | .359 | .365 | .371 | .377 | .383 | .389 | .395 | .400 | .34 |
| .35 | .356 | .363 | .369 | .375 | .381 | .387 | .393 | .398 | .404 | .409 | .35 |
| .36 | .366 | .373 | .379 | .385 | .390 | .396 | .402 | .407 | .413 | .418 | .36 |
| .37 | .376 | .382 | .388 | .394 | .400 | .406 | .411 | .417 | .422 | .427 | .37 |
| .38 | .386 | .392 | .398 | .404 | .410 | .415 | .421 | .426 | .431 | .436 | .38 |
| .39 | .396 | .402 | .408 | .413 | .419 | .425 | .430 | .435 | .440 | .445 | .39 |
| .40 | .406 | .412 | .417 | .423 | .429 | .434 | .439 | .444 | .450 | .455 | .40 |

# TABLE I (Continued).

## Real Cut-off Corresponding to Apparent Cut-off for Different Fractions of Clearance.

| Apparent Cut-off. | Clearance. | | | | | | | | | | Apparent Cut-off. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 | .10 | |
| .41 | .416 | .422 | .427 | .433 | .438 | .443 | .449 | .454 | .459 | .463 | .41 |
| .42 | .426 | .431 | .437 | .442 | .448 | .453 | .458 | .463 | .468 | .473 | .42 |
| .43 | .436 | .441 | .447 | .452 | .457 | .462 | .467 | .472 | .477 | .482 | .43 |
| .44 | .446 | .451 | .456 | .462 | .467 | .472 | .477 | .481 | .486 | .491 | .44 |
| .45 | .455 | .461 | .466 | .471 | .476 | .481 | .486 | .491 | .495 | .500 | .45 |
| .46 | .465 | .471 | .475 | .481 | .486 | .491 | .495 | .500 | .505 | .509 | .46 |
| .47 | .475 | .480 | .485 | .490 | .495 | .500 | .505 | .509 | .514 | .518 | .47 |
| .48 | .485 | .490 | .495 | .500 | .505 | .509 | .514 | .519 | .523 | .527 | .48 |
| .49 | .495 | .500 | .505 | .510 | .514 | .519 | .523 | .528 | .532 | .536 | .49 |
| .50 | .505 | .510 | .515 | .519 | .524 | .528 | .533 | .537 | .541 | .545 | .50 |
| .51 | .515 | .520 | .524 | .529 | .533 | .538 | .542 | .546 | .550 | .554 | .51 |
| .52 | .525 | .529 | .534 | .538 | .543 | .544 | .551 | .556 | .560 | .564 | .52 |
| .53 | .535 | .539 | .544 | .548 | .552 | .557 | .561 | .565 | .569 | .573 | .53 |
| .54 | .545 | .549 | .553 | .558 | .562 | .566 | .570 | .574 | .578 | .582 | .54 |
| .55 | .554 | .559 | .563 | .567 | .571 | .575 | .579 | .583 | .587 | .591 | .55 |
| .56 | .564 | .569 | .573 | .577 | .581 | .585 | .589 | .593 | .596 | .600 | .56 |
| .57 | .574 | .578 | .583 | .587 | .590 | .594 | .598 | .602 | .606 | .609 | .57 |
| .58 | .584 | .588 | .592 | .596 | .600 | .604 | .607 | .611 | .615 | .618 | .58 |
| .59 | .594 | .598 | .602 | .606 | .610 | .613 | .617 | .620 | .624 | .627 | .59 |
| .60 | .604 | .608 | .612 | .615 | .619 | .623 | .626 | .630 | .633 | .636 | .60 |
| .61 | .614 | .618 | .621 | .625 | .629 | .632 | .636 | .639 | .642 | .645 | .61 |
| .62 | .624 | .627 | .631 | .635 | .638 | .642 | .645 | .648 | .651 | .655 | .62 |
| .63 | .634 | .637 | .641 | .644 | .648 | .651 | .654 | .657 | .661 | .664 | .63 |
| .64 | .644 | .647 | .650 | .654 | .657 | .660 | .664 | .667 | .670 | .673 | .64 |
| .65 | .653 | .657 | .660 | .663 | .667 | .669 | .673 | .676 | .679 | .682 | .65 |
| .66 | .663 | .667 | .670 | .673 | .676 | .679 | .682 | .685 | .688 | .691 | .66 |
| .67 | .673 | .676 | .680 | .683 | .686 | .689 | .692 | .694 | .697 | .700 | .67 |
| .68 | .683 | .685 | .689 | .692 | .695 | .698 | .701 | .704 | .706 | .709 | .68 |
| .69 | .693 | .695 | .699 | .702 | .705 | .708 | .710 | .713 | .716 | .718 | .69 |
| .70 | .703 | .706 | .709 | .712 | .714 | .717 | .720 | .722 | .725 | .727 | .70 |
| .71 | .713 | .716 | .718 | .721 | .724 | .726 | .728 | .731 | .734 | .736 | .71 |
| .72 | .723 | .725 | .728 | .731 | .733 | .736 | .738 | .741 | .743 | .745 | .72 |
| .73 | .733 | .735 | .738 | .740 | .743 | .745 | .748 | .750 | .752 | .755 | .73 |
| .74 | .743 | .745 | .748 | .750 | .752 | .755 | .756 | .759 | .761 | .764 | .74 |
| .75 | .752 | .755 | .757 | .760 | .762 | .764 | .766 | .769 | .771 | .773 | .75 |

loss arising from the condensation of the live steam as it enters the cylinder and comes in contact with the surfaces of the latter which have just been cooled by contact with, and giving up their heat to, the exhaust steam. The complexity of this interchange of heat is such that thus far it has not been satisfactorily followed by calculation, and we must depend upon experiment for our knowledge of what constitutes the most economical point of cut-off with respect to the consumption of steam or fuel. This cut-off may be determined by means of an empirical formula representing the result of extensive and carefully conducted experiments by Mr. Charles Emery, namely,

$$\epsilon = \frac{1}{r} = \frac{1}{1 + \frac{p_1}{22}} = \frac{22}{22 + p_1} \qquad (7)$$

$\epsilon$ = real cut-off. $r$ = ratio of expansion. $p_1$ = absolute initial pressure.

Mr. Emery considers that the cut-offs given by this formula are "nearly correct for single engines of ordinary construction, and too large for the better class of compound engine." The first two columns of Table IV on page 18 contain the initial absolute pressure $p_1$ for various values of $\epsilon$ = real cut-off when $pv^{\frac{10}{9}}$ = constant represents the curve of expansion. Thus far the only element of economy considered in choosing the point of cut-off was economy of steam or of the fuel which produced it. But this is only one of the elements which influences the cost per hour of the *H. P.* employed. Other elements such as wages of attendants, interest on cost of engine and boilers, repairs on engine and boilers, insurance, depreciation, etc., influence the current cost of the *H. P.* employed. That cut-off which makes this cost a minimum is evidently the true economical point of cut-off, the one in which the engine owner is financially interested. But as this cost, per horse-power per day, does not vary much for a considerable range of cut-off, we will here consider only the principal factor in that cost, *i. e.*, steam.

We may make a sufficient approximation to the correct cut-off by taking a value of the latter greater than that given by Mr. Emery's empirical formula for greatest economy of steam or fuel, for it is evident from general principles that the influence of the other expenses will be to make the proper cut-off greater than that corresponding to the greatest economy of steam or fuel. As our initial pressure was above assumed to be 85 pounds (by guage) we get from Emery's formula $\frac{1}{r} < \frac{1}{5}$. If we now assume $\frac{1}{r} = \frac{1}{4}$ we shall have, both as regards the pressure and the cut-off, conditions such as are assumed by many high-speed engine builders.

## I c.

### Determination of Back Pressure and Amount of Compression.

For well-designed non-condensing engines the back pressure during the exhaust is usually taken at 16 pounds (above a vacuum) per □″, and in condensing engines at 2 pounds per □″. As regards the amount of compression or cushioning suitable for high-speed engines it may be said that while on the one hand compressing up to boiler pressure has the advantage of obviating the necessity of partly filling the clearance spaces with fresh steam at the beginning of each stroke, and also tends to diminish the condensation of this steam by heating up the cylinder walls, on the other hand it has the disadvantage of producing a reversal of pressure before the end of each stroke, which tends to produce a shock on the crank pin. If we wish to relieve the crank pin of all pressure at the end of the stroke we have only to compress the exhaust steam till its final pressure is equal to the sum of the terminal pressure (near end of the expansion line) and the pressure retarding the reciprocating parts. As in high speed engines the terminal pressure is practically reduced to a very small amount through expansion and an early exhaust, we may

call it 16 pounds and then we have the rule to compress up to the pressure

$$16 + \frac{F_0}{A} = \frac{p_1 + 16}{2} \text{ approximately,} \qquad (8)$$

$\frac{F_0}{A}$ being the mean retarding (or accelerating) pressure of the reciprocating parts at the end of the stroke. Stopping the compression short of the steam chest pressure has the incidental advantage of a good admission line to the indicator card, for there exists then a difference of pressure which can hurry the steam from the boiler into the cylinder. Accepting the latter rule as the one for our guidance we can draw the compression curve according to the law $pv =$ constant. When compression is carried up to boiler pressure, and cylinder is steam jacketed, we should use the law $pv^{\frac{10}{9}} =$ constant.

## I D.

## Method of Constructing Equilateral Hyperbola.

The use of the Mariotte curve or equilateral hyperbola, is so common, that an exact method of drawing it may not be out of place here, although it is already known to engineers. The equation is $pv = p_1v_1$ where $p_1$ and $v_1$ are respectively the absolute pressure and volume of the gas at some point where they are known, and $p$ and $v$ those at any other point. From the center O, Fig. 1, which is the intersection of the clearance and

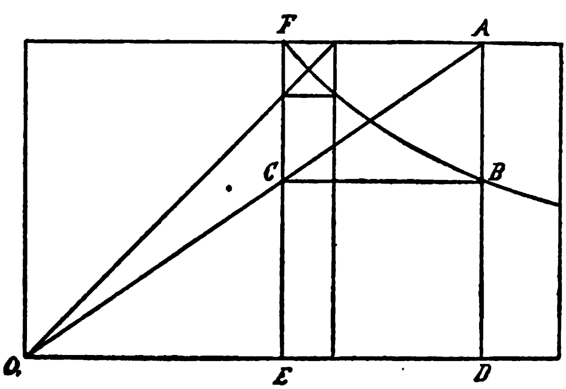



Fig. 1.

vacuum lines, whatever the degree of expansion employed, draw the line $OA$ and from the points $A$ and $C$, where it cuts the steam and cut-off lines let fall the perpendiculars $AB$ and $CB$. Their intersection at $B$ will give a point of the curve exactly. For from the similar triangles $AOD$ and $OCE$ we have $CE : OE :: AD : OD$ and consequently $BD : OE :: FE : OD$. But $OE$ and $FE$ represent respectively the volume and pressure at the point of cut-off $F$, and $OD$ and $BD$ those at the point $B$, hence $p : v_1 :: p_1 : v$ or $pv = p_1v_1$, therefore the point $B$ was correctly found. Other points may be found in the same way.

### Method of Determining Points, on the Curves $pv^n =$ Constant, by Tabulated Coordinates.

To find the coordinates for the curves by means of the following table we divide, in Fig. 2, the distance $HF = AC = v_1$ into

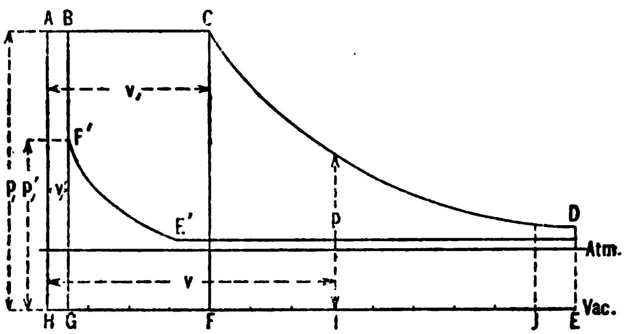



Fig. 2.

four equal parts and lay off one of these parts as many times as possible on $FE$. Then will for example $\frac{v}{v_1} = \frac{HI}{HF} = 1.75$, also $\frac{v^1}{v_1} = \frac{HJ}{FH} = 3$, etc. At the points of division on $FE$ erect ordinates and lay off on them the value of $\frac{p}{p_1} \times p_1$ corresponding to the

## TABLE II.

### VALUES OF THE RATIO $\frac{p}{p_1}$ OF THE ABSOLUTE PRESSURES OCCURRING IN FORMULA $p_1 v_1^n = pv^n =$ CONSTANT.

FOR $p$ IN LINEAR MEASURE MULTIPLY $\frac{p}{p_1}$ BY $p_1$ EXPRESSED IN LINEAR MEASURE.

| $\frac{v}{v_1}$ Ratio of assumed vol. $v$ to initial vol. $v_1$. | Exponent $n$ in formula $p_1 v_1^n = pv^n$. | | | | | | |
|---|---|---|---|---|---|---|---|
| | Equilateral hyperbola curve for air expanding with temperature constant. $n = 1$. | $n = 1.0646 = \frac{17}{16}$ nearly. Curve for steam, dry saturated during expansion. | $n = 1.125 = \frac{9}{8}$. Adiabatic curve for steam with 10% moisture. | $n = 1.135$. Adiabatic curve for steam, dry saturated at beginning of expansion. | $n = 1.250 = \frac{5}{4}$. Compression curve for high speed engine with steam jacketed cylinder. | $n = 1.333 = \frac{4}{3}$. Adiabatic curve for superheated steam. | $n = 1.408$. Adiabatic curve for air. |
| 1.125 | .8889 | .8822 | .8759 | .8748 | .8631 | .8547 | .8472 |
| 1.25 | .8000 | .7886 | .7780 | .7763 | .7569 | .7427 | .7304 |
| 1.50 | .6667 | .6494 | .6337 | .6312 | .6024 | .5824 | .5650 |
| 1.75 | .5714 | .5511 | .5328 | .5299 | .4968 | .4948 | .4548 |
| 2.00 | .5000 | .4781 | .4585 | .4550 | .4265 | .3969 | .3767 |
| 2.25 | .4444 | .4218 | .4016 | .3984 | .3629 | .3392 | .3192 |
| 2.50 | .4000 | .3771 | .3567 | .3535 | .3121 | .2947 | .2752 |
| 2.75 | .3636 | .3406 | .3204 | .3172 | .2824 | .2595 | .2407 |
| 3.00 | .3333 | .3105 | .2906 | .2874 | .2533 | .2311 | .2129 |
| 3.50 | .2857 | .2635 | .2443 | .2413 | .2089 | .1882 | .1714 |
| 4.00 | .2500 | .2286 | .2102 | .2073 | .1768 | .1575 | .1420 |
| 4.50 | .2222 | .2016 | .1841 | .1814 | .1526 | .1346 | .1203 |
| 5. | .2000 | .1803 | .1636 | .1548 | .1337 | .1170 | .1037 |
| 6. | .1657 | .1484 | .1332 | .1309 | .1065 | .0917 | .0802 |
| 7. | .1429 | .1260 | .1129 | .1099 | .0878 | .0747 | .0646 |
| 8. | .1250 | .1093 | .0964 | .0944 | .0743 | .0626 | .0535 |
| 9. | .1111 | .0964 | .0844 | .0826 | .0642 | .0534 | .0453 |
| 10. | .1000 | .0862 | .0750 | .0733 | .0562 | .0464 | .0391 |
| 11. | .0909 | .0779 | .0674 | .0658 | .0499 | .0409 | .0342 |
| 12. | .0833 | .0710 | .0611 | .0596 | .0450 | .0364 | .0302 |
| 13. | .0769 | .0653 | .0558 | .0544 | .0405 | .0327 | .0270 |
| 14. | .0714 | .0602 | .0514 | .0500 | .0369 | .0296 | .0243 |
| 15. | .0767 | .0562 | .0475 | .0463 | .0339 | .0270 | .0221 |
| 16. | .0625 | .0523 | .0442 | .0430 | .0313 | .0248 | .0206 |
| 18. | .0556 | .0461 | .0387 | .0376 | .0270 | .0212 | .0175 |
| 20. | .0500 | .0412 | .0344 | .0334 | .0236 | .0184 | .0147 |
| 22. | .0455 | .0372 | .0309 | .0300 | .0210 | .0162 | .0129 |
| 24. | .0417 | .0339 | .0280 | .0271 | .0188 | .0145 | .0114 |
| 26. | .0385 | .0312 | .0257 | .0248 | .0170 | .0130 | .0102 |
| 28. | .0357 | .0288 | .0236 | .0228 | .0155 | .0118 | .0092 |
| 30. | .0333 | .0268 | .0218 | .0211 | .0142 | .0107 | .0083 |

ratio of their abscissas $v$ to $v_1$. As stated in Table II, if we wish to obtain the value of the ordinate $p$ in linear measure we must multiply the ratios $\frac{p}{p_1}$ by $p_1$ expressed in linear measure. When the cut-off $\frac{AC}{HE}$ is very small we need not divide $HF$ into four equal parts, but can then simply lay off $HF$ as many times as possible on $FE$, thus getting a sufficient number of points to draw the curve $CD$ with accuracy. The compression curve $F^1E^1$, Fig. 2, can be drawn in a similar manner.

Construct two indicator diagrams, one for each end of the cylinder, the initial pressure being assumed equal to 85 pounds (by gauge), the real cut-off = ¼ and the clearance = 0.07. Leave space on sheet for four more indicator diagrams, two of which will represent the maximum, and two the minimum, power of the engine.

## I E.

## Determination of Mean Effective Pressure of Indicator and Other Diagrams.

The problem in this case is simply to find the average height of an irregular but closed figure. This may be done in four different ways. 1st, By John Coffin's modification of the planimeter (see catalogue of Ashcroft Manuf. Co). 2d, By dividing the enclosed area by equidistant ordinates and then calculating the area by Simpson's rule (see Nystrom, p. 114). 3d, By dividing the enclosed area by equidistant ordinates and changing the area included between each pair of ordinates into an equivalent rectangle and then measuring the height of the latter, the mean of all these heights will be that required. 4th, When as in indicator diagrams, the character of the bounding curves is known the average height may be calculated by means of the calculus and the results tabulated. The third method is most often used in "working up" indicator cards (diagrams), though the first is

also used when many cards are to be "worked up." Tabulated heights (or mean total pressures when clearance, back pressure and compression are neglected) are principally useful when laying out the indicator diagrams of proposed engines. The largest table of the kind with which the writer is acquainted is that prepared by Mr. Richard Buel for Du Bois' translation of Weisbach's *Mechanics*, Vol. II, (Heat Engines, etc.), pp. 479 and 482. The following Table III was taken from this work. The next Table, IV on page 18, was prepared by the writer and does not apply to all cases, but only to the case in which the cut-off is chosen with special reference to *economy of steam.* The body of the table contains the mean effective pressure $p_m$ of indicator diagrams for different amounts of clearance and real cut-offs $\epsilon$, or corresponding initial pressures $p_1$, prescribed by Mr. Emery's formula. The back pressure was assumed = 16 pounds and both expansion and compression curves were assumed to follow the law $pv^{\frac{17}{16}}$ = constant. Compression was supposed to be continued till the final pressure =

$$\frac{p_1 + 16}{2} = p_1 - \frac{F_0}{A} \tag{9}$$

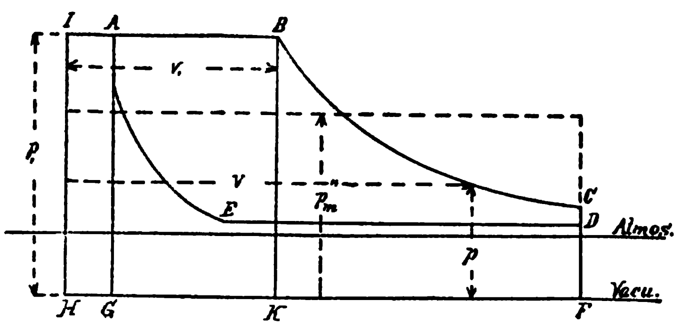



Fig. 3.

$\frac{IB}{HF}$ = real cut-off. $\frac{AB}{GF}$ = apparent cut-off. $IA$ = clearance.

$IH = BK$ = absolute initial pressure $p_1$. $p''_m = IBCFHI \div HF$.

# TABLE III.

## TABLE CONTAINING VALUES OF THE RATIO $\frac{p'_m}{p_1}$ FOR THE STEAM CURVES $pv$ AND $pv^{\frac{17}{16}}$.

| Real cut-off. | $pv$ = constant. Equilateral hyperbola. | $pv^{\frac{17}{16}}$ = constant. Saturation curve. | Real cut-off. | $pv$ = constant. Equilateral hyperbola. | $pv^{\frac{17}{16}}$ = constant. Saturation curve. | Real cut-off. | $pv$ = constant. Equilateral hyperbola. | $pv^{\frac{17}{16}}$ = constant. Saturation curve. | Real cut-off. | $pv$ = constant. Equilateral hyperbola. | $pv^{\frac{17}{16}}$ = constant. Saturation curve. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .01 | .0561 | .0500 | .26 | .6102 | .5959 | .51 | .8534 | .8463 | .76 | .9686 | .9668 |
| .02 | .0982 | .0884 | .27 | .6235 | .6594 | .52 | .8600 | .8532 | .77 | .9713 | .9696 |
| .03 | .1352 | .1245 | .28 | .6364 | .6226 | .53 | .8665 | .8599 | .78 | .9738 | .9723 |
| .04 | .1688 | .1566 | .29 | .6490 | .6355 | .54 | .8727 | .8664 | .79 | .9762 | .9749 |
| .05 | .1998 | .1866 | .30 | .6612 | .6479 | .55 | .8788 | .8727 | .80 | .9785 | .9773 |
| .06 | .2288 | .2148 | .31 | .6731 | .6601 | .56 | .8847 | .8789 | .81 | .9807 | .9796 |
| .07 | .2562 | .2415 | .32 | .6846 | .6719 | .57 | .8904 | .8848 | .82 | .9827 | .9817 |
| .08 | .2821 | .2669 | .33 | .6959 | .6835 | .58 | .8959 | .8906 | .83 | .9847 | .9838 |
| .09 | .3067 | .2912 | .34 | .7068 | .6947 | .59 | .9013 | .8962 | .84 | .9865 | .9857 |
| .10 | .3303 | .3145 | .35 | .7174 | .7056 | .60 | .9065 | .9017 | .85 | .9881 | .9874 |
| .11 | .3528 | .3368 | .36 | .7278 | .7163 | .61 | .9115 | .9069 | .86 | .9897 | .9891 |
| .12 | .3744 | .3583 | .37 | .7379 | .7267 | .62 | .9164 | .9120 | .87 | .9912 | .9906 |
| .13 | .3952 | .3790 | .38 | .7477 | .7368 | .63 | .9211 | .9169 | .88 | .9925 | .9920 |
| .14 | .4153 | .3990 | .39 | .7572 | .7466 | .64 | .9256 | .9217 | .89 | .9937 | .9933 |
| .15 | .4346 | .4183 | .40 | .7665 | .7562 | .65 | .9300 | .9263 | .90 | .9948 | .9945 |
| .16 | .4532 | .4370 | .41 | .7756 | .7656 | .66 | .9342 | .9307 | .91 | .9958 | .9956 |
| .17 | .4712 | .4552 | .42 | .7844 | .7747 | .67 | .9383 | .9350 | .92 | .9967 | .9965 |
| .18 | .4887 | .4727 | .43 | .7929 | .7835 | .68 | .9423 | .9391 | .93 | .9975 | .9973 |
| .19 | .5055 | .4897 | .44 | .8012 | .7921 | .69 | .9460 | .9431 | .94 | .9982 | .9981 |
| .20 | .5219 | .5062 | .45 | .8093 | .8005 | .70 | .9497 | .9469 | .95 | .9987 | .9987 |
| .21 | .5377 | .5223 | .46 | .8172 | .8087 | .71 | .9532 | .9506 | .96 | .9992 | .9991 |
| .22 | .5531 | .5378 | .47 | .8249 | .8166 | .72 | .9565 | .9541 | .97 | .9996 | .9995 |
| .23 | .5680 | .5530 | .48 | .8323 | .8244 | .73 | .9597 | .9575 | .98 | .9998 | .9998 |
| .24 | .5825 | .5677 | .49 | .8395 | .8319 | .74 | .9628 | .9607 | .99 | .9999 | .9999 |
| .25 | .5966 | .5820 | .50 | .8466 | .8392 | .75 | .9658 | .9638 | | | |

# TABLE IV.

## VALUES OF THE MEAN EFFECTIVE PRESSURES $p_m$ CORRESPONDING TO ECONOMICAL POINT OF CUT-OFF,

$$\varepsilon = \frac{22}{22 + p_1}.$$

COMPRESSION TAKES PLACE UP TO A PRESSURE $= \dfrac{p_1 + 16}{2} = p_1 - \dfrac{F_o}{A}.$

HERE ALL STEAM CURVES FOLLOW THE LAW $p_1 v_1^{\frac{17}{16}}$ = CONSTANT.

*See reference to this Table on page 16.*

| Real Cut-off $\varepsilon = \frac{1}{r}$. | Absolute pressures $p_r$ | Clearance. | | | | |
|---|---|---|---|---|---|---|
| | | .01 | .03 | .05 | .07 | .09 |
| .10 | 198. | 44.27 | 34.31 | 26.43 | 18.53 | 10.65 |
| .11 | 178. | 40.62 | 33.94 | 27.27 | 20.61 | 13.96 |
| .12 | 161.3 | 38.93 | 33.18 | 26.56 | 21.67 | 15.99 |
| .13 | 147.2 | 37.3 | 32.16 | 27.37 | 22.39 | 17.4 |
| .14 | 135.1 | 35.8 | 31.4 | 27.1 | 22.7 | 18.3 |
| .15 | 124.7 | 34.3 | 30.4 | 26.6 | 22.8 | 19.0 |
| .16 | 115.5 | 32.8 | 29.4 | 26.1 | 22.7 | 19.4 |
| .17 | 107.4 | 31.4 | 28.4 | 25.5 | 22.5 | 19.5 |
| .18 | 100.2 | 30.2 | 27.4 | 24.8 | 22.1 | 19.5 |
| .19 | 93.8 | 28.8 | 26.4 | 24.0 | 21.7 | 19.3 |
| .20 | 88.0 | 27.5 | 25.4 | 23.3 | 21.2 | 19.1 |
| .21 | 82.8 | 26.3 | 24.4 | 22.5 | 20.6 | 18.8 |
| .22 | 78.0 | 25.1 | 23.1 | 21.7 | 20.0 | 18.4 |
| .23 | 73.7 | 24.0 | 22.5 | 20.9 | 19.4 | 17.9 |
| .24 | 69.7 | 22.9 | 21.5 | 20.0 | 18.8 | 17.6 |
| .25 | 66.0 | 21.8 | 20.6 | 19.3 | 18.1 | 16.9 |
| .26 | 62.6 | 20.8 | 19.7 | 18.5 | 17.4 | 16.3 |
| .27 | 59.5 | 19.8 | 18.7 | 17.7 | 16.7 | 15.7 |
| .28 | 56.6 | 18.8 | 17.9 | 17.0 | 16.1 | 15.1 |
| .29 | 53.9 | 17.8 | 17.0 | 16.2 | 15.4 | 14.6 |
| .30 | 51.3 | 16.9 | 16.1 | 15.4 | 15.2 | 13.9 |

I F.

To determine the cut-off corresponding to a given horse-power $H_1$, when the mean effective pressure $p_m$ and the mean total pressure $p''_m$ (see Table III and Fig. 3) is known for another horse-power $H$ of the same engine running at the same speed, with the same initial pressure $p_1$ and *the same back pressure and compression.*

In engines with cut-off under the control of the governor, the speed (*i.e. R.p.m.*) remains constant. Hence we have

$$p_m : p_{m1} :: H : H_1 \text{ and } p_{m1} = \frac{H_1}{H} p_m. \qquad (10)$$

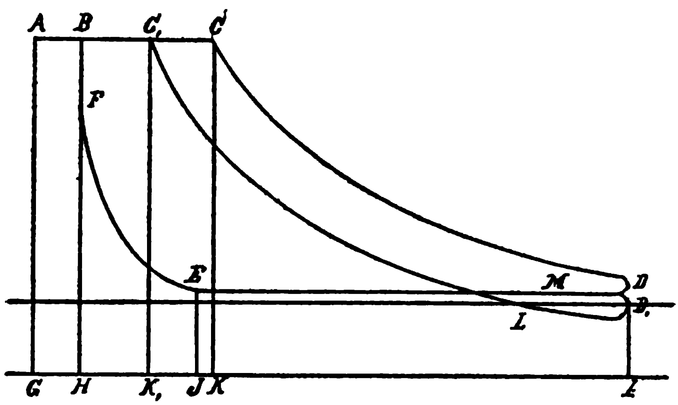



Fig. 4.

By means of Fig. 4, if we suppose the line of counter-pressures $DEF$ to remain the same for all variations of power, we can deduce the following relations:

$$\frac{\text{area } BCDEFB}{\text{stroke } HI} = p_m \qquad \frac{\text{area } BC_1LD_1MEFB}{\text{stroke } HI} = p_{m1} \qquad (11)$$

$$\frac{\text{area } ACDIGA}{\text{length } GI} = p''_m \qquad \frac{\text{area } AC_1LD_1IGA}{\text{length } GI} = p''_{m1} \qquad (12)$$

In the figure, $\frac{AB}{HI}$ = clearance $i$ and $AG$ = initial pressure $p_1$.

Table III on page 17 contains the values of $\frac{p''_m}{p_1}$ for various

real cut-offs and for the two steam curves $pv =$ constant and $p_1 v_1 \text{H} =$ constant.

$$p_4 = \frac{\text{area } ABFEMDIGA}{GI} \text{ or}$$

$$p_4 = \frac{\text{area } ACDIGA - \text{area } BCDMEFB}{GH + HI} = p''_m - \frac{p_m}{1 + i} \quad (13)$$

$$p''_{m1} \times GI = p_{m1} \times HI + p_4 \times GI$$

$$p''_{m1} = \frac{p_{m1}}{1+i} + p''_m - \frac{p_m}{1 + i}.$$

hence
$$\frac{p''_{m1}}{p_1} = \frac{p_{m1} - p_m}{(1 + i)p_1} + \frac{p''_m}{p_1} = \frac{p''_m}{p_1} + \left( \frac{\frac{H_1}{H} - 1}{1 + i} \right) \frac{p_m}{p_1}. \quad (14)$$

In a like manner it may be shown that

$$\frac{p''_{m2}}{p_1} = \frac{p''_m}{p_1} + \left( \frac{\frac{H_2}{H} - 1}{1 + i} \right) \frac{p_m}{p_1}$$

where $p''_{m2}$ is the corresponding mean total pressure for the horse-power $H_2$.

The quantities in the second members of these equations are either given directly or may be found from the diagrams already constructed or from the tables. This enables us to calculate $\frac{p''_{m1}}{p_1}$ and $\frac{p''_{m2}}{p_1}$ and from these, by means of Table III on page 17, we can get the corresponding cut-offs $\frac{1}{r_1}$ and $\frac{1}{r_2}$.

Calculate the cut-offs corresponding to 50 and 170 *I.H.P.*, the extremes of power for the proposed engine. Then draw two indicator diagrams for each of these powers.

## I G.

## DIAGRAMS OF EFFECTIVE STEAM PRESSURES.

Draw diagram of effective steam pressures on piston both for the forward and return strokes. We will here state what is to be understood by the forward and the return stroke. In all cases suppose the engine to lie horizontally, with the crank on the right hand and the cylinder on the left. The forward stroke will then be when the piston moves to the right or when the crank moves in its upper or lower semi-circle so that it first passes through the quadrant nearest the cylinder and then through the quadrant farthest from the cylinder. This does not agree with locomotive practice where the forward end of the cylinder is its front end, and the stroke of the piston toward that end the forward stroke, but it agrees with the practice of many engineers when treating of stationary engines. To avoid all ambiguity as

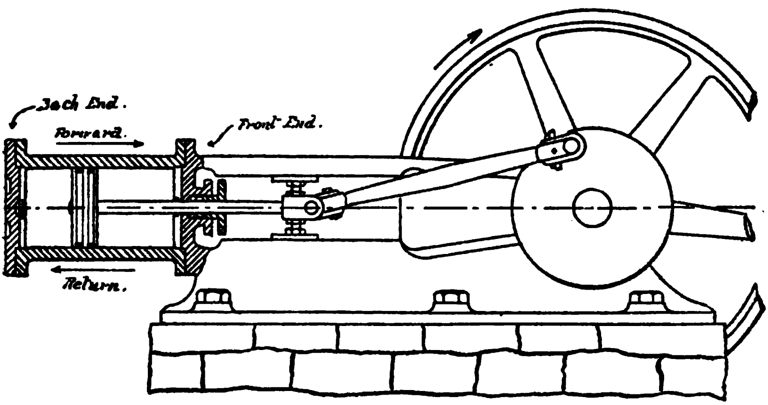



Fig. 5.

to which end of the cylinder is meant, it would be well when figures are not given to speak of the end in question as nearest to, or farthest from, the crank. In like manner we may speak of the stroke *toward* or *away from* the crank shaft, which will correspond respectively to forward and return strokes as defined above.

Fig. 6 and Fig. 7 represent respectively the indicator diagrams from back and front ends of the cylinder shown in Fig. 5. If we wish to ascertain only the power of an engine these indicator diagrams will be sufficient, for from them we can obtain in the usual manner, the power developed by the steam in each end of the cylinder, but if we wish to ascertain the real pressure

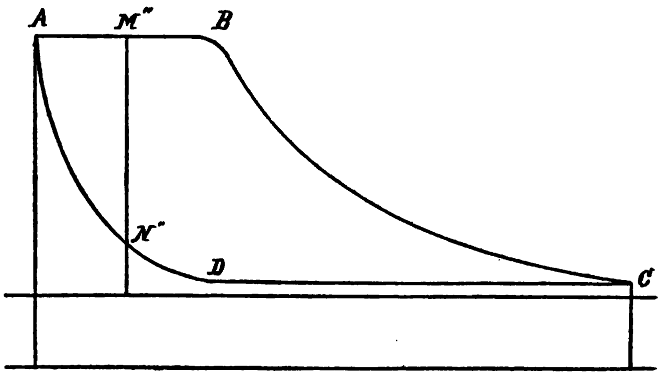



Fig. 6.

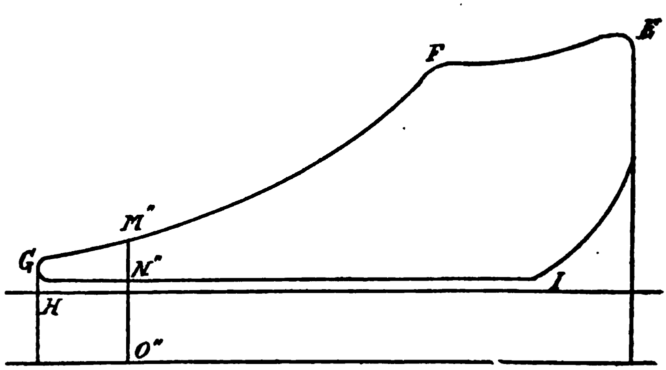



Fig. 7.

exerted by the steam upon the piston at any position of the latter we must combine both diagrams as follows.

The admission and expansion line of one end of the cylinder being placed upon the exhaust and compression line of the other end, Figs. 8 and 9, the portions of the vertical ordinates included

by the shaded portion of these figures will represent the difference between the absolute pressures of the steam on opposite sides of the piston and consequently will represent the effective pressure or driving effort exerted by the steam upon the piston for the various positions of the latter. For instance, for the piston position in Fig. 5, on the forward stroke, the effective or driving

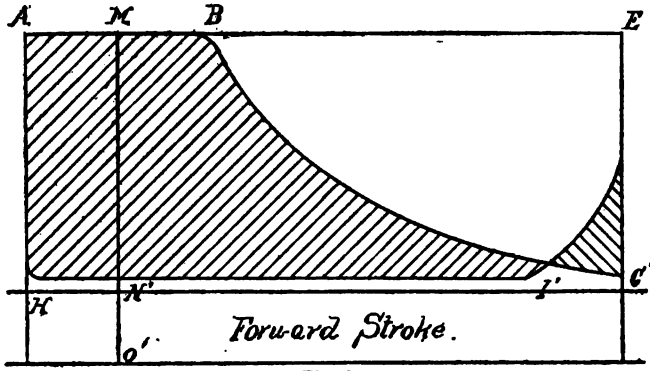



Fig. 8.

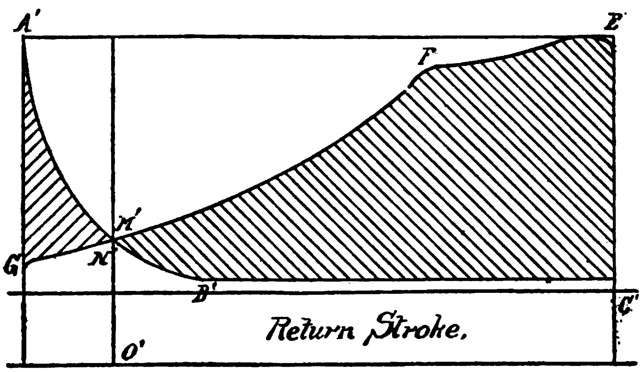



Fig. 9.

pressure on the piston will be $M^1N^1 = M^1O^1 - N^1O^1$, Fig. 8, $M^1O^1$ being the absolute pressure of steam in the back end of cylinder, and $N^1O^1$ the absolute pressure in the front end. The intercept $M^{11}N^{11}$, Fig. 6 or 7, corresponding to the same piston position is considerably different from $M^1N^1$, Fig. 8 or 9, and does

not represent the difference of two pressures existing simultaneously, in opposite ends of the cylinder, but represents the difference between two pressures separated by an interval of time corresponding to about half a revolution of the crank. The effective driving pressure on the piston for the position in Fig. 5, on the return stroke is equal to zero, as is shown at $M^1N^1$, Fig. 9, the pressure on opposite sides of piston being equal at that moment. The point where the lines cross represents, therefore, equilibrium between the opposing pressures; after this point is passed the resistances are in excess, and if it were not for the inertia of the reciprocating parts and fly-wheel, the motion of the piston would be reversed before it had completed its stroke. This change in the direction of the excess of pressure is represented in the diagram by a difference in shading. This reversal of pressures serves a most useful purpose in absorbing the inertia of the reciprocating parts, bringing them gradually to rest and thus preventing shocks and vibrations at the end of the stroke. Draw in accordance with the above, three pairs of diagrams of effective steam pressures on piston, corresponding to 50, 110 and 170 *I.H.P.*, the minimum, normal and maximum horse-powers of this engine.

## II.

## Determination of Ratio of Length of Connecting Rod to Length of Crank.

When the connecting rod is infinite in length (which is the case in the mechanism known as the normal double slider crank chain, Reuleaux's *Kinematics*, p. 314) the speed of the piston is the same for the crank positions $\omega$ and $180° - \omega$; the distance from the nearest end of the stroke is also the same for these two crank positions, for the piston will travel through the first and second halves of its stroke while the crank pin is sweeping through the first and second quadrants of its motion. The horizontal motion of the piston along the axis of the cylinder corres-

ponds exactly to the horizontal motion of the crank pin at every instant. Moreover, the pressure on slides at right angles to piston motion $= 0$, the obliquity of the connecting rod being 0.

But when the connecting rod is of finite length, the piston speed and distances from nearest end of stroke are different for the crank angles $\omega$ and $180° - \omega$, the paths traversed by piston while the crank is swinging through the first and second quadrants are unequal, and the pressure on the slide and consequently the friction on the slide is proportional to the ratio of the length of crank to connecting rod. In the following figure

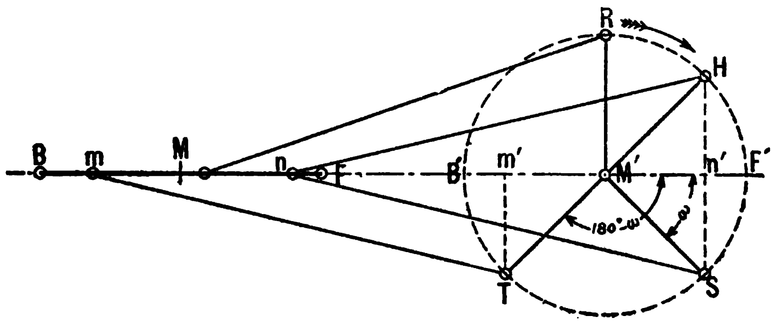



Fig. 10.

we have taken $\dfrac{\text{connecting rod}}{\text{crank}} = 2\frac{1}{2}$ to show the effect of a very short rod on piston speeds and positions.

If we supposse the motion of the crank pin uniform and equal to $v$ we shall have the piston speed $w = v \sin \omega$ for each of the crank positions $S$ and $T$ when the connecting rod is infinite in length, but when it is finite and equal to $2\frac{1}{2}$ times the crank, as in the figure, the velocity of piston for crank positions $S$ and $T$ will be respectively (see Fig. 10 for $\omega$)

$$w = v \sin \omega \left(1 + \frac{\cos \omega}{2.5}\right) \text{ for point } T$$

$$\text{and } w = v \sin \omega \left(1 - \frac{\cos \omega}{2.5}\right) \text{ for point } S. \qquad (16)$$

# TABLE V.

## DISTANCE OF PISTON FROM BEGINNING OF STROKE WHEN STROKE IS *towards* CRANK-SHAFT.

*To find Actual Distance of Piston from end of Stroke, multiply tabular quantity by Stroke.*

| Crank Angles. | Connecting-rod ÷ Crank = | | | | | | | | Crank Angles. | Connecting-rod ÷ Crank = | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 4.5 | 5 | 5.5 | 6 | 7 | 8 | ∞ | | 4 | 4.5 | 5 | 5.5 | 6 | 7 | 8 | ∞ |
| 2 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 | .0003 | .0003 | 92 | .5809 | .5737 | .5679 | .5632 | .5594 | .5533 | .5487 | .5174 |
| 4 | .0015 | .0015 | .0015 | .0014 | .0014 | .0014 | .0014 | .0012 | 94 | .5981 | .5909 | .5851 | .5805 | .5766 | .6706 | .5661 | .5349 |
| 6 | .0034 | .0033 | .0033 | .0032 | .0032 | .0031 | .0030 | .0027 | 96 | .6151 | .6079 | .6022 | .5977 | .5938 | .5878 | .5833 | .5523 |
| 8 | .0061 | .0060 | .0058 | .0057 | .0057 | .0056 | .0055 | .0049 | 98 | .6318 | .6247 | .6191 | .6145 | .6107 | .6048 | .6004 | .5696 |
| 10 | .0095 | .0092 | .0091 | .0089 | .0089 | .0087 | .0085 | .0076 | 100 | .6484 | .6414 | .6358 | .6313 | .6275 | .6216 | .6172 | .5868 |
| 12 | .0136 | .0133 | .0131 | .0129 | .0127 | .0126 | .0124 | .0109 | 102 | .6647 | .6578 | .6523 | .6478 | .6441 | .6384 | .6340 | .6040 |
| 14 | .0185 | .0181 | .0178 | .0175 | .0173 | .0170 | .0167 | ·0149 | 104 | .6807 | .6739 | .6685 | .6641 | .6605 | .6548 | .6505 | .6210 |
| 16 | .0241 | .0236 | .0232 | .0228 | .0225 | .0221 | .0218 | .0194 | 106 | .6964 | .6897 | .6845 | .6802 | .6766 | .6710 | .6668 | .6378 |
| 18 | .0304 | .0298 | .0293 | .0290 | .0285 | .0279 | .0275 | .0245 | 108 | .7119 | .7053 | .7002 | .6959 | .6924 | .6870 | .6829 | .6545 |
| 20 | .0375 | .0367 | .0360 | .0355 | .0350 | .0344 | .0339 | .0302 | 110 | .7270 | .7206 | .7115 | .7115 | .7080 | .7027 | .6987 | .6710 |
| 22 | .0452 | .0442 | .0434 | .0428 | .0423 | .0414 | .0408 | .0364 | 112 | .7418 | .7356 | .7307 | .7267 | .7233 | .7181 | .7143 | .6873 |
| 24 | .0536 | .0524 | .0515 | .0507 | .0501 | .0491 | .0484 | .0432 | 114 | .7562 | .7502 | .7455 | .7416 | .7383 | .7334 | .7296 | .7034 |
| 26 | .0627 | .0613 | .0602 | .0594 | .0586 | .0575 | .0566 | .0506 | 116 | .7703 | .7645 | .7599 | .7561 | .7530 | .7482 | .7445 | .7192 |
| 28 | .0724 | .0708 | .0696 | .0686 | .0677 | .0664 | .0654 | .0585 | 118 | .7841 | .7785 | .7740 | .7704 | .7674 | .7626 | .7591 | .7347 |
| 30 | .0827 | .0809 | .0796 | .0784 | .0774 | .0759 | .0748 | .0670 | 120 | .7974 | .7921 | .7878 | .7843 | .7814 | .7769 | .7735 | .7500 |
| 32 | .0936 | .0916 | .0901 | .0888 | .0877 | .0860 | .0848 | .0760 | 122 | .8104 | .8053 | .8012 | .7979 | .7951 | .7908 | .7875 | .7650 |
| 34 | .1051 | .1029 | .1012 | .0997 | .0985 | .0967 | .0953 | .0855 | 124 | .8230 | .8181 | .8142 | .8105 | .8084 | .8042 | .8011 | .7796 |
| 36 | .1172 | .1147 | .1128 | .1112 | .1099 | .1079 | .1063 | .0955 | 126 | .8352 | .8305 | .8268 | .8238 | .8213 | .8173 | .8144 | .7939 |
| 38 | .1298 | .1271 | .1250 | .1233 | .1218 | .1196 | .1179 | .1060 | 128 | .8470 | .8426 | .8361 | .8362 | .8338 | .8300 | .8273 | .8078 |
| 40 | .1430 | .1400 | .1317 | .1358 | .1342 | .1318 | .1297 | .1170 | 130 | .8584 | .8542 | .8509 | .8452 | .8459 | .8424 | .8398 | .8214 |
| 42 | .1566 | .1534 | .1509 | .1489 | .1471 | .1444 | .1424 | .1284 | 132 | .8694 | .8655 | .8623 | .8598 | .8577 | .8544 | .8519 | .8346 |
| 44 | .1707 | .1673 | .1646 | .1623 | .1605 | .1576 | .1557 | .1403 | 134 | .8799 | .8763 | .8733 | .8710 | .8690 | .8658 | .8635 | .8473 |
| 46 | .1853 | .1816 | .1787 | .1763 | .1743 | .1712 | .1689 | .1527 | 136 | .8901 | .8866 | .8839 | .8817 | .8798 | .8770 | .8751 | .8597 |
| 48 | .2003 | .1963 | .1922 | .1906 | .1885 | .1852 | .1827 | .1654 | 138 | .8998 | .8966 | .8941 | .8920 | .8903 | .8876 | .8856 | .8716 |
| 50 | .2156 | .2115 | .2081 | .2054 | .2032 | .1996 | .2970 | .1786 | 140 | .9090 | .9061 | .9038 | 9019 | .9003 | .8978 | .8959 | .8830 |
| 52 | .2314 | .2270 | .2234 | .2206 | .2182 | .2144 | .2117 | .1922 | 142 | .9178 | .9152 | .9130 | .9113 | .9098 | .9076 | .9059 | .8940 |
| 54 | .2474 | .2428 | .2391 | .2360 | .2335 | .2293 | .2266 | .2061 | 144 | .9262 | .9238 | .9218 | .9203 | .9189 | .9169 | .9153 | .9045 |
| 56 | .2638 | .2589 | .2550 | .2518 | .2492 | .2450 | .2419 | .2204 | 146 | .9342 | .9320 | .9302 | .9288 | .9276 | .9257 | .9243 | .9145 |
| 58 | .2805 | .2754 | .2713 | .2679 | .2652 | .2608 | .2575 | .2350 | 148 | .9417 | .9397 | .9381 | .9368 | .9358 | .9340 | .9328 | .9240 |
| 60 | .2974 | .2920 | .2878 | .2843 | .2814 | .2769 | .2735 | .2500 | 150 | .9487 | .9469 | .9455 | .9444 | .9435 | .9419 | .9408 | .9330 |
| 62 | .3146 | .3090 | .3046 | .3009 | .2979 | .2932 | .2897 | .2653 | 152 | .9553 | .9538 | .9825 | .9515 | .9507 | .9494 | .9484 | .9415 |
| 64 | .3320 | .3262 | .3215 | .3178 | .3147 | .3098 | .3061 | .2808 | 154 | .9617 | .9601 | .9590 | .9581 | .9574 | .9563 | .9554 | 9494 |
| 66 | .3495 | .3435 | .3387 | .3348 | .3316 | .3266 | .3228 | .2966 | 156 | .9671 | .9660 | .9651 | .9643 | .9637 | .9627 | .9620 | .9568 |
| 68 | .3672 | .3610 | .3561 | .3520 | .3487 | .3435 | .3397 | .3127 | 158 | .9724 | .9713 | .9706 | .9700 | .9694 | .9686 | .9678 | .9634 |
| 70 | .3850 | .3786 | .3735 | .3694 | .3660 | .3607 | .3567 | .3290 | 160 | .9772 | .9763 | .9757 | .9751 | .9747 | .9740 | .9735 | .9698 |
| 72 | .4028 | .3963 | .3911 | .3869 | .3834 | .3780 | .3739 | .3455 | 162 | .9815 | .9808 | .9803 | .9801 | .9795 | .9789 | .9785 | .9755 |
| 74 | .4208 | .4141 | .4088 | .4045 | .4009 | .3954 | .3912 | .3622 | 164 | .9854 | .9848 | .9844 | .9841 | .9838 | .9833 | .9830 | .9806 |
| 76 | .4388 | .4319 | .4266 | .4222 | .4185 | .4128 | .4085 | .3790 | 166 | .9888 | .9884 | .9881 | .9878 | .9878 | .9872 | .9869 | .9851 |
| 78 | .4568 | .4498 | .4444 | .4399 | .4362 | .4304 | .4260 | .3960 | 168 | .9918 | .9913 | .9912 | .9910 | .9909 | .9907 | .9905 | .9891 |
| 80 | .4747 | .4677 | .4622 | .4576 | .4539 | .4480 | .4436 | .4132 | 170 | .9943 | .9941 | .9939 | .9938 | .9937 | .9935 | .9933 | .9924 |
| 82 | .4927 | .4857 | .4799 | .4754 | .4716 | .4656 | .4612 | .4304 | 172 | .9964 | .9962 | .9961 | .9960 | .9959 | .9958 | .9957 | .9951 |
| 84 | .5105 | .5034 | .4977 | .4930 | .4892 | .4832 | .4787 | .4477 | 174 | .9979 | .9979 | .9978 | .9978 | .9977 | .9977 | .9976 | .9973 |
| 86 | .5283 | .5211 | .5154 | .5107 | .5069 | .5008 | .4963 | .4651 | 176 | .9990 | .9990 | .9990 | .9990 | .9990 | .9990 | .9990 | .9988 |
| 88 | .5460 | .5388 | .5330 | .5283 | .5245 | .5185 | .5139 | .4826 | 178 | .9998 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 |
| 90 | .5635 | .5563 | .5505 | .5453 | .5420 | .5359 | .5314 | .5000 | 180 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

# TABLE VI.

## DISTANCE OF PISTON FROM BEGINNING OF STROKE WHEN STROKE IS *away from* CRANK-SHAFT.

*To find Actual Distance of Piston from end of Stroke, multiply tabular quantity by Stroke.*

| Crank Angles. | Connecting-rod ÷ Crank = | | | | | | | | Crank Angles. | Connecting-rod ÷ Crank = | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 4 | 4.5 | 5 | 5.5 | 6 | 7 | 8 | ∞ | | 4 | 4.5 | 5 | 5.5 | 6 | 7 | 8 | ∞ |
| 2 | .0002 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | 92 | .4540 | .4612 | .4670 | .4717 | .4755 | .4715 | .4861 | .5174 |
| 4 | .0010 | .0010 | .0010 | .0010 | .0010 | .0010 | .0010 | .0012 | 94 | .4717 | .4789 | .4846 | .4893 | .4931 | .4992 | .5037 | .5349 |
| 6 | .0021 | .0021 | .0022 | .0022 | .0023 | .0023 | .0024 | .0027 | 96 | .4895 | .4966 | .5023 | .5070 | .5108 | .5168 | .5213 | .5523 |
| 8 | .0036 | .0038 | .0039 | .0040 | .0041 | .0042 | .0043 | .0049 | 98 | .5073 | .5143 | .5201 | .5246 | .5284 | .5344 | .5388 | .5696 |
| 10 | .0057 | .0059 | .0061 | .0062 | .0063 | .0065 | .0067 | .0076 | 100 | .5253 | .5323 | .5378 | .5424 | .5461 | .5520 | .5564 | .5868 |
| 12 | .0082 | .0087 | .0088 | .0090 | .0091 | .0093 | .0095 | .0109 | 102 | .5432 | .5502 | .5556 | .5601 | .5638 | .5696 | .5740 | .6140 |
| 14 | .0112 | .0116 | .0119 | .0122 | .0122 | .0128 | .0131 | .0149 | 104 | .5612 | .5681 | .5734 | .5778 | .5815 | .5872 | .5915 | .6210 |
| 16 | .0146 | .0152 | .0156 | .0159 | .0162 | .0167 | .0170 | .0194 | 106 | .5792 | .5859 | .5912 | .5955 | .5991 | .6046 | .6088 | .6378 |
| 18 | .0185 | .0192 | .0197 | .0199 | .0205 | .0211 | .0215 | .0245 | 108 | .5972 | .6037 | .6089 | .6131 | .6166 | .6220 | .6261 | .6545 |
| 20 | .0228 | .0237 | .0243 | .0248 | .0253 | .0260 | .0265 | .0302 | 110 | .6150 | .6214 | .6265 | .6306 | .6340 | .6393 | .6533 | .6710 |
| 22 | .0276 | .0286 | .0294 | .0300 | .0306 | .0314 | .0322 | .0366 | 112 | .6328 | .6390 | .6439 | .6480 | .6513 | .6565 | .6603 | .6873 |
| 24 | .0329 | .0340 | .0349 | .0367 | .0363 | .0373 | .0380 | .0432 | 114 | .6505 | .6565 | .6613 | .6652 | .6684 | .6734 | .6772 | .7034 |
| 26 | .0385 | .0399 | .0410 | .0419 | .0426 | .0437 | .0446 | .0506 | 116 | .6680 | .6738 | .6785 | .6822 | .6853 | .6902 | .6939 | .7192 |
| 28 | .0447 | .0462 | .0475 | .0485 | .0493 | .0506 | .0516 | .0585 | 118 | .6844 | .6910 | .6954 | .6991 | .7021 | .7068 | .7103 | .7347 |
| 30 | .0513 | .0531 | .0545 | .0556 | .0565 | .0581 | .0592 | .0670 | 120 | .7026 | .7060 | .7122 | .7157 | .7186 | .7231 | .7265 | .7500 |
| 32 | .0583 | .0603 | .0619 | .0632 | .0642 | .0660 | .0671 | .0760 | 122 | .7195 | .7245 | .7287 | .7321 | .7348 | .7392 | .7425 | .7650 |
| 34 | .0658 | .0680 | .0698 | .0712 | .0724 | .0748 | .0757 | .0855 | 124 | .7362 | .7411 | .7450 | .7482 | .7508 | .7550 | .7581 | .7796 |
| 36 | .0738 | .0762 | .0782 | .0797 | .0811 | .0831 | .0847 | .0955 | 126 | .7526 | .7572 | .7609 | .7640 | .7665 | .7705 | .7734 | .7939 |
| 38 | .0822 | .0848 | .0870 | .0887 | .0902 | .0924 | .0941 | .1060 | 128 | .7686 | ,7730 | .7766 | .7794 | .7818 | .7856 | .7883 | .8078 |
| 40 | .0910 | .0939 | .0962 | .0981 | .0997 | .1022 | .1041 | .1170 | 130 | .7844 | .7885 | .7919 | .7919 | .7969 | .8004 | .8030 | .8214 |
| 42 | .1002 | .1034 | .1059 | .1060 | .1097 | .1124 | .1144 | .1284 | 132 | .7997 | .8037 | .8068 | .8094 | .8115 | .8148 | .8173 | .8346 |
| 44 | .1099 | .1134 | .1161 | .1183 | .1202 | .1230 | .1249 | .1403 | 134 | .8147 | .8184 | .8213 | .8237 | .8257 | .8288 | .8311 | .8473 |
| 46 | .1201 | .1237 | .1267 | .1290 | .1310 | .1342 | .1365 | .1527 | 136 | .8293 | .8327 | .8354 | .8377 | .8395 | .8424 | .8443 | .8597 |
| 48 | .1306 | .1348 | .1377 | .1402 | .1423 | .1456 | .1481 | .1654 | 138 | .8434 | .8469 | .8491 | .8511 | .8529 | .8556 | .8576 | .8716 |
| 50 | .1416 | .1458 | .1491 | .1518 | .1541 | .1576 | .1607 | .1786 | 140 | .8570 | .8600 | .8623 | .8642 | .8658 | .8682 | .8703 | .8830 |
| 52 | .1530 | .1574 | .1609 | .1638 | .1662 | .1700 | .1727 | .1922 | 142 | .8702 | .8729 | .8750 | .8767 | .8782 | .8804 | .8821 | .8940 |
| 54 | .1648 | .1695 | .1732 | .1762 | .1787 | .1827 | .1856 | .2061 | 144 | .8828 | .8853 | .8882 | .8888 | 8901 | .8921 | .8937 | .9045 |
| 56 | .1769 | .1819 | .1858 | .1804 | .1916 | .1958 | .1989 | .2204 | 146 | .8949 | .8971 | .8988 | .9003 | .9015 | .9033 | .9047 | .9145 |
| 58 | .1896 | .1947 | .1988 | .2021 | .2049 | .2092 | .2125 | .2350 | 148 | .9064 | .9084 | .9099 | .9112 | .9123 | .9140 | .9152 | .9240 |
| 60 | .2026 | .2079 | .2122 | .2157 | .2186 | .2231 | .2295 | .2500 | 150 | .9173 | .9191 | .9204 | .9216 | .9226 | .9241 | .9252 | .9330 |
| 62 | .2159 | .2215 | .2260 | .2296 | .2326 | .2324 | .2409 | .2653 | 152 | .9276 | .9292 | .9304 | .9314 | .9323 | .9336 | .9346 | .9415 |
| 64 | .2297 | .2355 | .2401 | .2439 | .2470 | .2518 | .2555 | .2808 | 154 | .9373 | .9387 | .9398 | .9406 | .9414 | .9425 | .9434 | .9494 |
| 66 | .2438 | .2498 | .2545 | .2584 | .2617 | .2666 | .1704 | .2966 | 156 | .9464 | .9476 | .9485 | .9493 | .9499 | .9509 | .9516 | .9568 |
| 68 | .2582 | .2644 | .2693 | .2733 | .2767 | .2819 | .2857 | .3127 | 158 | .9548 | .9558 | .9566 | .9572 | .9577 | .9586 | .9592 | .9636 |
| 70 | .2730 | .2794 | .2844 | .2885 | .2929 | .2973 | .3013 | .3290 | 160 | .9625 | .9633 | .9640 | .9645 | .9650 | .9656 | .9661 | .9698 |
| 72 | .2881 | .2947 | .2998 | .3041 | .3076 | .3130 | .3171 | .3455 | 162 | .9696 | .9752 | .9707 | .9710 | .9715 | .9721 | .9725 | .9755 |
| 74 | .3036 | .3103 | .3155 | .3198 | .3234 | .3290 | .3332 | .3622 | 164 | .9759 | .9764 | .9767 | .9772 | .9775 | .9779 | .9782 | .9806 |
| 76 | .3193 | .3261 | .3315 | .3359 | .3395 | .3452 | .3495 | .3790 | 166 | .9815 | .9819 | .9822 | .9825 | .9827 | .9830 | .9883 | .9851 |
| 78 | .3353 | .3422 | .3477 | .3522 | .3559 | .3616 | .3660 | .3960 | 168 | .9864 | .9867 | .9869 | .9871 | .9873 | .9874 | .9876 | .9861 |
| 80 | .3516 | .3586 | .3642 | .3687 | .3725 | .3784 | .3828 | .4132 | 170 | .9905 | .9908 | .9909 | .9911 | .9912 | .9913 | .9915 | .9924 |
| 82 | .3682 | .3753 | .3809 | .3855 | .3893 | .3952 | .3996 | .4304 | 172 | .9939 | .9940 | .9942 | .9943 | .9943 | .9944 | .9945 | .9951 |
| 84 | .3849 | .3921 | .3978 | .4024 | .4062 | .4122 | .4167 | .4477 | 174 | .9966 | .9967 | .9967 | .9968 | .9968 | .9969 | .9970 | .9973 |
| 86 | .4019 | .4091 | .4149 | .4195 | .4234 | .4294 | .4349 | .4651 | 176 | .9985 | .9985 | .9985 | .9986 | .9986 | .9986 | .9986 | .9988 |
| 88 | .4191 | .4263 | .4321 | .4368 | .4406 | .4467 | .4013 | .4826 | 178 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 | .9997 | .9997 |
| 90 | .4365 | .4437 | .4495 | .4547 | .4580 | .4641 | .4686 | .5000 | 180 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

With respect to distance of piston from end of stroke it is evident from the figure that the distances $mB$ and $nF$, corresponding to the crank positions $T$ and $S$, are unequal. If the connecting rod were infinite $m^1B^1$ and $n^1F^1$ would respectively represent the distances of piston from nearest end of stroke for the crank positions $T$ and $S$.

The following figure, drawn to scale, shows graphically the inequalities of the paths traversed by piston during the first and

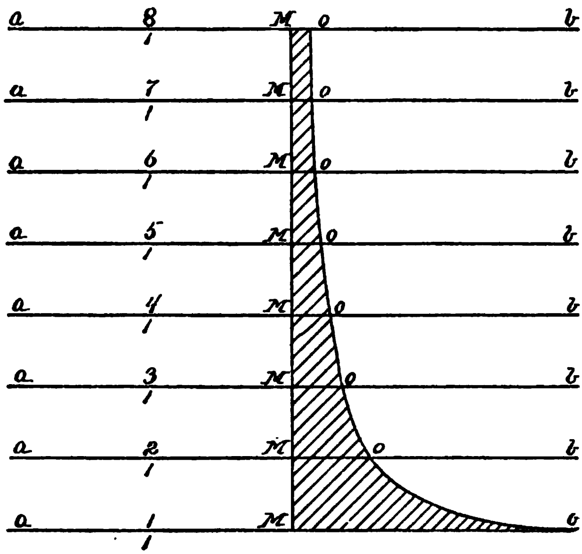



Fig. 11.

second quadrants of crank motion for different ratios of connecting rod to crank, the horizontal position of stroke (= $ab$) included by shaded figure representing the distance $Mo$ of the piston from its middle position $M$ (see preceding figure and Tables V and VI).

When length of crank = $R$ = 1 and $\omega = 90°$

| $\frac{L}{R}$ = | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|
| *Mo.* = | .063 | .072 | .084 | .101 | .127 | .172 | .268 | 1.0 |

Fig. 11 shows clearly that as the ratio of connecting rod to crank diminishes the inequality of the paths $ao$ and $ob$ respec-

tively described by piston while crank passes through the first and second quadrant, increases. This inequality tends to heap up the work in the first and fourth quadrants, as will now be more fully shown.

In order to show how the length of connecting rod influences the tangential components of the pressures on crank pin, we will assume the horizontal pressure on cross-head pin constant, and

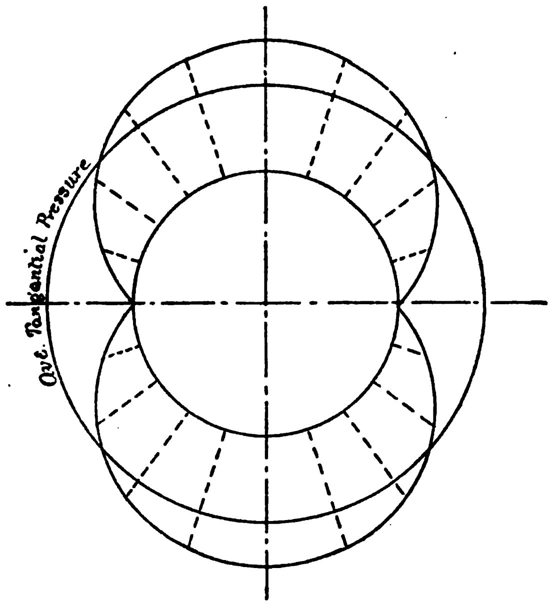



Fig. 12.

will lay off the tangential pressures on prolongation of the radii of the crank-pin circle, Figs. 12 and 13. The curve drawn through the extremities of these radial ordinates will show the variation of the tangential pressure for the different crank positions; the area included between this curve and the crank-pin circle measures, approximately, the work done in one revolution. The two diagrams drawn correspond respectively to $\frac{\text{connecting rod}}{\text{crank}} = \text{infinity}$ and $\frac{\text{connecting rod}}{\text{crank}} = 2\frac{1}{2}$ as above. A glance at these diagrams shows

at once that while with the infinite rod the work done in each of the quadrants is equal, with the finite rod the principal part of the work is done in quadrants I and IV. The average tangential pressures can easily be found by remembering that if friction is neglected the work done on piston must equal the work done on crank pin, and that therefore the average tangential pressure $p_t$

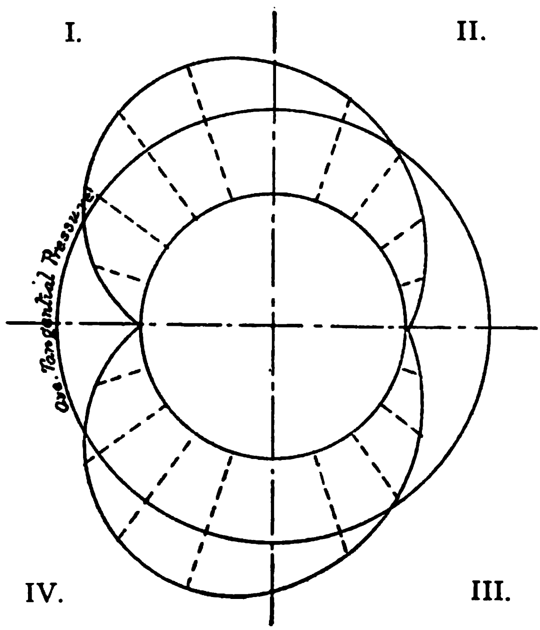



Fig. 13.

must be to the average piston pressure $p_m$ as the path $2S$ ($S =$ stroke) of piston in one revolution is to the path $\pi S$ of the crank pin, or

$$p_t : p_m :: 2S : \pi S \quad \text{hence} \quad p_t = \frac{2}{\pi} p_m = 0.6366\, p_m. \qquad (17)$$

Attention has already been called to the fact that the pressure and friction on the slide increase with the ratio of crank to connecting rod, then the

$$\text{max. friction on slides} = \mu P \times \frac{\text{crank}}{\text{connecting rod}}. \qquad (18)$$

$\mu =$ coefficient of friction, $P =$ pressure on cross-head pin.

But the ratio of crank to connecting rod affects not only the distribution of work and the magnitude of friction, it also affects the distribution of steam by increasing the duration of admission and reducing that of exhaust at one end of the cylinder, while it diminishes the period of admission and increases the duration of exhaust at other end.

That it thus affects the admission of steam can be roughly shown by means of Fig. 10, page 25. Since the ratio of eccentric radius to length of eccentric rod is small, even when the ratio

$\frac{\text{crank}}{\text{connecting rod}}$ is great, we may neglect the obliquity of eccentric

rod and treat the latter as if it were of infinite length, consequently the angular positions of the eccentric radius for the points of cut-off will be diametrically opposite, and since eccentric and crank are keyed to the same shaft, the crank positions corresponding to points of cut-off will also be directly opposite. Let $H$ and $T$, Fig. 10, page 25, represent the two crank positions corresponding to cut-off, it will then be evident from the figure that the period of admission $Bm$ for the end of the cylinder farthest from the crank will be greater than the admission $Fn$ for end of cylinder nearest crank. When the ratio of crank to connecting rod is large this distribution of steam will be injurious, but when it is equal to, or less than $\frac{1}{4}$ this irregularity of admission and exhaust at the two ends will not only not be excessive but will be a positive advantage in high-speed engines, because it gives the greater compression (*i.e.* diminished period of exhaust) at the end farthest from the crank where it is most needed. This will be proved later when we come to consider the proper period of compression. Moreover, with such proportions of crank to connecting rod, the valve setting obtained from Zeuner's simple valve diagram will give very satisfactory results practically. The

indicator diagrams for the two ends of the cylinder obtained from such distribution of steam will of course not be identical in form, but they will be nearly equal in area, and as we have already mentioned will give the greater compression where it is most needed. In marine engines where engine space is limited the ratio is sometimes as low as $1\frac{1}{2}$, while in other well designed stationary engines it is sometimes as high as 8. Decide upon the proper ratio for the present case.

## III.

### Determination of that Mean Accelerating Force (per $\square''$ of Piston) Necessary to Start Reciprocating Parts, which Corresponds to the Most Uniform Tangential Pressure on Crank Pin.

That the weight and speed of the reciprocating parts have an important influence on the steadiness with which an engine can be run, will be evident from the following consideration: Steadiness of running, other things being equal, results from the driving effort, or tangential pressure on crank pin, being as nearly constant as possible for all positions of the crank; the magnitude of the tangential pressure ($= DF$ in Fig. 14) depends upon the position of the crank $CD$, and upon the direction and intensity of the pressure $DB' = EB$ exerted by the connecting rod upon the crank pin; this latter pressure in turn depends upon the angular position of the connecting rod and upon the magnitude of the force $K = EA$. Let us now suppose the mass $M$ of the reciprocating parts (piston, piston rod, cross-head and connecting rod)* to be concentrated in a ring around the cross-head

* According to Weisbach-Herrmann, (Vol. III, Section II, p. 867) the work stored up in a connecting rod of uniform section is the same as if one-third of its mass were concentrated at crank pin and the remaining two-thirds at cross-head pin. This result is itself an approximation and as sections are generally not uniform the accuracy of the assumption is still further diminished. Usually rod tapers from wrist to crank pin.

pin, and that the resistance offered by this mass to a change in its state of motion, is represented by $F$, we will then have $F = M\theta$ where $\theta$ represents the acceleration of the center of the cross-head pin, $M =$ mass of reciprocating parts and $F =$ force necessary to give the mass $M$ the acceleration $\theta$.

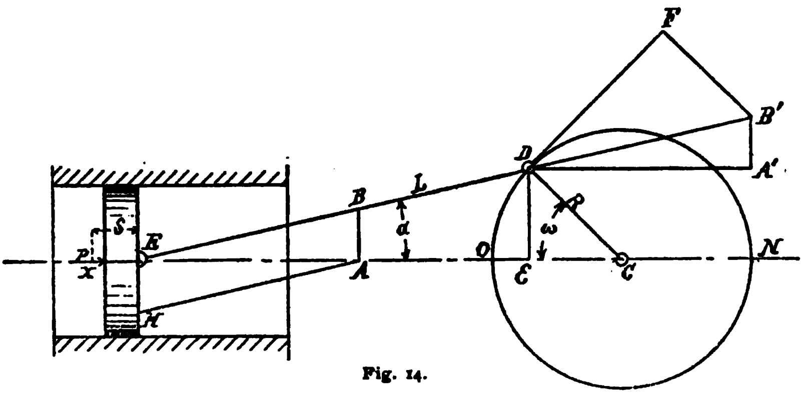



Fig. 14.

The accelerating force $F$ is of course part of the steam pressure $P$, the difference between them, $P - F = K = EA$ (see figure), is that part of the steam pressure which is transmitted through the piston rod to the lower end of connecting rod and from these to the crank pin. When the point $E$ (*i.e.*, the piston) has reached its maximum velocity, acceleration ceases, and we have $F = 0$ and $P = K = EA = DA'$, but retardation now begins (for the reciprocating parts must be brought to rest at the end of the stroke) and no part of the steam pressure being absorbed in producing acceleration, the whole of the steam pressure $P$ is transmitted to the cross-head pin. This $P$ is however not the only pressure to which the crank pin is subjected, for the reciprocating parts having once attained their maximum velocity, tend to keep it, and tend to move faster (horizontally) than the crank pin, and will thus exert a pressure $F$ upon the latter, over and above that received from the steam ($= P$) hence

$$P + F = K = EA = DA'.$$

The general expression for the horizontal pressure exerted upon the crank pin or cross-head pin will then be

$$DA^1 = EA = K = P \pm F$$

the *upper* sign of $F$ being employed when reciprocating parts are retarded, and the *lower* sign, when the reciprocating parts are accelerated, *$F$ being the accelerating or retarding force necessary to produce in the mass $M$ the acceleration or retardation $\theta$*, that is,

$$F = M\theta = \frac{W}{g}\theta \qquad (19)$$

so that $F$ increases with the weight $W$ of the reciprocating parts and their acceleration.

Before determining the value of $\theta$ for the different piston and crank positions, we will call attention to, and emphasize the point, that the horizontal pressures on crank pin or cross-head pin are not directly obtainable from the diagram of effective steam pressures on piston but can be obtained indirectly from the latter by *subtracting* the *accelerating* pressures from the steam pressures given by the diagram *for the earlier part of the stroke*, and *adding* the *retarding* pressures to the steam pressures given by the diagram *for the later part of the stroke*. From this we already see that the mass and motion of the reciprocating parts tend to equalize the horizontal pressures upon the crank pin by diminishing the larger pressures near the beginning and increasing the smaller pressures near the end of expansion, the energy of the steam absorbed by the reciprocating parts near the beginning, being given out again to the crank pin toward the end, of the expansion. With too great a speed, however, this equalizing tendency will be more than neutralized by the transfer of the whole of the steam pressures to the end of the stroke and will tend to cause shocks and dangerous stresses on all the working pieces. This point will be more fully described further on.

We will now find an expression for the acceleration $\theta$ and then substitute in

$$F = \frac{W}{g}\theta. \qquad (19)$$

The distance $s$ traversed by cross-head pin $E$ from the beginning $X$ of its stroke (see Fig. 14) is equal to

$$s = XE = XC - EC = (R + L) - (L \cos \alpha + R \cos \omega)$$

$$s = R(1 - \cos \omega) + L(1 - \cos \alpha). \qquad (20)$$

This equation holds for both strokes, provided we estimate $\omega$ from 0° to 360° and $s$ from the same dead point (left hand dead point $X$, Fig. 14). But if we estimate $\omega$ and $s$ from the opposite center or dead point for the return stroke, and this is commonly done, we must use

$$s = R(1 - \cos \omega) - L(1 - \cos \alpha).$$

The cosine of the angle made by connecting rod with axis of cylinder can be obtained as follows

$$DG = L \sin \alpha = R \sin \omega$$

hence

$$\sin \alpha = \frac{R}{L} \sin \omega \qquad (21)$$

and $\cos \alpha = \sqrt{1 - \frac{R^2}{L^2} \sin^2 \omega} = 1 - \frac{1}{2}\frac{R^2}{L^2} \sin^2 \omega$ very nearly (22)

Introducing this value of cosine $\alpha$ in equation (20) we obtain

$$s = R\left[(1 - \cos \omega) + \frac{1}{2}\frac{R}{L} \sin^2 \omega\right] \qquad (23)$$

For the return stroke, and estimating $s$ and $\omega$ from the right hand dead point this equation becomes

$$s = R\left[(1 - \cos \omega) - \frac{1}{2}\frac{R}{L} \sin^2 \omega\right]$$

From this general formula for the travel, $s$, of the piston we can obtain the velocity, $w$, of the piston or of the point $E$ when the crank has a uniform velocity.

According to Elementary Mechanies we have

$$\text{for linear velocity, } w = \frac{ds}{dt}$$

$$\text{and for acceleration, } \theta = \frac{dw}{dt}$$

Now from (23)

$$ds = R(\sin \omega + \tfrac{1}{2}\frac{R}{L} \sin 2\omega)d\omega$$

therefore

$$w = R\left(\sin \omega + \tfrac{1}{2}\frac{R}{L} \sin 2\omega\right)\frac{d\omega}{dt} \qquad (24)$$

is velocity of piston.

If the crank pin has a uniform circumferential velocity $v$ we will have

$$Rd\omega = vdt \text{ or } \frac{d\omega}{dt} = \frac{v}{R}. \qquad (25)$$

Substituting this value of $\frac{d\omega}{dt}$ in (24) we obtain

$$w = v\left(\sin \omega + \tfrac{1}{2}\frac{R}{L} \sin 2\omega\right); \qquad (26)$$

For return stroke and estimation of $s$, $\omega$ and $w$ from the right hand dead point, we have instead

$$w = v\left(\sin \omega - \tfrac{1}{2}\frac{R}{L} \sin 2\omega\right).*$$

Differentiating (26) we have

$$dw = v\left(\cos \omega d\omega + \tfrac{1}{2}\frac{R}{L} \cos 2\omega\ 2d\omega\right),$$

reducing and dividing both members by $dt$ we obtain

$$\frac{dw}{dt} = v\left(\cos \omega + \frac{R}{L} \cos 2\omega\right)\frac{d\omega}{dt}.$$

Now since the acceleration of a unit of mass is $\theta = \frac{dw}{dt}$ we can substitute $\theta$ in preceding equation, getting

$$\theta = v\left(\cos \omega + \frac{R}{L} \cos 2\omega\right)\frac{d\omega}{dt}, \qquad (27)$$

---

* An exact expression is :

$$w = v\left[\sin \omega \pm \tfrac{1}{2}\frac{R}{L}\frac{\sin 2\omega}{\sqrt{1 - \frac{R^2}{L^2}\sin^2 \omega}}\right]$$

For exact ratios of $w$ to $v$ see Table VIII, p. 60.

but we found from (25) that $\frac{d\omega}{dt} = \frac{v}{R}$, therefore,

$$\theta = \frac{v^2}{R}\left[\cos\omega + \frac{R}{L}\cos 2\omega\right].^* \qquad (28)$$

For return stroke, and estimation of $s$, $w$, $\omega$ and $\theta$ from the right hand dead point, this becomes

$$\theta = \frac{v^2}{R}\left(\cos\omega - \frac{R}{L}\cos 2\omega\right).$$

At this point we should note that the factor $\frac{v^2}{R}$ is identical with the acceleration of a unit of mass rotating in the crank-pin circle, and when the connecting rod $L$ is infinite the value of $\theta$ for $\omega = o$ and $\omega = 180°$ is $\theta = \pm\frac{v^2}{R}$, that is with infinite connecting rod the acceleration of reciprocating parts, when the crank is on the dead centers $O$ and $N$, is identical with that possessed by a unit of mass rotating in the crank pin circle. But when the connecting rod is of finite length, $\theta$ is no longer equal to $\frac{v^2}{R}$, either on dead center $O$ or on $N$, but we have instead when $\omega = o$

$$\theta = \frac{v^2}{R}\left(1 + \frac{R}{L}\right) \qquad (29)$$

and when $\omega = 180°$

$$\theta = \frac{v^2}{R}\left(1 - \frac{R}{L}\right) \qquad (30)$$

---

* An exact expression for $\theta$ is:

$$\theta = \frac{v^2}{R}\left[\cos\omega \mp \frac{\sin^2\omega}{\sqrt{\frac{L^2}{R^2} - \sin^2\omega}} \pm \frac{L^2}{R^2}\frac{\cos^2\omega}{\left(\frac{L^2}{R^2} - \sin^2\omega\right)^{\frac{3}{2}}}\right]$$

Table VII on p. 39 was not computed from this formula, but from bracketed part of equation 28. The table, to be perfectly exact, should have been computed from the bracketed part of the equation just given. The difference is so slight, however, that it can be neglected in the problems arising in practice. The upper signs correspond to forward stroke.

the numerical mean of these two values is, however, equal to $\frac{v^2}{R}$; we call this mean $\theta_m$, or

$$\theta_m = \frac{\frac{v^2}{R}\left(1 - \frac{R}{L}\right) + \frac{v^2}{R}\left(1 + \frac{R}{L}\right)}{2} = \frac{v^2}{R} \tag{31}$$

Returning now to the expression deduced above for $\theta$ and substituting in

$$F = M\theta = \frac{W}{g}\theta \tag{32}$$

we get
$$F = \frac{W}{g}\frac{v^2}{R}\left(\cos\omega + \frac{R}{L}\cos 2\omega\right). \tag{33}$$

Dividing both members of the equation by $A$, area of piston in square inches, we get

$$\frac{F}{A} = \frac{W}{g} \times \frac{v^2}{R} \times \frac{1}{A}\left(\cos\omega + \frac{R}{L}\cos 2\omega\right) \tag{34}$$

If the weight $W$ of the reciprocating parts were rotating in the crank pin circle, the centrifugal force would be

$$F_0 = M\frac{v^2}{R} = \frac{W}{g}\frac{v^2}{R}; \tag{35}$$

substituting this in the above expression for $\frac{F}{A}$ we get

$$\frac{F}{A} = \left(\cos\omega + \frac{R}{L}\cos 2\omega\right)\frac{F_0}{A} = c\frac{F_0}{A} \tag{36}$$

For return stroke, and estimation of $s$, $\omega$, $w$, and $\theta$ from the right hand dead point, this becomes

$$\frac{F}{A} = \left(\cos\omega - \frac{R}{L}\cos 2\omega\right)\frac{F_0}{A} = c\frac{F_0}{A}.$$

It is evident that the value of $c$ will be the same numerically for angle $\omega$ of the forward stroke and angle $180° - \omega$ for the return stroke, but its algebraic sign will be different in these two cases.

Table VII on following page contains values of $c$ for different crank angles $\omega$ and different values of $\frac{L}{R}$.

# TABLE VII.

## ACCELERATION OF RECIPROCATING PARTS $= c\dfrac{F_o}{A}$.

TABLE CONTAINING VALUES OF $c = \left(\cos\omega \pm \dfrac{R}{L}\cos 2\omega.\right)$

The $\left\{\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right\}$ signs in formula relate respectively to $\left\{\begin{matrix}\text{forward}\\ \text{return}\end{matrix}\right\}$ stroke.

The algebraic signs in the table relate to forward stroke only; for return stroke, signs opposite to those there given must be used.

| Crank Angles. | | Connecting-rod ÷ Crank = | | | | | Crank Angles. | | Connecting-rod ÷ Crank = | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Forward. | Return. | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 | Forward. | Return. | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 |
| 0° | 180° | 1.250 | 1.222 | 1.200 | 1.182 | 1.167 | 90° | 90° | -.250 | -.222 | -.200 | -.182 | -.167 |
| 3° | 177° | 1.248 | 1.220 | 1.198 | 1.180 | 1.165 | 93° | 87° | -.301 | -.272 | -.250 | -.233 | -.218 |
| 6° | 174° | 1.240 | 1.212 | 1.191 | 1.173 | 1.158 | 96° | 84° | -.350 | -.322 | -.301 | -.283 | -.268 |
| 9° | 171° | 1.226 | 1.199 | 1.178 | 1.161 | 1.141 | 99° | 81° | -.394 | -.367 | -.346 | -.329 | -.315 |
| 12° | 168° | 1.206 | 1.181 | 1.161 | 1.144 | 1.130 | 102° | 78° | -.436 | -.411 | -.391 | -.374 | -.360 |
| 15° | 165° | 1.183 | 1.158 | 1.139 | 1.124 | 1.110 | 105° | 75° | -.476 | -.449 | -.430 | -.415 | -.401 |
| 18° | 162° | 1.153 | 1.131 | 1.113 | 1.098 | 1.086 | 108° | 72° | -.511 | -.489 | -.471 | -.456 | -.444 |
| 21° | 159° | 1.120 | 1.099 | 1.083 | 1.069 | 1.058 | 111° | 69° | -.544 | -.523 | -.516 | -.491 | -.482 |
| 24° | 156° | 1.081 | 1.063 | 1.048 | 1.036 | 1.026 | 114° | 66° | -.574 | -.556 | -.541 | -.529 | -.519 |
| 27° | 153° | 1.038 | 1.022 | 1.009 | .998 | .989 | 117° | 63° | -.601 | -.585 | -.572 | -.561 | -.552 |
| 30° | 150° | .991 | .977 | .966 | .957 | .949 | 120° | 60° | -.625 | -.611 | -.600 | -.591 | -.583 |
| 33° | 147° | .941 | .929 | .920 | .913 | .907 | 123° | 57° | -.647 | -.635 | -.626 | -.619 | -.616 |
| 36° | 144° | .886 | .878 | .871 | .865 | .861 | 126° | 54° | -.665 | -.657 | -.650 | -.644 | -.640 |
| 39° | 141° | .829 | .823 | .819 | .815 | .812 | 129° | 51° | -.681 | -.675 | -.671 | -.667 | -.664 |
| 42° | 138° | .769 | .766 | .764 | .762 | .760 | 132° | 48° | -.695 | -.692 | -.689 | -.688 | -.686 |
| 45° | 135° | .707 | .707 | .707 | .707 | .707 | 135° | 45° | -.707 | -.707 | -.707 | -.707 | -.707 |
| 48° | 132° | .643 | .646 | .648 | .650 | .652 | 138° | 42° | -.717 | -.720 | -.722 | -.724 | -.726 |
| 51° | 129° | .577 | .583 | .587 | .591 | .594 | 141° | 39° | -.725 | -.731 | -.735 | -.739 | -.742 |
| 54° | 126° | .511 | .519 | .526 | .532 | .536 | 144° | 36° | -.732 | -.740 | -.747 | -.753 | -.758 |
| 57° | 123° | .443 | .455 | .464 | .469 | .477 | 147° | 33° | -.737 | -.749 | -.758 | -.765 | -.771 |
| 60° | 120° | .375 | .389 | .400 | .409 | .417 | 150° | 30° | -.741 | -.755 | -.766 | -.775 | -.783 |
| 63° | 117° | .307 | .323 | .336 | .347 | .356 | 153° | 27° | -.744 | -.761 | -.773 | -.784 | -.793 |
| 66° | 114° | .240 | .259 | .273 | .285 | .296 | 156° | 24° | -.747 | -.765 | -.780 | -.792 | -.803 |
| 69° | 111° | .172 | .193 | .209 | .223 | .234 | 159° | 21° | -.748 | -.769 | -.785 | -.798 | -.810 |
| 72° | 108° | .107 | .129 | .147 | .162 | .174 | 162° | 18° | -.749 | -.771 | -.789 | -.804 | -.816 |
| 75° | 105° | .032 | .067 | .086 | .102 | .115 | 165° | 15° | -.750 | -.774 | -.793 | -.809 | -.822 |
| 78° | 102° | -.020 | .005 | .025 | .032 | .056 | 168° | 12° | -.750 | -.775 | -.795 | -.810 | -.826 |
| 81° | 99° | -.082 | -.055 | -.034 | -.017 | -.003 | 171° | 9° | -.750 | -.777 | -.797 | -.815 | -.830 |
| 84° | 96° | -.140 | -.112 | -.090 | -.073 | -.058 | 174° | 6° | -.750 | -.778 | -.799 | -.817 | -.832 |
| 87° | 93° | -.197 | -.169 | -.147 | -.139 | -.114 | 177° | 3° | -.750 | -.778 | -.800 | -.818 | -.833 |
| 90° | 90° | -.250 | -.222 | -.200 | -.182 | -.167 | 180° | 0° | -.750 | -.778 | -.800 | -.818 | -.833 |

From the above expression for $\dfrac{F}{A}$ we can deduce an approximate, graphical, method sufficiently accurate for most purposes.

If we make $\omega = 90°$ and $270°$ we get, respectively,

$$\frac{F}{A} = -\frac{R}{L}\frac{F_o}{A} \text{ and } +\frac{R}{L}\frac{F_o}{A}; \tag{37}$$

for $\omega = o^\circ$ we have

$$\frac{F}{A} = \left(1 + \frac{R}{L}\right)\frac{F_o}{A} \tag{38}$$

and for $\omega = 180^\circ$ we have

$$\frac{F}{A} = -\left(1 - \frac{R}{L}\right)\frac{F_o}{A}. \tag{39}$$

Suppose that we now draw a circle with radius $CG = \frac{F_o}{A}$ and lay off $CB, C'B'$ and $C''B''$ each equal to $\frac{RF_o}{LA}$ and then through

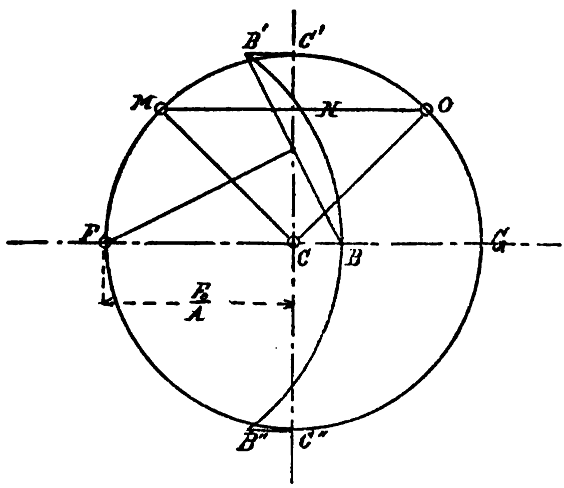



Fig. 15.

the three points $B', B, B''$ thus found pass the circle $B'BB''$ the middle of the chord $B'B$ will always fall on $CC'$ and the center of the circle on $GBF$.* Then the acceleration for any crank angle $FCM$ will be represented by $MN = \frac{F}{A}$ and the retardation for the crank angle $FCO$ by $ON\ \frac{F}{A}$. For any

* The center $F$ of the arc $B'BB''$ falls on the circumference $C'MC''$ only when $\frac{R}{L} = \frac{1}{4}$.

other crank angle the horizontal line included between the two circles will measure the corresponding accelerating or retarding force.

## EXACT GRAPHICAL METHOD OF FINDING ACCELERATING OF PISTON AND OF ANY POINT OF THE ROD.

The point *P* in Fig. 10 is the instantaneous center for the relative motion of connecting rod to engine bed. The velocity of crank pin *B* is to that of slide *A* as the instantaneous radius *PB*

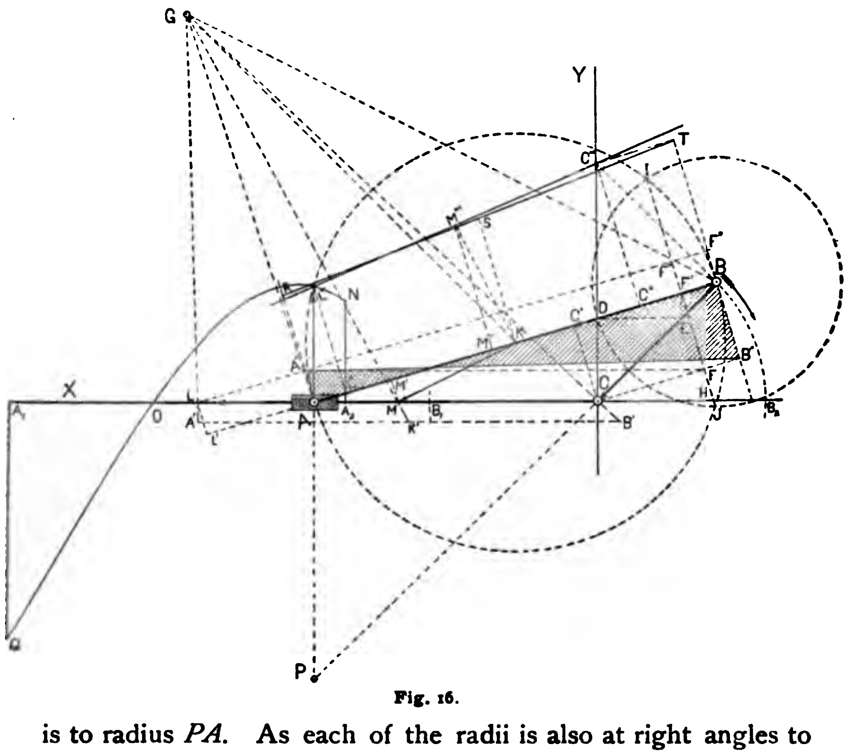


Fig. 16.

is to radius *PA*. As each of the radii is also at right angles to the direction of the velocity they may respectively be taken as the representatives of the velocities of points *B* and *A*. As velocities may be combined and resolved like forces, we may regard *APB* as a triangle of velocities and either side as the resultant of the other two. Thus *PA* may be considered as the resul_

tant of $PB$ and $BA$. That is, the velocity of point $A$ may be considered as the resultant of the velocity of point $B$ about $P$ and of the velocity of point $A$ about $B$ as a center (the angular velocity about $B$ being then equal to that of $B$ around $P$).*

If we suppose the angular velocity of pin $B$ about center $C$ to be uniform and equal to unity, the crank radius $CB$ will represent the velocity of its pin and, to the same scale, the distance $CD$, the velocity of the slide $A$ at the same instant. Then the sides of triangle $BCD$ will replace the sides of $BPA$ as the representatives of the velocities. The length $DB$ thus represents on our scale the aforesaid velocity of point $A$ of the rod when turning about its point $B$.

We are now ready to determine the acceleration of point $A$. When the direction of an acceleration and the intensity of one of its two components and the direction of both are known, we can easily find the intensity of the acceleration itself by means of the triangle of accelerations, (which triangle is analogous to that of velocities or forces. In this case we know the direction of the acceleration of the point $A$ to be along its path $CAL$. We may suppose it to be resolved into two components, one along the rod $AB$ and the other at right angles thereto. We will determine the one along the rod $AB$. It was shown above that the motion of $A$ was compounded of its own motion about $B$ and of $B$'s motion. Now when the crank has a uniform angular velocity equal to unity its acceleration is represented in direction and intensity by $BC$. If we resolve $BC$ into components along and at right angles to $BA$, the one along the rod will be $BC'$. The part of the acceleration of $A$ which is due to its rotation about $B$ as a center is also made up of two components, one along $AB$ and the other at right angles to $AB$. We need to know only the former,

*This can also be regarded as an example of the resolution of a rotation about one axis $P$ into an equivalent rotation about another parallel axis $B$ plus a circular translation. See Weisbach-Herrmann's Machinery of Transmission, Vol. III, Sect. II, §4 of the Introduction.

and this is $\frac{(\text{velocity})^2}{AB}$. But it was shown above that this velocity $= BD$, hence the component along $AB$ is equal to $\frac{BD^2}{AB}$. This quotient can be found geometrically by describing a circle on the rod $AB$ as a diameter and finding its intersections with an arc struck from $B$ as a center and $BD$ as a radius. The common chord $IJ$ joining these intersections will cut from $BA$ the distance $BF' = \frac{BD^2}{AB}$ and will also cut from the horizontal through $C$, a distance $CH$ exactly equal to the desired total acceleration of slide $A$. This can now be easily shown, for the component $BF'$ just found has a direction from $F'$ to $B$ and the component $BC'$ of $B$ along the rod has evidently the direction $B$ to $C'$. Therefore $BC' - F'B = F'C' =$ that one of the two components of the total accelaration of $A$ which acts along rod $AB$. Through $C$ draw $CF$ equal and parallel to $C'F'$ and to it at $F$ erect a perpendicular $FH$; this cuts from $ACB_2$, the direction of the total acceleration of $A$, a distance $HC$ equal to the direction and intensity of the total acceleration of slide $A$.*

The point $F'$ can also be found by drawing through $D$, the intersection of the rod $AB$ with $CY$ the perpendicular to the stroke, the parallel $DE$ and where this meets the crank draw $EF'$ parallel to $CY$; this parallel cuts the rod or its prolongation in the desired point $F'$. The proof of this construction is:

$$BF':BD = BE:BC = BD:BA, \text{ that is, } BF' = \frac{BD^2}{AB}.$$

* This construction is not only the simplest, but it is also applicable to all slider-crank mechanisms, whether the stroke of the slide $A$ does or does not pass through the crank center $C$. In the latter as in the former case the intercept must be taken on a line $CH$ through $C$ and parallel to the slide stroke. It is also applicable to variable crank-pin velocity when scale is such that latter is represented by crank $AB$. We have then only to draw a parallel to stroke through the end of crank-pin acceleration (which in this case does not fall at center $C$ of shaft). On this parallel the intercept included between the end of acceleration and chord $IJ$, Fig. 14, will be the exact acceleration of slide $A$.

But this construction of $F'$ evidently fails when the crank is at either dead point.

The determination of the exact acceleration of $A$ will enable us to construct its curve of acceleration $NOQ$, which is drawn on the stroke of $A$ as a base. This suffices for most of the cases arising in practice. Greater accuracy is of course obtainable if we ascertain the acceleration of each point of the rod, multiply this by the elementary mass at that point and then find the resultant of all these elementary forces of inertia. This resultant can be resolved into two components, one acting at wrist pin $A$ and the other at crank pin $B$. These can then be combined with the other forces acting at $A$ and $B$ and the exact tangential pressure on crank pin found. We will postpone this combination of forces for the present and confine ourselves to finding an easy method of constructing the acceleration of each point of the rod and its components parallel to, and perpendicular to, the rod.

We will first show that the end of the acceleration $KM$ of any point $K$ of the center line of the rod lies on the straight line joining the ends $C$ and $L$ of $BC$ and $AL$, the accelerations of the two points $B$ and $A$. To do this we must make use of the properties of the center of acceleration $G$.* This center is that point of the moving rod which has no acceleration. The acceleration of any point $K$ is directly proportional to the distance $GK$ of this point

* For method of finding the center of acceleration $G$ when the directions of the acceleration of two points are given and also their ratio, see Weisbach-Herrmann's Machinery of Transmission, Vol. III., Section I, § 21. But for this, most common case, in which the crank has uniform rotation, we would suggest connecting crank-pin center $B$ with point $H$ (the intersection of line $IJ$ with horizontal $CB_2$ Fig. 14) and on this connecting line laying off from $B$ a distance equal to the length of connecting rod; then through the end of this distance draw a horizontal till it cuts the prolongation of the crank. The triangle thus obtained will be similar to $CBH$ and when this triangle is swung around $B$ till its base coincides with $BA$ its vertex will fall at $G$ and give exactly the position of the center of acceleration. The modification necessary for cases of variable rotation of crank are evident; the triangles must be similar.

from the center of acceleration $G$, and for the instant in question the acceleration of each point makes the same angle with its own instantaneous radius of acceleration. For example, let us suppose the center of acceleration $G$ known, and let $BC$, $AL$ and $KM$ represent respectively the accelerations of points $B$, $A$ and $K$ in direction and intensity.

$$\text{Then will } BC : AL : KM = GB : GA : GK$$
$$\text{and angles } CBG = LAG = MKG.$$

From these properties, it follows that the triangles $GBC$, $GKM$ and $GAL$ are all similar and hence their corresponding angles $BGC$, $KGM$ and $AGL$ are equal to each other. If we turn the whole system of points $AKB$ about the center $G$ through the angle $BGC = KGM = AGL$, the points $B$, $K$ and $A$ will fall on $B'$, $K'$ and $A'$. Now if we can show that $MC$ is parallel to $K'B'$ and $ML$ to $K'A'$, we shall have proved that $MC$ and $ML$ are one and the same straight line because $K'B'$ and $K'A'$ constitute one straight line. From the similarity of the triangles $GBC$, $GKM$ and $GAL$, we have

$$GC:GM = GB:GK = GB':GK'$$
$$GM:GL = GK:GA = GK':GA'$$

and therefore $MC$ and $ML$ are respectively parallel to $K'B'$ and $K'A'$ and we have proved that the ends of all the accelerations of the points of one straight line lie on one and the same straight line.

This is true of any line of the system and hence of any combination of lines belonging to or constituting the system. Moreover it is evident that any line in the plane of rotation will be reduced in the ratio of $GC$ to $GB$ so that

$$\frac{CL}{BA} = \frac{GC}{GB}.$$

The ends of the accelerations of all points in the plane figure constituting the rod will form a reduced *image* of the shape of the rod which will be exactly similar (in the plane of motion) to the original rod. This similarity extends to every detail.

A corollary from this proposition is that the ends of the accelerations of the various points divide the distance $CL$ in the same ratio as do their corresponding points the rod or distance $BA = B'A'$. For instance, we have

$$CM:ML = B'K':K'A' = BK:KA.$$

This corollary enables us to determine the acceleration of any point of the rod $AB$ (without the help of the center of acceleration $G$) when we know the direction and intensity of the accelerations of any two points on that line. For example, if $K$ is half way between $A$ and $B$, the end $M$ of its acceleration $KM$ will be half way between the ends $C$ and $L$ of the known accelerations $BC$ and $AL$.

If the total accelerations of each of the points of the rod be resolved into two components, one parallel and the other perpendicular to the rod, and these components be laid off as ordinates, (taking the rod as an axis of abscissas), one set of components on one side of the rod and the other set on the opposite side of the rod, then will the components of each set terminate on the same straight line. That is, the components of the set which are at right angles to $AB$ will terminate on the straight line $A''B''$, and the components of the set which coincide with $AB$, will (when revolved through 90°) terminate on the straight line $RST$.

This can be proved as follows: Drop from $C$, $M$ and $L$ the respective perpendiculars $CC'$, $MM'$ and $LL'$. Because they are a series of parallels they will divide the distance $C'L'$ in the same ratio as $CL$, and since $CL$ has been shown to be divided by the acceleration in the same ratio as $BA$, it follows that $C'L'$ is divided in the same ratio as $BA$. But the ordinates $C'C$, $M'M$, $L'L$, etc. erected on $C'L'$, by construction, terminate in a straight line, namely $CML$. As $AF'$ is equal to $L'C'$ we may suppose these self-same ordinates to be laid off from the former and then they will all terminate on the straight line $A''F$. Now if we suppose the base $AF'$ of this set of ordinates to be stretched till it is equal to $AB$ in length, the ordinates remaining equidistant and chang-

ing only their position and not their magnitude, then will they all terminate on the straight line $A''B''$. For before stretching, the ratio $\frac{dy}{dx} = a$, of the difference of any two adjacent ordinates to the difference of their abscissas, was equal to a constant $a$, which is the characteristic of a straight line. After stretching, the ratio of these differences is $\frac{dy}{dx^1} = a^1 =$ a constant also, because $\frac{dx^1}{dx} = \frac{AB}{AF^1} =$ a constant for the points of this rod and this position of the mechanism.

If we revolve the accelerations $BC$, $KM$, $AL$, etc., through 90° so that they occupy positions $BC'''$, $KM'''$, $AL'''$, etc., these new positions will also make a constant angle with their instantaneous radii of acceleration, and we may show as before that the points $L'''M'''C'''$ lie on a straight line. If from their ends $C'''$, $M'''$ and $L'''$ we drop on $AB$ the perpendiculars $C'''C''$, $M'''M''$, $L'''L''$, etc., and erect these as ordinates at the corresponding points of the base $AB$ we can show as before that these all terminate on one straight line $RST$. But as these perpendiculars $C'''C''$, etc., are respectively equal to the second set of components $BC'$, etc., the second part of our proposition has been established. The forces of inertia due to this second set of components do not however act transversely on the rod, but longitudinally. Their sum or resultant combined with the resultant of all the bending forces that are due to inertia will give a total resultant, due to inertia of rod, that can be resolved into two forces acting at pins $A$ and $B$.*

Exactly how this resultant is found will be discussed later on; at present the shaded area and area $ABTR$ furnish the components for constructing the acceleration of any point belonging to the center line of rod. For this purpose however area $ABTR$ would suffice, for if its component be laid off on the rod, say $KM' = KS$, a perpendicular erected at $M'$ will cut the line $CA$ in $M$ giving at once $KM$ as the acceleration of the point $K$.

* This must not be interpreted as giving the total inertia resistance of rod for distributed mass.

## III A.

### ACCELERATION DIAGRAMS.

Obtain by means of the preceding diagrams or tables, or both, the accelerating and retarding forces corresponding: to an assumed value of $\frac{F_o}{A}$, to a given ratio of connecting rod to crank, and to certain crank angles. Then find the corresponding piston positions as in the preceding plate. At these positions erect ordinates equal to the accelerating or retarding forces the former being laid off below, and the latter above, the line of piston positions. It is evident from the figure that the acceleration curve for one stroke can be obtained from that of the other by revolving the latter 180° about the central line.

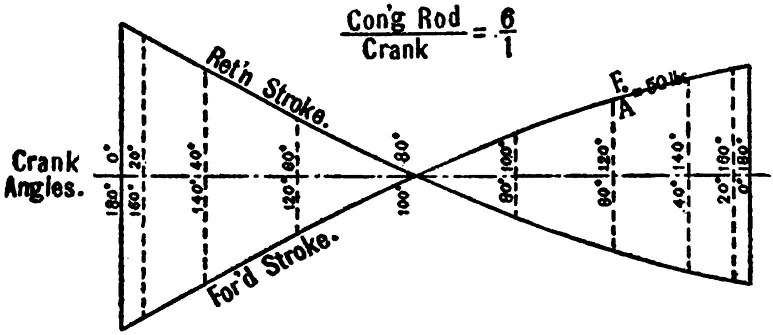



Fig. 17.

Construct on the admission line of each of the six diagrams of effective steam pressure on piston, two acceleration diagrams (in the manner shown in Fig. 17) which correspond respectively to $\frac{F_o}{A} = \frac{p_1 - 16}{2} =$ say, 40 lbs. and $\frac{F_o}{A} = 2p_2 =$ say, 50 lbs.; $p_2$ is the terminal pressure at the end of the expansion curve. Assume also $\frac{F_o}{A} = 30$.

## III B.

### DETERMINATION OF PRESSURE ON CROSS-HEAD PIN, WHEN WEIGHT OF ROD AND FRICTION ARE NEGLECTED, BUT INERTIA TAKEN INTO ACCOUNT BY ASSUMING WHOLE MASS OF ROD AND RECIPROCATING PARTS CONCENTRATED ON PISTON.

To obtain this diagram we must combine the diagram of effective steam pressures on piston with the diagram of accelerating and retarding pressures. Moreover the combination must be so effected that the accelerating pressures will be subtracted from the effective steam pressures, and the retarding forces added. This can be done in two ways: by subtracting and adding the ordinates of the acceleration diagram, Fig. 17, to the upper lines *ABC* and *EFG* of Figs. 8 and 9, representing the pressures on driving side of piston, and getting as a result the horizontal pressure diagrams *VTSI'H* and *G'POLA'DC'*, Figs. 18 and 19, or by subtracting and adding the ordinates of Fig. 17 to the lower lines *HI'E* and *A'D'C'*, Figs. 8 and 9, representing the counter pressures, and getting as a result the horizontal pressure diagrams *A'B'C'MN* and *E'F'G'HKE'*, Figs. 20 and 21. A comparison of the two sets of diagrams will show that the corresponding ordinates included by the shaded portions are equal.

The shaded area in each case is bounded on the one side by the resultant line (due to the combination of expansion line or line of counter-pressure with acceleration curve) and on the other side by that line of Fig. 8 or 9 which did not help to form the resultant line.

It should also be noticed that where the resultant line crosses the unused line (given by Fig. 8 or 9) the horizontal pressures on cross-head pin become equal to zero, and after passing this point, see Fig. 21, the direction of the horizontal pressures on cross-head pin is reversed before the piston reaches the end of its stroke. In Fig. 20 the steam and retarding pressures are so related as to com-

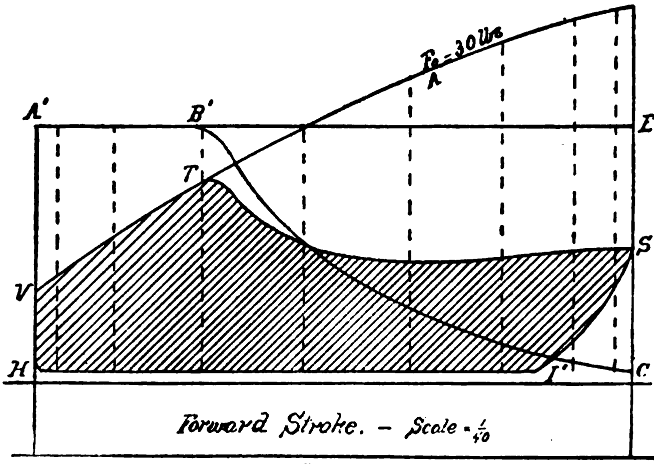



Fig. 18.

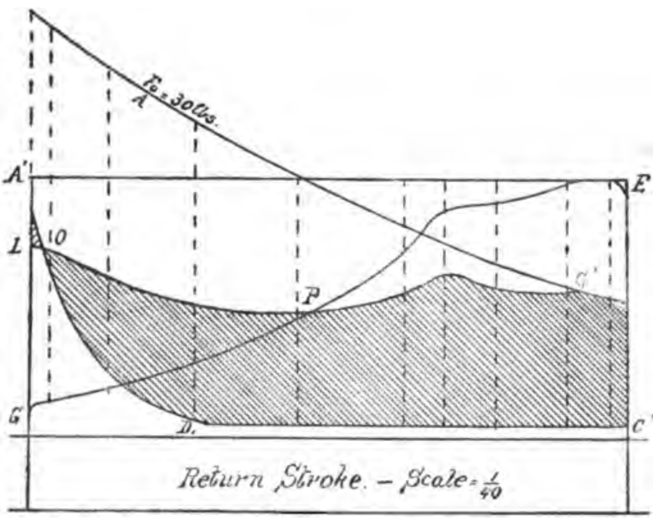



Fig. 19.

pletely unload the crank-pin at the end of the stroke. This is not so favorable as it seems, for at the beginning of the return stroke there is suddenly applied a great pressure in the opposite direction. A little more compression *I'S*, Fig. 18, would have changed this. It is probable that the arrangement shown in

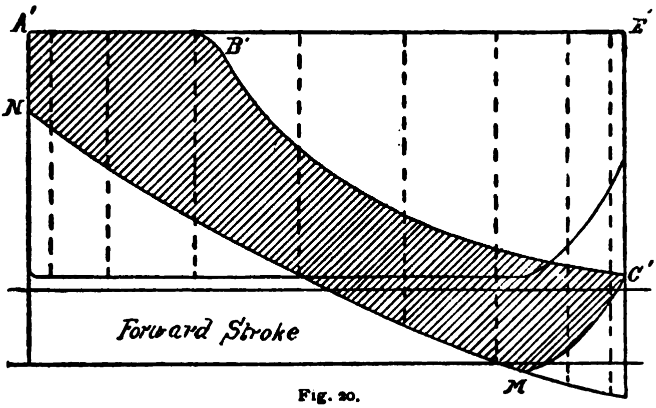



Fig. 20.

Fig. 21, is less likely to cause pounding than that in Fig. 18, and this in spite of the fact that in the case of Fig. 21 the reversal of pressure near end of stroke occurs while piston is in motion. A

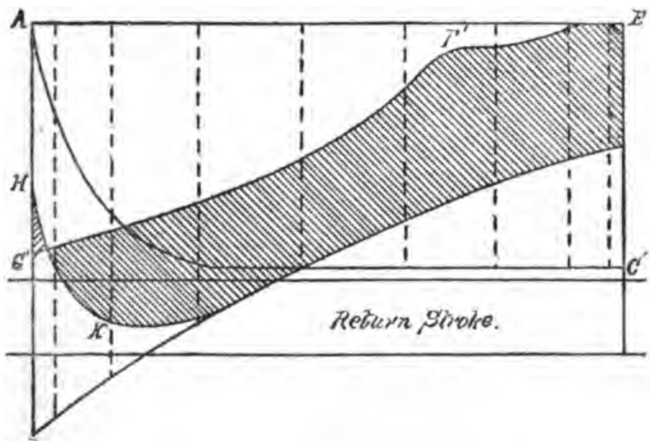



Fig. 21.

less lead in the case of Fig. 19 would be advantageous in cutting down the large initial pressure.

Another construction used by Grashof, is well worth noting. It is to lay off these resultant pressures at cross-head pin on the rectified crank-pin circle. The straight base will then represent time, and the rate at which the pressure changes can then be

easily seen. This will be particularly useful in judging of the rapidity with which the reversals of pressure occur.

We shall hereafter only make use of horizontal pressure or wrist-pin diagrams similar to those shown in Figs. 18 and 19.

## III c.

## Effect of Valve Setting on the Horizontal Pressure on Cross-Head Pin.

We give two indicator diagrams, Figs. 22 and 23, from opposite ends of the same cylinder to show the influence respectively of a late

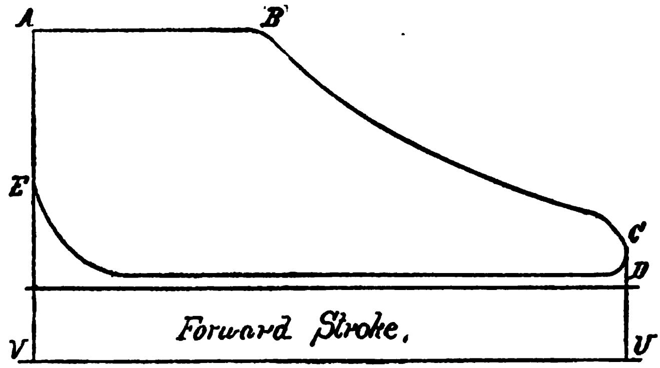



Fig. 22.

admission and too late an exhaust. By combining the diagram *ABCUV*, Fig. 22, with diagrams of counter-pressure *HIJK*, Fig. 23, we get the effective steam pressure diagram *ABCKJIH*, Fig. 24.

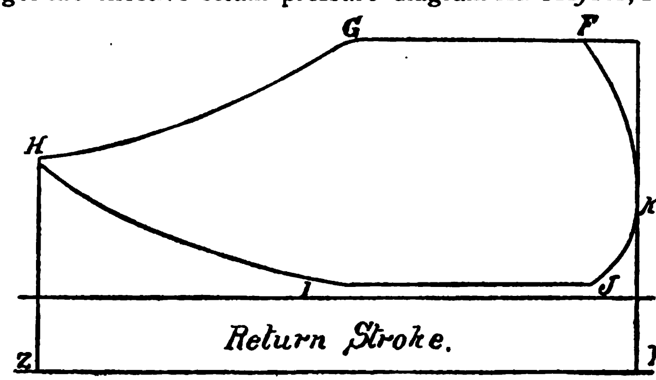



Fig. 23.

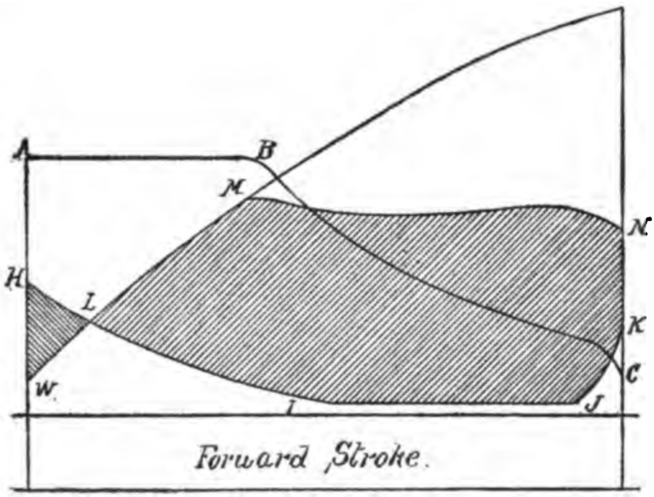



Fig. 24.

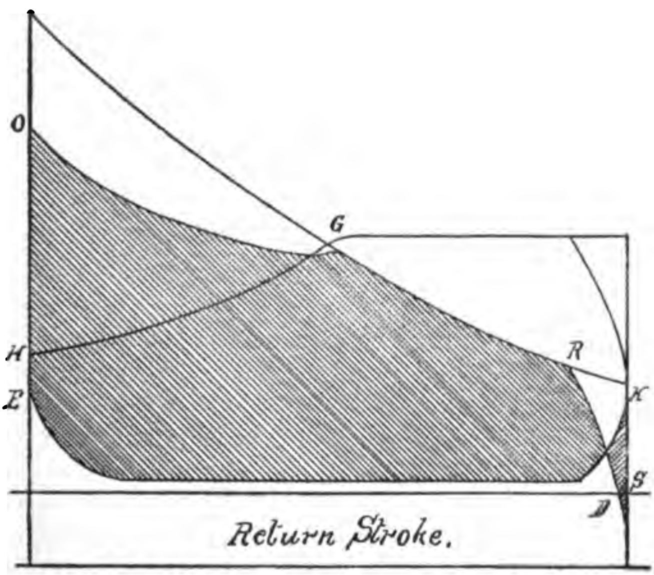



Fig. 25.

In like manner by combining the counter-pressure diagram *EDUV*, Fig. 22, with *HGFKYZ*, Fig. 23, we get the effective pressure diagram *KDEHG*, Fig. 25. By combining these real

pressure diagrams with the acceleration diagrams we get the horizontal pressures on cross-head pin, shown in Figs. 24 and 25 by shaded areas. Fig. 24 shows that, owing to the late exhaust *HI*, Fig. 23, there is a drag instead of a push on the crank during the first portion of the stroke, and a reversal of pressure on the crank while the piston is in rapid motion at *L*. This reversal will cause a shock or knock if there is any play in the joints of the mechanism. In like manner Fig. 25 shows a drag instead of a push on crank at beginning of stroke and a reversal of pressure at *D*, but here the piston motion is considerably slower than at *L*, so there will probably be no shock. The final pressure, *KS* Fig. 25, on piston at beginning of return stroke is about equal to that, *KN* Fig. 24, at end of forward stroke, and is a desirable result. It would be better however to accomplish the same result by more compression and earlier admission. Fig. 25 also shows the very unequal distribution of the horizontal pressures which attends a large cut-off when unaccompanied by suitable compression. By having the proper amount of compression at *S*, Fig. 18, not only may the reciprocating parts be brought to rest without shock, but all load can be taken off the crank pin while it is passing the dead center. This is not always done on account of the decided advantages gained by compressing nearly up to the boiler pressure, some of which advantages are the filling and heating of the hurtful space by compressed steam instead of the live, fresh, steam and the avoidance of such admission lines as *K,F* Fig. 23. Moreover, the reversal of pressure which accompanies considerable compression is not necessarily an evil, provided it takes place when the piston is moving slowly, *i. e.*, near the end of stroke where reversal naturally takes place, The ideal case is to have the piston unloaded while moving slowly (near end of stroke) and then let the driving pressure on opposite side increase gradually till after the reverse stroke has begun. Considerable compression and even a slight negative lead may be used to effect this desirable result, desirable, because of its freedom from pounding.

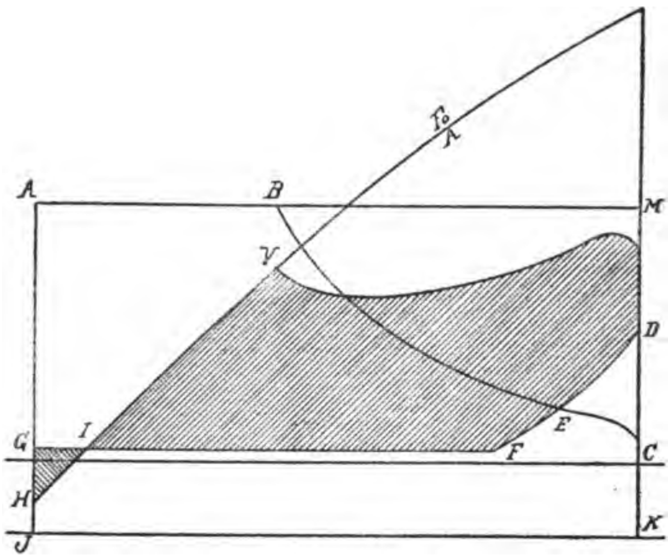



Fig. 26.

In Fig. 26 the ordinate $AH$ at the beginning of the acceleration curve $\frac{F_0}{A}$ is greater than the corresponding effective steam pressure $AG$; the result is that instead of there being a push upon the crank in the direction of its motion at the beginning of the stroke, there is a drag upon the crank, this condition continuing until the piston reaches the position $I$, where a reversal of pressure takes place causing shock and vibrations injurious to the durability of the machine, because at that end of the stroke there is considerable piston speed at point $I$.

In Fig. 27 the condition of affairs is still worse, for two reversals of pressure take place at $I'$ and $I''$, causing two shocks in rapid succession. In this last case, although the effective pressure $A'G'$ is greater than the initial pressure of acceleration $A'H'$, the ordinates $\left(= \frac{F}{A} = \frac{F_0}{A}(\cos \omega + \frac{R}{L} \cos 2\omega)\right)$ of the acceleration

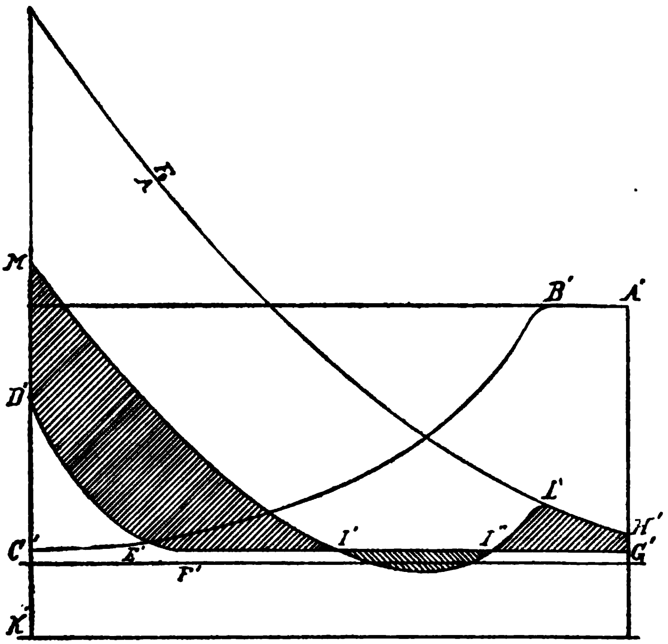



Fig. 27.

curve $\frac{F_0}{A}$ are still too large for the short cut-off $\frac{A'B'}{G'C'}$ and the small initial pressure $A'G'$. It is evident from the figures that if the effective pressure diagrams $ABCDEFGA$ and $A'B'C'D'E'F'G'A'$ of Figs. 26 and 27 are to remain unchanged, that is, if the mean effective pressure pin is to remain unchanged, the only remedy for the evils exhibited in Figs. 26 and 27 is to diminish the value of $\frac{F_0}{A}$.

When the engine varies between wide limits of cut-off, as in the present case, the value of $\frac{F_0}{A}$ is chosen so as to favor as much as possible the particular horse-power at which the engine ordinarily runs; in such a case it is desirable to know what is the minimum cut-off at which reversal of pressure will be avoided. This may be ascertained by the cut and try method, but it may

also be calculated with sufficient accuracy if we make the following suppositions, namely, that the curve of expansion is an equilateral hyperbola, that the best value for $\frac{F_0}{A}$ is

$$\frac{F_0}{A} = \frac{p_1 - 16}{2} *$$

and that the crank angle is about 60° when the horizontal pressure curve *EFG* (Fig. 28), is tangent to the back pressure line

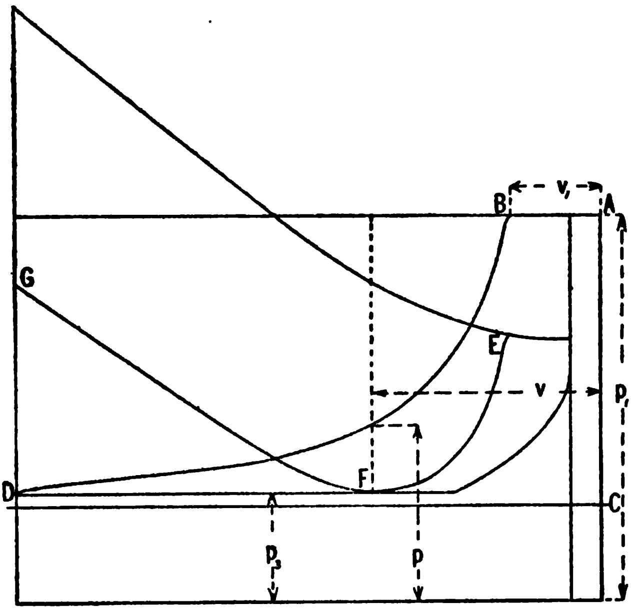



Fig. 28.

*DF.* As the reversals in question are more apt to occur during the return stroke we must use a formula

$$p - p_3 = \frac{F_0}{A}\left(\cos\omega - \frac{R}{L}\cos 2\omega\right) = \frac{v_1}{v}p_1 - p_3 \qquad (40)$$

corresponding to that stroke and introduce the assumptions just made.

---

* Radinger holds that $\frac{F_0}{A} = 2p_2$ = twice the terminal pressure at end of expansion will give the steadiest running of engine.

By deriving a value for $v$ from the piston travel $s$ for return stroke and allowing for clearance, we find that under the average conditions of running of high-speed engines, the minimum, real, cut-off should be greater than 0.10 to avoid reversals of pressure when piston is running swiftly. The approximate formula for this cut-off is :

$$\frac{AB}{DC} > \frac{1}{8p_1}\frac{F_0}{A}\left(\frac{R}{L}+1\right)+\frac{p_3}{4p_1} = \left(\frac{1}{16}-\frac{1}{p_1}\right)\left(\frac{R}{L}+1\right)+\frac{4}{p_1} \quad (41)$$

To find the number of horse-powers corresponding to this minimum, real, cut-off we look in Table III for value of $\frac{p''_{m1}}{p_1}$ and then transforming, Eq. 14, we get

$$H_1 = \left[1-(1+i)\left(\frac{p''_m}{p_1}-\frac{p''_{m1}}{p_1}\right)\frac{p_1}{p_m}\right]H$$

$$=\left[1-(1+i)\left(\frac{p''_m-p''_{m1}}{p_m}\right)\right]H$$

All the terms of the second member being known we can calculate readily the minimum horse-power at which the engine can be safely run at the given speed and pressure.

Find minimum cut-off and horse power for present engine. Construct on each of the six diagrams of effective steam pressures on piston two diagrams of resultant pressures on cross-head pin corresponding to $\frac{F_0}{A} = \frac{p_1 - 16}{2}$ and $\frac{F_0}{A} = 2p_2$, *i.e.* $\frac{F_0}{A} = 50$ and 40 lbs.

## III D.

## Conversion of Horizontal Pressures of Crank Pin into Tangential Pressures.

The horizontal component of the pressure of the rod against the crank-pin is evidently equal to the horizontal pressure of the cross-head pin. In this problem we are to find the rotative effect of a horizontal pressure on crank-pin.

Let $P^1$ represent horizontal pressure on cross-head pin.

$T$ represent force along connecting rod.

$t$ represent tangential pressure on crank pin.

then will
$$T = \frac{P^1}{\cos a} \tag{42}$$

and
$$t = T \sin(\omega + a) = P^1 \frac{\sin(\omega + a)}{\cos a} \tag{43}$$

$$\frac{t}{P^1} = \frac{\sin(\omega + a)}{\cos a} \tag{44}$$

If we substitute in this equation $\sin a = \frac{R}{L} \sin \omega$ and $\cos a = \sqrt{1 - \frac{R^2}{L_2} \sin^2 \omega}$ (see Eqs. 21 and 22) we shall have the means of calculating the ratios $\frac{t}{P^1}$ from the crank angle $\omega$. Table VIII on page 60 contains these ratios.

If we wish to determine the tangential pressures from given horizontal pressures by graphical means we can proceed as follows, Fig. 29.

In triangle $AKC$, Fig. 29, we have
$$\frac{KC}{AC} = \frac{KC}{R} = \frac{\sin(\omega + a)}{\cos a} \tag{45}$$

or
$$\frac{t}{P^1} = \frac{KC}{R} = \frac{w}{v} \tag{46}$$

hence if on any scale, say 20 lbs. to the inch, we lay off $DE = P^1$, join $K$ with $D$ and prolonging $KD$ till it intersects at $F$ the perpendicular to $DE$ erected at $E$, we will have in the two similar triangles, $DKC$ and $DFE$, the proportion $FE : P^1 :: KC : R$, *i. e.* $t = FE$.

# TABLE VIII.

## ROTATIVE EFFECT OF A UNIT OF HORIZONTAL PRESSURE ON CRANK.

To find actual tangential pressure (or rotative effect) on crank, multiply tabular quantity by resultant horizontal pressure on crank or cross-head pin.

Forward stroke is towards, and return stroke away from, crank shaft.

To find wrist-pin velocity multiply tabular values by crank-pin velocity.

| Crank Angles. | | Connecting Rod ÷ Crank = | | | | | |
|---|---|---|---|---|---|---|---|
| Forw'd. | Return. | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 | ∞ |
| 5 | 175 | .1089 | .1064 | .1045 | .1030 | .1016 | .0832 |
| 10 | 170 | .2164 | .2117 | .2079 | .2047 | .2022 | .1737 |
| 15 | 165 | .3215 | .3145 | .3089 | .3054 | .3005 | .2588 |
| 20 | 160 | .4227 | .4136 | .4065 | .4019 | .3957 | .3420 |
| 25 | 155 | .5189 | .5081 | .4995 | .4925 | .4866 | .4226 |
| 30 | 150 | .6091 | .5968 | .5870 | .5791 | .5724 | .5000 |
| 35 | 145 | .6923 | .6788 | .6682 | .6596 | .6523 | .5736 |
| 40 | 140 | .7675 | .7533 | .7421 | .7329 | .7253 | .6428 |
| 45 | 135 | .8341 | .8195 | .8081 | .7988 | .7910 | .7071 |
| 50 | 130 | .8914 | .8771 | .8657 | .8564 | .8488 | .7660 |
| 55 | 125 | .9392 | .9253 | .9144 | .9055 | .8982 | .8192 |
| 60 | 120 | .9769 | .9641 | .9540 | .9458 | .9390 | .8660 |
| 65 | 115 | 1.0046 | .9932 | .9842 | .9769 | .9709 | .9063 |
| 70 | 110 | 1.0224 | 1.0127 | 1.0052 | .9990 | .9939 | .9397 |
| 75 | 105 | 1.0304 | 1.0228 | 1.0169 | 1.0121 | 1.0082 | .9659 |
| 80 | 100 | 1.0289 | 1.0237 | 1.0199 | 1.0164 | 1.0137 | .9848 |
| 85 | 95 | 1.0186 | 1.0160 | 1.0139 | 1.0127 | 1.0109 | .9962 |
| 90 | 90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 95 | 85 | .9738 | .9764 | .9785 | .9797 | .9816 | .9962 |
| 100 | 80 | .9407 | .9460 | .9500 | .9532 | .9559 | .9848 |
| 105 | 75 | .9016 | .9091 | .9150 | .9198 | .9237 | .9659 |
| 110 | 70 | .8571 | .8667 | .8743 | .8804 | .8855 | .9397 |
| 115 | 65 | .8080 | .8194 | .8285 | .8357 | .8418 | .9063 |
| 120 | 60 | .7552 | .7680 | .7781 | .7863 | .7931 | .8660 |
| 125 | 55 | .6992 | .7130 | .7239 | .7328 | .7401 | .8192 |
| 130 | 50 | .6407 | .6550 | .6664 | .6756 | .6833 | .7660 |
| 135 | 45 | .5801 | .5946 | .6061 | .6155 | .6232 | .7071 |
| 140 | 40 | .5181 | .5323 | .5435 | .5527 | .5603 | .6428 |
| 145 | 35 | .4549 | .4683 | .4790 | .4876 | .4949 | .5736 |
| 150 | 30 | .3909 | .4032 | .4130 | .4209 | .4276 | .5000 |
| 155 | 25 | .3264 | .3371 | .3458 | .3528 | .3586 | .4226 |
| 160 | 20 | .2614 | .2704 | .2776 | .2821 | .2884 | .3420 |
| 165 | 15 | .1962 | .2032 | .2088 | .2123 | .2171 | .2588 |
| 170 | 10 | .1309 | .1356 | .1394 | .1426 | .1451 | .1737 |
| 175 | 5 | .0655 | .0679 | .0698 | .0714 | .0727 | .0872 |

The value of $t$ will be on the same scale as $P^1$. If we determine in like manner, for the same pressure $P' = DE$, the tangential pressures corresponding to the various crank positions of Fig. 29 and lay them off on the extreme left or right hand ordinates of

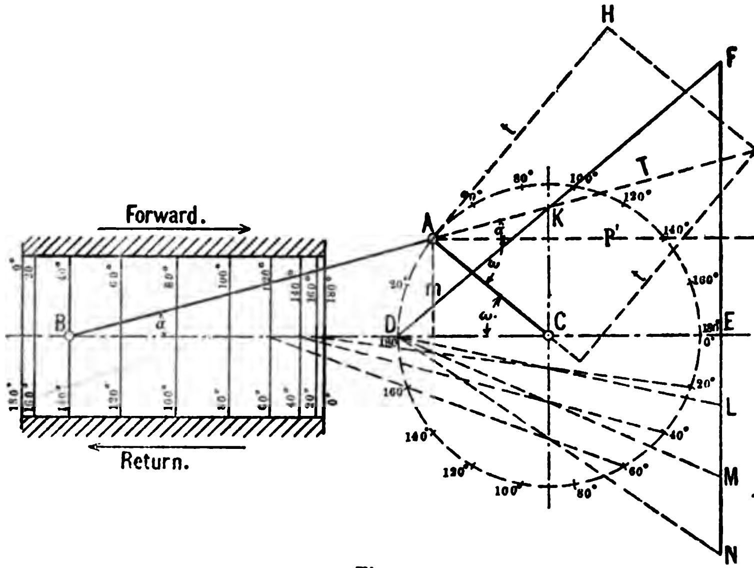
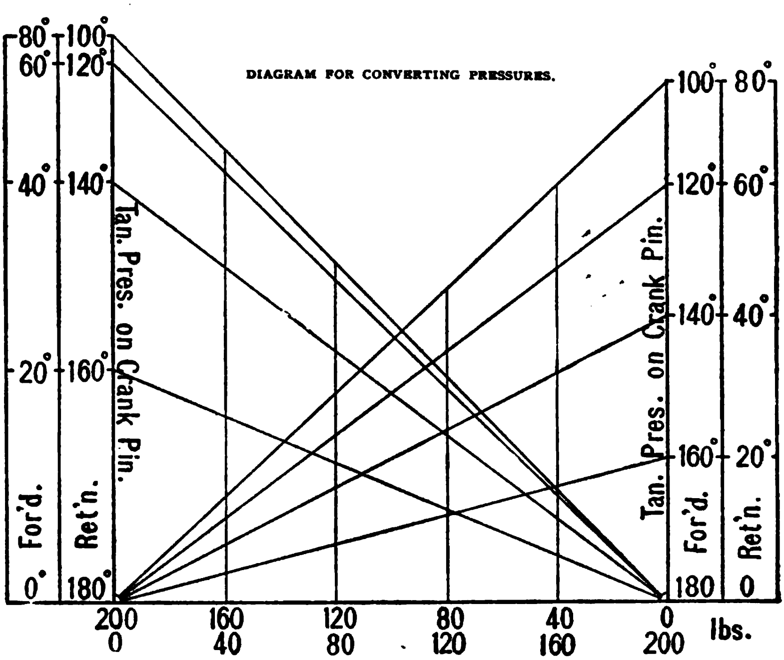



Fig. 29.

the section paper employed, then drawing diagonals to the assumed zero of cross-head pin pressures we will get a diagram similar to Fig. 30 from which we can determine the tangential pressures for any value of $P^1$. In Fig. 29 the ratio of length of connecting rod to length of crank was taken at $2\frac{1}{2}$. In the diagram to be drawn on section paper assume this ratio to be $= 6$, also $DE = P^1 = 200$ lbs.

Lay off, in Fig. 30, the tangential pressures on a scale of 20 lbs. to the inch, and the horizontal or cross-head pressures on a scale of 20 lbs. to the inch. The tangential pressures can also be obtained in the ordinary manner by resolving the force acting along the connecting rod into two components respectively tan-

gent and normal to the path of the crank pin. But this would involve a good deal more labor than the present method. Figs. 29 and 30 are quite accurately drawn and the method of obtaining 30 from 29 can therefore be easily followed. Thus *EL*, *EM* and *EN* are the tangential pressures corresponding respectively to the angular positions 20°, 40° and 60° of crank for ***return***




Horizontal pressure on Cross-head pin.
Resultant of effective steam pressure and inertia of reciprocating parts.

Fig. 30.

*stroke* and to 160°, 140° and 120° for the *forward stroke*. The quickest method of obtaining a diagram like Fig. 30 would be to calculate the tangential pressures from Table VIII, assuming a constant horizontal pressure of 200 lbs. on cross-head pin. It is sufficiently evident from Fig. 30 how the tangential pressures for other and smaller resultant pressures on cross-head pin can be obtained.

## III E.

## CONSTRUCTION OF DIAGRAM OF TANGENTIAL PRESSURES ON CRANK PIN.

We are now ready to construct the tangential pressure diagrams from the diagrams of resultant pressures on cross-head pin corresponding to various values of $\frac{F_o}{A}$. We proceed as in the construction of Fig. 13. By means of the diagram, Fig. 30, we first transform the horizontal pressures into tangential pressures, and then lay off these tangential pressures as prolongations of the radii of the crank-pin circle (or of any other convenient circle), the latter forming the base of the radial ordinates. A curve is next drawn through the extremities of these ordinates and then a circle concentric with—and outside of—the crank-pin circle, is drawn, the difference between the two circumferences is equal to the mean tangential pressure, which as before is equal to $p_t = 0.6366\, p_m$, Eq. 17. The deviations of the curve (drawn through the extremities of the ordinates representing the tangential pressures) from the circle of tangential pressures will show the irregularity of the driving power. The more closely the tangential pressure curve approximates to the mean-tangential-pressure-circle, the steadier will the engine run. Figs. 31 and 32 show two different methods of representing the tangential pressures. In the former figure the tangential pressures are laid off from the crank-pin circle itself; in the latter from the rectified semi-circle. Fig. 31 has the advantage of showing the phases of resistance *CMNO* and *AQS* and the curve of tangential pressures in a continuous manner, but it is subject to the following very slight disadvantage, viz: that the area enclosed by *AQRMCBA* does not represent with great accuracy the work done in a semi-revolution, the inaccuracy being due to the divergence of the radial ordinates. The objection does not apply to Fig. 32, where the area *AVECBA* is exactly equal to the

area $AF'DCA$ = work done in one semi-revolution; moreover by adding the shaded portions $CD'E'$ $(=CDE)$ and $AF'G'$ as shown in Fig. 32, the objection that this mode of representation does not represent the phases of resistance in a continuous man-

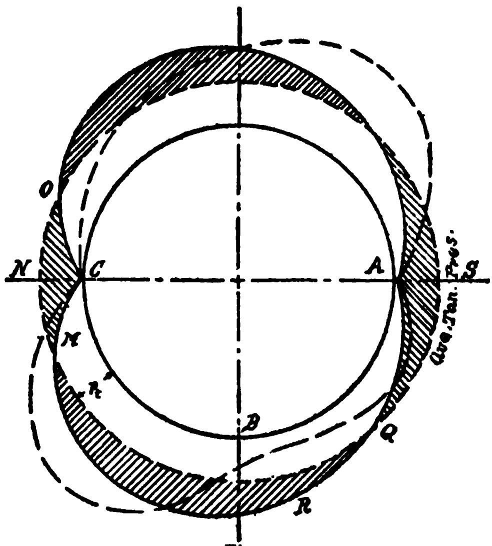



Fig 31.

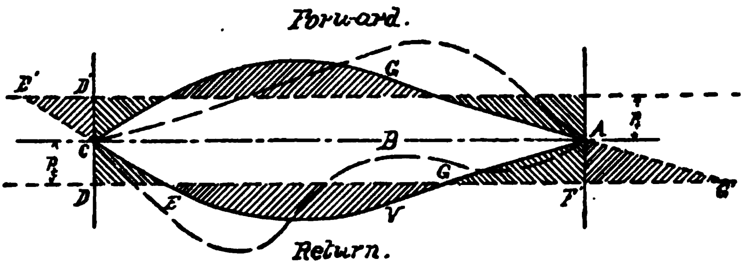



Fig. 32.

ner disappears. If the work has been done correctly it will be found that in Fig. 31 the area of the shaded portion $QRM$ = area $CMN$ + area $QAS$, nearly, and that in Fig. 32 the phase of excess $GVE = AGF' + DEC$ exactly.

Inspection of either of these figures shows that the tangential pressure line (shown by full line) represents steadier running than

the tangential pressures corresponding to greater piston speed or heavier reciprocating parts (shown by broken line).

Draw for the minimum, rated and maximum power of the proposed engine, separately two tangential pressure diagrams, one like Fig. 31 and the other like Fig. 32. On each diagram draw two tangential pressure curves corresponding to the resultant, or horizontal, pressure on cross-head pin which were drawn for the assumed values: $\frac{F_o}{A} = \frac{p_1 - 16}{2} =$ about 40 and $\frac{F_o}{A} = 2p_2 =$ about 50 lbs. When these have all been drawn, inspect the result and see if a different value than those chosen for $\frac{F_o}{A}$ would not give a more uniform tangential pressure—*i. e.* steadier motion. If so, take the new value of $\frac{F_o}{A}$ and draw corresponding acceleration curves for both the forward and the return stroke, and superimpose these upon the diagrams of effective steam pressures already drawn and combine them as before to form diagrams of resultant pressures on cross-head pin. Then convert these last diagrams into tangential pressure diagrams and again inspect the latter diagrams to see if any improvement can be effected. This process is to be continued till that value for $\frac{F_o}{A}$ has been found which gives the most uniform tangential pressure for the normal, rated, horse-power of the proposed engine. This most favorable value of $\frac{F_o}{A}$ for the rated power should however be examined with reference to the minimum power required, to see whether it then causes reversals of pressure. When this is the case some smaller value of $\frac{F_o}{A}$ less favorable to the normal or rated power should be chosen for the engine. In this connection we may quote the following from *Rigg's Steam Engine.*

## CHOICE OF SPEED AND LOAD.

"There are, then, three elements which can be adjusted to each other, namely, steam pressure, including the rate of expansion, weight of reciprocating parts, and number of revolutions per minute, or piston velocity; any of these elements can be altered at the option of the designer of the engine, and the problem to be solved is not to transfer the strains from one end of the stroke to the other, not to work steadily with the highest expansion, not to produce a regular uniform horizontal pressure on the crank. But the end and object of all these calculations and changes in the old acknowledged rules for the construction of engines is, by working with that amount of expansion which practical experience proves economical, to obtain as nearly as possible a uniform tangential pressure on the crank. When this is done an engine will drive its load steadily and well, and the influence of the reciprocating parts has a most direct bearing on this most important subject.

In applying the foregoing reasoning it is necessary to exercise a judicious choice in balancing the evils which arise from a too low or too high speed, and so to decide a rate at which an engine can be run to the best advantage. When the load upon the engine is regular it is comparatively easy to do this, but in a great majority of cases the load is often varying, and the speed is bound to remain constant. Thus, it is necessary to select a load on the engine which shall meet most requirements, and then the indicator diagram either actually taken or assumed will form the principal datum required."

To these considerations might be added another, namely that to avoid pounding or shocks at crank- or wrist-pin, the driving pressures should change as gradually as possible from one side to the other of piston, the change taking place as near the dead point as possible. By using considerable compression or even a slightly negative lead this gradual change may be accomplished for both ends of the stroke.

## IV.

## Determination of Diameter of Cylinder $D$, Length of Stroke $S$, Revolutions per Minute $N$ or Piston Speed per Minute $w^1$ and Weight of Reciprocating Parts $W$ by Means of the Tables or Formulas.

$$\frac{F_0}{A} = 0.0000142 \frac{W}{A} SN^2 = .000085 \frac{W}{A} Nw^1$$

$$\frac{F_0}{A} = 3.63 \frac{H}{p_m} \frac{W}{A} \frac{N}{D^2} = 21.8 \frac{w^1}{SD^2} \frac{W}{A} \frac{H}{p_m}. \tag{47}$$

$$\frac{H}{p_m} = 0.0000238\, w^1 D^2 \tag{48}$$

$$\frac{H}{p_m} = 0.00000396\, NSD^2 \tag{49}$$

$$\frac{\frac{F_0}{A}}{\frac{H}{p_m}} = 3.63 \frac{W}{A} \frac{N}{D^2} = 21.8 \frac{W}{A} \frac{w^1}{SD^2} \tag{50}$$

$H$ = number of indicated H. P.

$S$ = length of stroke in inches.

$\frac{W}{A}$ = weight of reciprocating parts per □″ of piston, weight of rod included.

$D$ = diameter of cylinder in inches.

$w^1$ = average speed of piston in feet per minute.

$N$ = revolutions per minute.

$p_m$ = mean effective pressure per □″ of piston.

$\frac{F_0}{A}$ = average force accelerating reciprocating parts when crank is on dead centers.

# TABLE IX.*

## VALUE OF $\frac{H}{p_m}$ OR INDICATED H. P. FOR EACH POUND OF MEAN EFFECTIVE PRESSURE PER □″ OF PISTON AREA.

Quantities in table were obtained by neglecting area of piston rod and assuming that the same mean effective pressure ($p_m$) existed on both sides of the piston area; consequently the tabular quantities are about 1½ % and ½ % too large for small and large cylinders respectively. To correct for piston rod multiply tabular quantities by $[1 - \frac{1}{2}\left(\frac{d^1}{D}\right)^2]$ where $D$ = diameter of cylinder and $d^1$ = diameter of piston rod.

| Diameter of Cylinder. | Average speed of piston in feet per minute. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 700 | 750 | 800 | 850 | 900 | 950 | 1000 | 1050 | 1100 | 1150 | 1200 |
| 6 | .600 | .643 | .685 | .728 | .771 | .814 | 857 | .900 | .942 | .985 | 1.028 |
| 6½ | • .704 | .754 | .804 | .855 | .905 | .955 | 1.006 | 1.056 | 1.106 | 1.156 | 1.207 |
| 7 | .816 | .875 | .933 | .991 | 1.050 | 1.108 | 1.116 | 1.225 | 1.283 | 1.341 | 1.399 |
| 7½ | .937 | 1.004 | 1.071 | 1 138 | 1.255 | 1.272 | 1 339 | 1.406 | 1 473 | 1.540 | 1.607 |
| 8 | 1.066 | 1.142 | 1 219 | 1.295 | 1.371 | 1.447 | 1.523 | 1.599 | 1.676 | 1.752 | 1.828 |
| 8½ | 1.204 | 1.290 | 1.376 | 1.462 | 1.548 | 1.634 | 1.719 | 1.806 | 1.892 | 1.977 | 2.063 |
| 9 | 1 349 | 1.446 | 1.542 | 1.639 | 1.735 | 1 831 | 1 928 | 2 021 | 2.121 | 2 217 | 2.313 |
| 9½ | 1.504 | 1.611 | 1.718 | 1.826 | 1.933 | 2.041 | 2.148 | 2 255 | 2.363 | 2.470 | 2.578 |
| 10 | 1.666 | 1.785 | 1 904 | 2.023 | 2.142 | 2.261 | 2 380 | 2.499 | 2.618 | 2.737 | 2.856 |
| 10½ | 1.874 | 2.008 | 2.142 | 2.276 | 2.420 | 2.544 | 2.678 | 2.811 | 2.945 | 3.079 | 3.213 |
| 11 | 2.016 | 2.160 | 2.304 | 2.448 | 2 592 | 2.736 | 2.880 | 3.024 | 3.168 | 3.312 | 3.456 |
| 11½ | 2.203 | 2.361 | 2.518 | 2.675 | 2.833 | 2.990 | 3.148 | 3.305 | 3.462 | 3.620 | 3.777 |
| 12 | 2.399 | 2.570 | 2.742 | 2.913 | 3.084 | 3 256 | 3.427 | 3.599 | 3 770 | 3.941 | 4.113 |
| 13 | 2.816 | 3-017 | 3.218 | 3.419 | 3.620 | 3.821 | 4 022 | 4.223 | 4.424 | 4.626 | 4.827 |
| 14 | 3.265 | 3.500 | 3.732 | 3 965 | 4.198 | 4 432 | 4.665 | 4.896 | 5.231 | 5.365 | 5.597 |
| 15 | 3.749 | 4.016 | 4.284 | 4.552 | 4.820 | 4 987 | 5.355 | 5.623 | 5.891 | 6.158 | 6.426 |
| 16 | 4.265 | 4.570 | 4;774 | 5.179 | 5 484 | 5.788 | 6.093 | 6.397 | 6.696 | 7.007 | 7.311 |
| 17 | 4.815 | 5.159 | 5.503 | 5.846 | 6.190 | 6.534 | 6.878 | 7.222 | 7.566 | 7.810 | 8.254 |
| 18 | 5 397 | 5.783 | 6.169 | 6.555 | 6.940 | 7.326 | 7.711 | 8 087 | 8.482 | 8.868 | 9.253 |
| 20 | 6.664 | 7 140 | 7 616 | 8.092 | 8.568 | 9.044 | 9.520 | 9.996 | 10.472 | 10.948 | 11.424 |
| 22 | 8.063 | 8.639 | 9.215 | 9.791 | 10.367 | 10.943 | 11.519 | 12 095 | 12.577 | 13.147 | 13.823 |
| 24 | 9.596 | 10 282 | 10.967 | 11.652 | 12.338 | 13.023 | 13.709 | 14.394 | 15.080 | 15.765 | 16.451 |
| 26 | 11.162 | 12.067 | 12.871 | 13.675 | 14.480 | 15.284 | 16.089 | 16.893 | 17.998 | 18.502 | 19.307 |
| 28 | 13.061 | 13.984 | 14.927 | 15.860 | 16.793 | 17.726 | 18.659 | 19.592 | 20.525 | 21.458 | 22.391 |
| 30 | 14.994 | 16.065 | 17.136 | 18.207 | 19.278 | 20.349 | 21.420 | 22.491 | 23.562 | 24.633 | 25.704 |
| 32 | 17.060 | 18.278 | 19.497 | 20.716 | 21.934 | 23.153 | 24.371 | 25.590 | 26.708 | 28.027 | 29.245 |
| 34 | 19.259 | 20.635 | 22.010 | 23.386 | 24.762 | 26.137 | 27.513 | 28.888 | 30.264 | 31.640 | 33.035 |
| ·36 | 21.591 | 22.134 | 24.676 | 26.218 | 27.760 | 29.303 | 30.845 | 32.380 | 33.929 | 35.472 | 37.014 |
| 38 | 24.057 | 25.775 | 27.494 | 29.212 | 30.930 | 32.649 | 34.367 | 36.086 | 37.804 | 39.532 | 41.339 |
| 40 | 26.656 | 28.560 | 30.464 | 32.368 | 34.272 | 36.176 | 38.080 | 39.984 | 41.888 | 43.792 | 45.696 |

* Table IX continued on page 69.

## VALUE OF $\frac{H}{p_m}$ OR INDICATED H. P. FOR EACH POUND OF MEAN EFFECTIVE PRESSURE PER □″ OF PISTON AREA.

*This part taken from Riggs' Steam Engine.*

| Diameter of Cylinder. | Average speed of piston in feet per minute. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 240 | 300 | 350 | 400 | 450 | 500 | 550 | 600 | 650 | 750 |
| 4 | .091 | .114 | .133 | .162 | .171 | .19 | .209 | .228 | .247 | .285 |
| 4½ | .115 | .144 | .168 | .192 | .216 | .24 | .264 | .288 | .312 | .36 |
| 5 | .144 | .18 | .21 | .24 | .27 | .30 | .33 | .36 | .39 | .45 |
| 5½ | .173 | .216 | .252 | .288 | .324 | .36 | .396 | .432 | .468 | .54 |
| 6 | .205 | .256 | .299 | .342 | .385 | .428 | .471 | .513 | .555 | .641 |
| 6½ | .245 | 307 | .391 | .409 | .461 | .512 | .563 | .614 | .698 | .800 |
| 7 | .279 | .348 | .408 | .466 | .524 | .583 | .641 | .699 | .756 | .874 |
| 7½ | .321 | .401 | .468 | .534 | .602 | .669 | .735 | .802 | .869 | 1.002 |
| 8 | .365 | .456 | .532 | .608 | .685 | .761 | .837 | .912 | .989 | 1.121 |
| 8½ | .413 | .516 | .602 | .688 | .774 | .86 | .946 | 1.032 | 1.118 | 1.29 |
| 9 | .462 | .577 | .674 | .770 | .866 | .963 | 1.059 | 1.154 | 1.251 | 1.444 |
| 9½ | .515 | .644 | .751 | .859 | .966 | 1.074 | 1.181 | 1.288 | 1.395 | 1.610 |
| 10 | .571 | .714 | .833 | .952 | 1.071 | 1.190 | 1.309 | 1.428 | 1.547 | 1.785 |
| 10½ | .63 | .787 | .919 | 1.050 | 1.181 | 1.313 | 1.444 | 1.575 | 1.706 | 1.969 |
| 11 | .691 | .864 | 1.008 | 1.152 | 1.296 | 1.44 | 1.584 | 1.728 | 1.872 | 2.160 |
| 11½ | .754 | .943 | 1.1 | 1.257 | 1.414 | 1.572 | 1.729 | 1.886 | 2.043 | 2.357 |
| 12 | .820 | 1.025 | 1.195 | 1.366 | 1.540 | 1.708 | 1.880 | 2.050 | 2.222 | 2.564 |
| 13 | .964 | 1.206 | 1.407 | 1.608 | 1.809 | 2.01 | 1.211 | 2.412 | 2.613 | 3.015 |
| 14 | 1.119 | 1.398 | 1.631 | 1.864 | 2.097 | 2.331 | 2.564 | 2.797 | 3.029 | 3.495 |
| 15 | 1.285 | 1.606 | 1.873 | 2.131 | 2.409 | 2.677 | 2.945 | 3.212 | 3.479 | 4.004 |
| 16 | 1.461 | 1.827 | 2.131 | 2.436 | 2.741 | 3.045 | 3.349 | 3.654 | 3.958 | 4.567 |
| 17 | 1.643 | 2.054 | 2.396 | 2.739 | 3 081 | 3.424 | 3.766 | 4.108 | 4.450 | 5.135 |
| 18 | 1.849 | 2.312 | 2 697 | 3.083 | 3.468 | 3.854 | 4.239 | 4.624 | 5.009 | 5.78 |
| 19 | 2.061 | 2.577 | 3.006 | 3.436 | 3.865 | 4.295 | 4.724 | 5.154 | 5.583 | 6.442 |
| 20 | 2.292 | 2.855 | 3.331 | 3.807 | 4.285 | 4 759 | 5.234 | 5.731 | 6.186 | 7.138 |
| 21 | 2.518 | 3.148 | 3.672 | 4.197 | 4.722 | 5.247 | 5.771 | 6 296 | 6.820 | 7.869 |
| 22 | 2.764 | 3.455 | 4.031 | 4.607 | 5.183 | 5.759 | 6.334 | 6.911 | 7.486 | 8.638 |
| 23 | 3.021 | 3.776 | 4.404 | 5.035 | 5.664 | 6.294 | 6.923 | 7.552 | 8.181 | 9.44 |
| 24 | 3.289 | 4.111 | 4.797 | 5.482 | 6.167 | 6.853 | 7.538 | 8.223 | 8.908 | 10.279 |
| 25 | 3.569 | 4.461 | 5.105 | 5.948 | 6.692 | 7.436 | 8.179 | 8.923 | 9.566 | 11.053 |
| 26 | 3.861 | 4.826 | 5.630 | 6.435 | 7.239 | 8.044 | 8.848 | 9.652 | 10.456 | 12.065 |
| 27 | 4.159 | 5.199 | 6.066 | 6.932 | 7.799 | 8.666 | 9.532 | 10.399 | 11.265 | 12.998 |
| 28 | 4.477 | 5.596 | 6.529 | 7.462 | 8.395 | 9.328 | 10.261 | 11.193 | 12.125 | 13.991 |
| 29 | 4.805 | 6.006 | 7.007 | 8.008 | 9.009 | 10.01 | 11.011 | 12.012 | 13.013 | 15.015 |
| 30 | 5.141 | 6.426 | 7.497 | 8.568 | 9.639 | 10.71 | 11.781 | 12.852 | 13.923 | 16.065 |
| 31 | 5.486 | 6.865 | 8.001 | 9.144 | 10.287 | 11.43 | 12.573 | 13.716 | 14.866 | 17.145 |
| 32 | 5.846 | 7 308 | 8.526 | 9.744 | 10.962 | 12.18 | 13.398 | 14.616 | 15.834 | 18.270 |
| 33 | 6.216 | 7.770 | 9.065 | 10.360 | 11.655 | 12.959 | 14.245 | 15.54 | 16.835 | 19.425 |
| 34 | 6.59 | 8.238 | 9 611 | 10.984 | 12.357 | 13.73 | 15.103 | 16.476 | 17.849 | 20.595 |
| 35 | 6.993 | 8.742 | 10.199 | 11.656 | 13.113 | 14.57 | 16 027 | 17.484 | 18.941 | 21.855 |
| 36 | 7.401 | 9.252 | 10.794 | 12.336 | 13.878 | 15.42 | 16 962 | 18.504 | 20.046 | 23.130 |
| 37 | 7.819 | 9.774 | 11.403 | 13.033 | 14.861 | 16.29 | 17.919 | 19.548 | 21.177 | 24.435 |
| 38 | 8.246 | 10.308 | 12.026 | 13.744 | 15.462 | 17.18 | 18.896 | 20.616 | 22.334 | 25.770 |
| 39 | 8.648 | 10.86 | 12.67 | 14.48 | 16.29 | 18.1 | 19.91 | 21.62 | 23.53 | 27.15 |
| 40 | 9.139 | 11.424 | 13.328 | 15.232 | 17.136 | 19.04 | 20.944 | 22.848 | 24.752 | 28.560 |
| 41 | 9.604 | 12.006 | 14.007 | 16.008 | 18.009 | 20.00 | 22.011 | 24.012 | 26.013 | 30.015 |
| 42 | 10.065 | 12.594 | 14.693 | 16.792 | 18.901 | 20.99 | 23.089 | 25.188 | 27.287 | 31.485 |
| 43 | 10.56 | 13.20 | 15.4 | 17.6 | 19.8 | 22.0 | 24.2 | 26.4 | 28.6 | 33.0 |
| 44 | 11.046 | 13.818 | 16.121 | 18.424 | 20.727 | 23.03 | 25.333 | 27.636 | 29.939 | 34.545 |
| 45 | 11.563 | 14.454 | 16.863 | 19.272 | 21.681 | 24.09 | 26.399 | 28.908 | 31.317 | 36.135 |
| 46 | 12.086 | 15.128 | 17.626 | 20.144 | 22.662 | 25.18 | 27.698 | 30.216 | 32.754 | 37.770 |
| 47 | 12.614 | 15.768 | 18.396 | 21.024 | 23.652 | 26.28 | 28.908 | 31.536 | 34.164 | 39.420 |
| 48 | 12.846 | 16.446 | 19.187 | 21.928 | 24.669 | 27.41 | 30.151 | 32.152 | 35.633 | 41.115 |
| 49 | 12.913 | 17.142 | 19.999 | 22.856 | 25.713 | 28.57 | 31.427 | 34.284 | 37.141 | 42.855 |
| 50 | 14.28 | 17.85 | 20.825 | 23.8 | 26.775 | 29.75 | 32.725 | 35.7 | 38.675 | 44.625 |
| 51 | 14.832 | 18.54 | 21.665 | 24.76 | 27.855 | 30.95 | 34.045 | 37.08 | 40.205 | 46.425 |
| 52 | 15.437 | 19.296 | 22.512 | 25.728 | 28.944 | 32.16 | 35.376 | 38.592 | 41.808 | 48.240 |
| 53 | 16.041 | 20.052 | 23.394 | 26.736 | 30.078 | 33.42 | 36.762 | 40.104 | 43.446 | 50.13 |
| 54 | 16.656 | 20.82 | 24.29 | 27.76 | 31.23 | 34.7 | 38.17 | 41.64 | 45.11 | 52.05 |
| 55 | 17.275 | 21.594 | 25.193 | 28.792 | 32.391 | 35.99 | 39.589 | 43.188 | 46.787 | 53.985 |
| 56 | 17.909 | 22.386 | 26.117 | 29.848 | 33.579 | 37.31 | 41.041 | 44.772 | 48.503 | 55.965 |
| 57 | 18.557 | 23.296 | 27.062 | 30.928 | 34.794 | 38.66 | 42.526 | 46.392 | 50.258 | 57.99 |
| 58 | 19.214 | 24.018 | 28.021 | 32.024 | 36.027 | 40.03 | 44.033 | 48.036 | 52.039 | 60.045 |
| 59 | 19.902 | 24.852 | 28.994 | 33.136 | 37.278 | 41.42 | 45.562 | 49.704 | 53.846 | 62.13 |
| 60 | 20.558 | 25.698 | 29.981 | 34.264 | 38.547 | 42.83 | 47.113 | 51.396 | 55.679 | 64.245 |

The formula shows that for a given horse-power ($= H$) and size ($= S \times D$) of engine, the value of $\frac{F_0}{A}$ may be made to vary with the three factors $p_m$, $\frac{W}{A}$ and $w^1$ or $N$.

The principal quantities which effect the value of $p_m$ are the initial pressure $p_1 = AJ$ and the cut-off $\frac{AB}{JK}$; the most important considerations governing the choice of these two quantities will be given a little later. The value $\frac{W}{A}$ cannot become smaller than a certain value $\frac{W^1}{A}$ prescribed by the strength and stiffness of the reciprocating parts. The following limits employed in practice for high speed engines may be of service:

| | | | | |
|---|---|---|---|---|
| $w'$ | varies | ordinarily | from | 600 to 1000 ft. per m. |
| $\frac{W}{A}$ | " | " | " | 2 to 6 lbs. |
| $\frac{S}{D}=$ | " | " | " | 0.8 to 2. |

The larger values of $w'$ and the smaller values of $\frac{S}{D}$ are usually employed for large engines. In using the following tables we first assume a diameter and with our given value of $\frac{H}{p_m}$ (= I. H. P. for each pound of mean effective pressure per □″ of piston area) we find from table IX the corresponding piston speed, or we assume a given piston speed and find from it and $\frac{H}{p_m}$ the corresponding diameter. If we now assume a given ratio of $\frac{S}{D}$ we can get from Table X the number of revolutions $N$ and $\frac{W}{A}$, or if

we assume $\frac{W}{A}$ we can, from the same table, get $S$ and $N$. It should be noticed that within certain limits we can diminish $N$ and yet increase $S$ without disturbing other established quantities.

## V.

## Preliminary Estimate of Dimensions of Reciprocating Parts.

Calculate the weight of hollow piston head, the proportions being assumed like those given in Reuleaux's *Constructeur*, Fig. 798, p. 746, but with an additional plate at the lower end so that the head will be closed at both ends.

Also estimate the weight of piston rod from the table on p. 751 of Reuleaux's *Constructeur*, remembering that the length of piston rod is greater than the stroke by, length of piston head, length of stuffing box, length of cylinder cover and amount that enters cross-head. The weight of the cross-head in the present design is small, and in this rough estimate may be taken at 35 lbs. The weight of the connecting rod may also be got at roughly by supposing it to be of uniform rectangular cross-section throughout, its dangerous cross-section being calculated from the formulas given at the end of Klein's *Elements of Machine Design*. Assume the ratio of depth of connecting rod to its width to be $\frac{h}{b} = 2$. Adding together the weight of piston head, piston rod and the connecting rod, and then dividing the sum by the area of the piston we get the quantity $\frac{W'}{A}$ which must be equal or less than the quantity $\frac{W}{A}$ prescribed, by the chosen dimensions and by the speeds or value $\frac{F_0}{A}$.

# TABLE X.

VALUES OF $\frac{W}{A} \div \frac{F_o}{A} = \frac{W}{F_o} = \frac{1}{.0000142SN^2}$

TO FIND $\frac{W}{A}$ FROM TABLE MULTIPLY BY ASSUMED VALUE OF $\frac{F_o}{A}$.

$\frac{W}{A}$ = weight of reciprocating parts per □″ of piston.

$S$ = length of stroke in inches. $N$ = revolution per minute.

$\frac{F_o}{A}$ = average accelerating force per □″ of piston.

QUANTITIES BRACKETED IN TABLE ARE LENGTH OF STROKE = $S$, FOR DIFFERENT VALUES OF $N$ AND PISTON SPEED $\left(w^1 = \frac{2NS}{12}\right)$.

| Piston speed in ft. per minute. | Revolutions per minute = $N$. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 | 500 | 600 |
| 600 | (36.0) .1955 | (24.0) .1303 | (18.0) .0978 | (14.4) .0782 | (12.0) .0652 | (10.29) .0559 | (9.0) .0489 | (8.0) .0435 | (7.2) .0391 | (6.0) .0326 |
| 650 | (39.0) .1788 | (26.0) .1186 | (19.5) .0889 | (15.6) .0712 | (13.0) .0593 | (11.14) .0508 | (9.75) .0444 | (8.67) .0395 | (7.8) .0356 | (6.5) .0297 |
| 700 | (42.0) .1676 | (28.0) .1118 | (21.0) .0838 | (16.8) .0670 | (14.0) .0559 | (12.0) .0479 | (10.5) .0419 | (9.33) .0372 | (8.4) .0335 | (7.0) .0281 |
| 725 | (43.5) .1618 | (29.0) .1078 | (21.75) .0809 | (17.4) .0647 | (14.5) .0539 | (12.43) .0462 | (10.88) .0405 | (9.66) .0360 | (8.7) .0324 | (7.25) .0270 |
| 750 | (45.0) .1564 | (30.0) .1067 | (22.5) .0782 | (18.0) .0626 | (15.0) .0521 | (12.76) .0447 | (11.25) .0391 | (10.0) .0348 | (9.0) .0313 | (7.5) .0261 |
| 775 | (46.5) .1510 | (31.0) .1010 | (23.25) .0767 | (18.6) .0606 | (15.5) .0505 | (13.31) .0433 | (11.63) .0379 | (10.33) .0337 | (9.3) .0301 | (7.75) .0252 |
| 800 | (48.0) .1466 | (32.0) .0981 | (24.0) .0733 | (19.2) .0587 | (16.0) .0489 | (13.71) .0419 | (12.0) .0369 | (11.66) .0326 | (9.6) .0293 | (8.0) .0244 |
| 825 | (49.50) .1422 | (33.0) .0948 | (24.75) .0710 | (19.8) .0570 | (16.5) .0474 | (14.4) .0406 | (12.38) .0356 | (11.0) .0316 | (9.9) .0284 | (8.25) .0237 |
| 850 | (51.0) .1380 | (34.0) .0920 | (25.5) .0690 | (20.4) .0553 | (17.0) .0460 | (14.57) .0394 | (12.75) .0345 | (11.33) .0307 | (10.2) .0276 | (8.5) .0231 |
| 875 | (52.5) .1341 | (35.0) .0893 | (26.25) .0670 | (21.0) .0536 | (17.5) .0447 | (14.71) .0383 | (13.13) .0335 | (11.66) .0298 | (10.5) .0268 | (8.75) .0223 |
| 900 | (54.0) .1303 | (36.0) .0868 | (27.0) .0652 | (21.6) .0521 | (18.0) .0434 | (15 43) .0377 | (13.5) .0326 | (12.0) .0290 | (10.8) .0261 | (9.0) .0217 |
| 950 | (57.0) .1235 | (38.0) .0823 | (28.5) .0618 | (22.8) .0506 | (19.0) .0412 | (16.29) .0353 | (14.25) .0309 | (12.66) .0274 | (11.4) .0247 | (9.5) .0211 |
| 1000 | (60 0) .1172 | (40.0) .0782 | (30.0) .0587 | (24.0) .0469 | (20.0) .0381 | (17.14) .0335 | (15.0) .0293 | (13.33) .0261 | (12.0) .0235 | (10 0) .0196 |
| 1050 | (63.0) .1117 | (42.0) .0745 | (31.5) .0553 | (25.2) .0447 | (21.0) .0373 | (18.0) .0319 | (15.75) .0280 | (14.0) .0241 | (12.6) .0224 | (10.5) .0186 |
| 1100 | (66.0) .1066 | (44.0) .0711 | (33.0) .0533 | (26.4) .0427 | (22.0) .0356 | (18.86) .0305 | (16.5) .0267 | (14.66) .0237 | (13.2) .0213 | (11.0) .0182 |
| 1150 | (69.0) .1020 | (46.0) .0680 | (34.5) .0510 | (27.6) .0418 | (23.0) .0340 | (19.71) .0292 | (17.25) .0254 | (15.33) .0221 | (13.8) .0204 | (11.5) .0174 |
| 1200 | (72.0) .0978 | (48.0) .0652 | (36.0) .0489 | (28.8) .0391 | (24.0) .0326 | (20.57) .0279 | (18.0) .0244 | (16.0) .0217 | (14.4) .0196 | (12.0) .0163 |

## VI.

### Determination of Weight of Fly-wheel Rim.*

Let $m_0$ represent the mass of the fly-wheel, $W_0$ its weight, $V_1$ its maximum, $V_2$ its minimum, and $V = \frac{V_1 + V_2}{2}$ its average speed of rim in feet per second. Then will maximum variation of energy equal

$$\frac{m_0}{2}(V_1^2 - V_2^2) = \frac{m_0}{2}(V^1 + V_2)(V_1 - V_2) \qquad (51)$$

the coefficient of unsteadiness is

$$f = \frac{\overline{V_1} - \overline{V_2}}{V} \qquad (52)$$

hence variation of energy =

$$f\, m_0\, V^2 = f\frac{w_0}{g}\; V^2 = .000085\, f\, D_0^2 N^2 w_0 \qquad (53)$$

$D_0$ being diameter of rim in feet. $N =$ Revolutions per minute From the phase of greatest variation of tangential pressuse diagrams we get $\overline{Mm}$ and $\overline{MON}$ then if $A =$ area of piston in □″ and $R' =$ length of crank in feet we have variation of energy =

$$\overline{Mm} \times A \times \frac{\overline{MON}}{360} \times 2\pi R' = .000085\, f\, D_0^2 N^2 w_0 \qquad (54)$$

$$w_0 = 205\,\frac{A \times \overline{MON} \times \overline{Mm} \times R'}{f\, D_0^2 N^2} \qquad (55)$$

According to *Der Taschenbuch des Ingenieurs* the coefficient of unsteainess $f = \frac{V_1 - V_2}{V}$ where $V = \frac{V_1 + V_2}{2}$ varies as follows.

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* The figure and formulas of this article were taken from Rigg's Treatise on the Steam Engine.

For machines which will permit a very uneven motion, hammers, etc., $f = \frac{1}{5}$

For machines which permit some irregularity, pumps, shearing machines, etc., $f = \frac{1}{20}$ to $\frac{1}{30}$

For machines which require approximation to uniform speed, as in flour mills, $f = \frac{1}{25}$ to $\frac{1}{35}$

For machines with tolerably uniform speed as weaving and paper making, $f = \frac{1}{30}$ to $\frac{1}{40}$

For cotton-spinning machinery requiring very uniform speed, $f = \frac{1}{40}$ to $\frac{1}{60}$

For the spinning machinery of very high yarn numbers,* $f = \frac{1}{100}$

The areas (see figure) included between mean tangential pressure circle and that portion of the tangential pressure curve lying outside of the mean tangential pressure circle we will call phases of excess of power; the areas included between the mean circle and that portion of tangential pressure curve within the circle of average resistance we will call the excess of resistance. These phases may be numbered as in figure by Roman numerals, and the angles at the center which they subtend, in degrees, The average excess or deficiency of pressure of any phase may be found tentatively by drawing arcs subtending the angle of each phase as in figure, the area included between these arcs and the mean tangential circle being equal to the areas of their corresponding phases.

$k$ is the ratio, $\dfrac{\text{Excess of power or resistance during any phase}}{\text{Total power exerted during a revolution}}$

$$= \frac{\overline{Mm}}{\overline{Aa}} \times \frac{\overline{MON}}{360} = \text{coefficient of variation of energy.}$$

* For electric lighting, $f = \frac{1}{100}$, $k = \frac{\Delta E}{\int P ds}$.

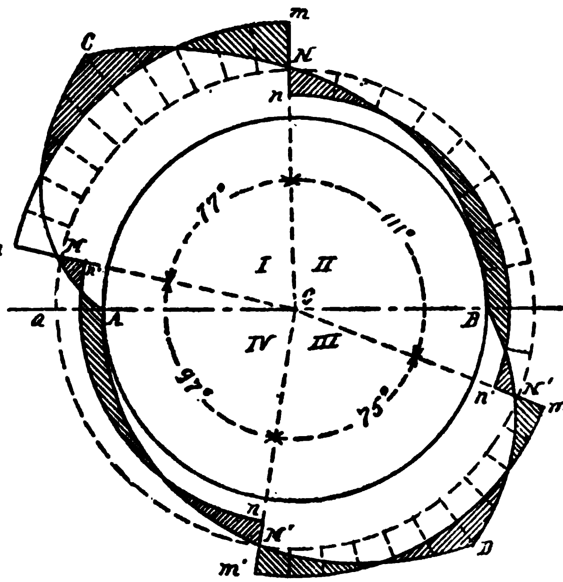


Fig. 33.

$f$ = coefficient of unsteadiness $= \dfrac{V_1 - V_2}{V}$ when $V = \dfrac{V_1 + V_2}{2}$

= mean velocity of rim. $f = \dfrac{1}{50}$ in present case.

$A$ = area of piston in □″.

$R$ = length of crank in inches.

$\overline{Mm}$ = excess of pressure in lbs. (see figure).

$\overline{MON}$ = angle (in degrees) of phase of greatest variation.

$D_0$ = diameter of fly-wheel in feet.

$N$ = revolutions per minute.

$H'$ = indicated horse-power of engine when the greatest variations of power or resistance occur.

$\overline{V}$ = velocity of rim per second $\leqq$ 80 feet. Assume = 64 feet in present case.

$W_0$ = weight of fly-wheel rim. (It may be diminished 8% for arms and boss.)

$$W_\circ = 17.1 \frac{AR \ \overline{Mm} \ \overline{MON}}{fD_\circ^2 N^2} \qquad (56)$$

or

$$W_\circ = 388000000 \frac{kH'}{fD_\circ^2 N^3} \qquad (57)$$

If in the figure on the preceding page $\overline{Mm} = 2.9$ $\overline{Nn} = 1.9$, $\overline{N'm} = 1.9$, $\overline{M'n} = 1.6$ and $\overline{Aa} = p_t = 3.1$, the areas of the various phases will be measured by the following products

| | | | |
|---|---|---|---|
| I | Excess of Power | $= 2.9 \times 77° = 223.3$ | |
| II | " " Resistance = | | $1.9 \times 111° = 210.9$ |
| III | " " Power | $= 1.9 \times 75° = 142.5$ | |
| IV | " " Resistance = | | $1.6 \times 97° = 155.2$ |
| | | Excess of Power = $\overline{365.8}$ | Excess of Resistance = $\overline{366.1}$ |

Had the work been perfectly accurate the two results would have been exactly equal. It is evident that in the present example I is the phase of greatest variation, consequently its average ordinate $\overline{Mm}$ and angle $\overline{MON}$ should be substituted in formula for weight of fly-wheel rim. The ratio which the measure 223.3 of phase I bears to the measure $\overline{Aa} \times 360 = 3.1 \times 360 = 1116$ of the total work done in one revolution is $\frac{223.3}{1116} = 0.22 = k$.

Find from the three tangential pressure diagrams for $H, H_1, H_2$, the phase of greatest variation and substitute in formula for fly-wheel.

In a manner similar to that detailed on preceding page we can find the phase of greatest variation from the tangential pressure diagrams drawn on the rectified crank-pin circle. Instead of multiplying the average pressure of each phase by the size of the phase expressed in degrees, we multiply by the distance between the two intersections of each phase with the mean tangential pressure line.

## VII.

## Balancing the Reciprocating Parts by Counter-weight on Crank.

We have already seen that only a portion $P'$ of the effective steam pressure $P$ is transmitted to the crank pin during the first portion of the stroke, the remainder $(P - P')$ being engaged in accelerating the reciprocating parts. The absorption of pressure by the moving parts disturbs the statical equilibrium of the forces acting on the engine bed $BCDA$. The steam pressure $P$ acting on the rigid frame at $A$ being greater than $P'$ the horizontal force at $B$, the tendency is to shift the engine bed in the direction $CD$

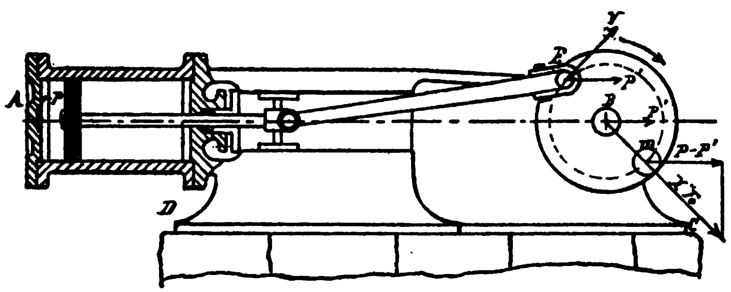


Fig. 34.

of the greater force $P$, the shifting force being equal to $P - P'$. Similar considerations will show that as the piston approaches the end of its stroke this shifting force changes in direction and tends to slide the engine bed on its foundations in the opposite direction $DC$. In order that this effect may be wholly or partially neutralized by other means than the employment of numerous bolts for holding the engine bed to its foundations, a counter-weight $m$ or its equivalent is attached to the crank as shown in the figure. The horizontal component $P - P^1$ of the centrifugal force $xF_0$ developed by this mass $m$ when revolving in the crank-pin circle with velocity $V$ will then be transmitted to the point $B$ of the engine bed and together with $P'$ will make a force equal to $P$ and will balance the steam pressure $P$ acting at the other end $A$ of the frame.

The horizontal component of the centrifugal force is easily obtained from the expression

$$x F_0 \cos \omega = m \frac{v^2}{R} \cos \omega = \frac{\mathfrak{w} v^2}{g R} \cos \omega \tag{58}$$

where $m$ represents the mass engaged in balancing the reciprocating parts reduced to a point on the crank-pin circle diametrically opposite to the crank pin, $v =$ velocity of crank pin, $R =$ radius of crank, $\omega =$ crank angle, we have already shown that the force $F$ accelerating or retarding the reciprocating parts is given by the expression

$$F = \frac{W}{g} \frac{v^2}{R} \left( \cos \omega + \frac{R}{L} \cos 2\omega \right) \tag{59}$$

where $W$ represents the weight of the reciprocating parts. The frame will be in statical equilibrium and will have no tendency whatever to shift on its foundations

when $$\frac{\mathfrak{w}}{g} \frac{v^2}{R} \cos \omega = \frac{W}{g} \frac{v^2}{R} \left( \cos \omega + \frac{R}{L} \cos 2\omega \right) \tag{60}$$

that is when $$\mathfrak{w} = \frac{W\left( \cos \omega + \frac{R}{L} \cos 2\omega \right).}{\cos \omega} \tag{61}$$

It is evident from this equation that equilibrium cannot exist for all values of $\omega$ unless the counter-weight $\mathfrak{w}$ also varies with the crank angle $\omega$. This it is of course not practicable to do, hence there will always be a shifting force acting on frame equal in amount to

$$\begin{aligned} &\frac{W}{g} \frac{v^2}{R} \left( \cos \omega + \frac{R}{L} \cos 2\omega \right) - \frac{\mathfrak{w}}{g} \frac{v^2}{R} \cos \omega \\ &\quad = F_0 \left( \cos \omega + \frac{R}{L} \cos 2\omega - x \cos \omega \right) \\ &\qquad = F_0 \left[ (1 - x) \cos \omega + \frac{R}{L} \cos 2\omega \right] \end{aligned} \tag{62}$$

$x$ being equal to $\frac{\mathfrak{w}}{W}$.

The maximum shifting effort when reciprocating parts are not balanced by counter-weight corresponds to $\omega = 0$ and $x = 0$; hence maximum shifting force equals

$$F_0\left(1 + \frac{R}{L}\right)$$

and when the reciprocating parts are balanced the maximum shifting force corresponds again to $\omega = 0$ and is equal to

$$F_0\left(1 - x + \frac{R}{L}\right).$$

When $x =$ unity, that is when counter-weight $w = W =$ weight of reciprocating parts, the ratio of these two maximum shifting forces is

$$\frac{R}{L} \div \left(1 + \frac{R}{L}\right).$$

It will generally suffice to make $x = .5$ to $.8$; the unbalanced portion being resisted by the foundations.

Calculate the value of the vertical components of the mass $m$.

Instead of employing the counter-weight as shown in the preceding figure it is customary to employ a crank disk of the following shape.

The crank proper is balanced by a counter-weight of similar shape shown in dotted lines, there remain therefore only the two portions $ABCDE$ and $A'B'C'D'E'$ having the depth $w$ which can be utilized as counter-weight for balancing the reciprocating parts, the dotted lines $A'F'G'E'$ and $AFGE$ representing the counter-balance for part (if any) of the weight of connecting rod, which part is supposed to be concentrated at crank pin. It is evident from the figure that the center of gravity $c$ of the portions $ABCDEGF$ and $A'B'C'D'E'F'$ does not fall upon the crank-pin circle, consequently this deficiency of radius must be made good by increasing the masses $ABCDEGF$ and $A'B'C'D'E'G'F'$. Let the sum

of these larger masses be represented by $M'$, its radius by $\mathfrak{r}$, then we must have

$$\frac{M'(v')^2}{\mathfrak{r}} = \frac{m'v^2}{R}$$

and since $\dfrac{v'}{v} = \dfrac{\mathfrak{r}}{R}$ we have $M' = \dfrac{R}{\mathfrak{r}}m'$. (63)

The value of $\mathfrak{r}$ can be found experimentally by suspending the figure $ABCDEGF$ from two of its corners and determining the

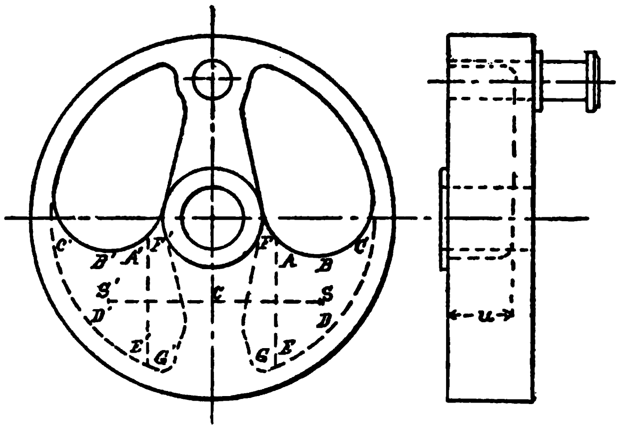

Fig. 35.

intersection of the two lines of suspension, or it may be balanced over a knife edge in two different positions. $M'$ can then be computed.

The dimensions of the simple crank can be calculated from the formula given by Reuleaux.

To find room on the crank disk for the large mass $M'$ of counterweight, the outside circumference of crank disk, must be taken considerably larger than crank-pin circle. By casting hollows in the disk and filling them with lead, all the counter-weight desired can be placed on crank disk. When double cranks or disks are employed the disposal of counter-weight becomes an easy matter.

## VIII.

### Effect of Frictional Resistances, of the Weight of the Rod and Exact Values of the Forces of Inertia, on the Rotative Effort, on the Pin Pressures and on the Force Shaking the Engine Bed.

The effect of these three influences on the rotative effort and energy of the fly-wheel has been very fully considered by Prof. D. S. Jacobus* in two papers, published in Vol. XI, Transaction of American Society of Mechanical Engineers.

In these papers very accurate formulas were established which took account of these three influences. They were applied to four different engines representing a wide range of practice. The exact diagrams constructed for these engines showed that the fluctuation of energy, $\Delta E$, of the fly-wheel due to the variation of the rotative effort differed by an insignificant amount from the approximate diagrams found by supposing all the reciprocating parts (inclusive of rod) concentrated on piston. A similar result was obtained for the crank-pin pressures, the nearly exact ones differing but slightly in direction and intensity from those found by assuming the mass of the rod to be all concentrated at wrist-pin. The following tables are given by Prof. Jacobus and furnish the data of the four engines and the results of the numerical computations.

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* Prof. Jacobus credits Prof. J. B. Webb with valuable assistance in the preparation of these two papers.

# TABLE XI.

## DIMENSIONS AND SPEED OF ENGINES TO WHICH THE FORMULAS HAVE BEEN APPLIED.

| | Class of Engine. | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| | Small horizontal high-speed. | Large horizontal high-speed Locomotive. | Large horizontal slow-speed of revolution. Harris—Corliss. | Westinghouse. |
| Revolutions of crank shaft per minute . . | 300 | 250 | 60 | 320 |
| Length of stroke, in inches . . . . . . | 12 | 24 | 60 | 10 |
| Diameter of cylinder, in inches . . . . . | 10 | 18½ | 26⅛ | 11 |
| Length of connecting-rod, in inches . . . | 36 | 92 | 150 | $5R$ |
| Distance from the wrist-pin to the center of gravity of rod, in inches . . . . . . | 20.15 | 55 | 78 | $3.22\ R$ |
| Distance from the center of the crank shaft to the line of travel of the wrist-pin, in inches . . . . . . . . . . . . | 0 | 0 | 0 | $0.5\ R$ |
| Principal radius of gyration, in inches . . . | 15.00 | 34.1 | 48 | $2.07\ R$ |
| Weight of piston, piston-rod, and cross-head, in pounds . . . . . . . . . . . | 90. | 474 | 1300 | 100 |
| Weight of connecting-rod, in pounds . . . | 70. | 307 | 1200 | 50 |
| Indicated horse-power . . . . . . . . | 57 | 345* | 346 | 66 |

* For one cylinder.

## TABLE XII.

RESULTS OF NUMERICAL APPLICATIONS TO SEVERAL STANDARD CLASSES OF ENGINES.

| Class of Engine. | Conditions assumed. | $\frac{\Delta E}{\int P ds}$ | | | Pressure acting on crank pin | | |
|---|---|---|---|---|---|---|---|
| | | Exact. | Approximate. | Not including accelerating forces. | Exact. | Approximate. | Not including accelerating forces. |
| I | Not including the effects of friction and weight | .184 | .174 | .238 | 58.1 | 57.3 | 83.0 |
| I | Not including friction but including weight . . | .180 | . . . | . . . | 58.0 | . . . | . . . |
| I | Including both friction and weight , . . . | .171 | . . . | . . . | 56.8 | . . . | . . . |
| II | Not including friction and weight . . . . . . | .181 | .172 | .275 | 56.8 | 55.5 | 108.0 |
| III | Not including friction and weight . . . . . . | .142 | .131 | .185 | 76.1 | 75.5 | 89.4 |
| IV | Not including friction and weight . . . . . . | .160 | .147 | .247 | 61.0 | 60.8 | 75.5 |

The term approximate in these tables refers to the assumption that the whole mass of the rod is concentrated at wrist-pin. The expresssion $\frac{\Delta E}{\int P ds}$ is the ratio of the periodical excess or deficiency of energy $\Delta E$, to the whole energy exerted per revolution, $\int P ds$. The particular crank-pin pressure given in table is the maximum one and is in pounds per square inch of piston area.

It is evident from these tables that for most practical work the aforesaid approximation, used in the preceding sections, is sufficiently accurate. The remainder of this section may therefore be omitted by those who do not care to examine the refinements of this subject.

We have not reproduced here any of Prof. Jacobus' formulas for the components of the forces acting on the pins, though they are easily applied, and, after the constants have been computed and introduced, lead quickly to the desired results. We have preferred graphical determinations because they are simple, and naturally accompany designing. We will therefore follow graphical methods in finding exact values for each of the three influences under consideration.

The order in which we will take them up is:

*a.* The friction between a pin and its bearing.

*b.* The direction of internal stress in a rod, when friction is taken into account, but gravity and inertia neglected.

*c.* The direction of internal stress in a rod, when friction and some other force, say, resultant of gravity and inertia-resistance, are considered.

*d.* Determination of the exact accelerating force of the rod corresponding to its motion in the slider-crank chain.

*e.* Combination of weight of rod with its force of inertia.

*f.* The components of this total force at wrist- and crank-pin, when friction is neglected.

*g.* The components of this force at these pins, when friction is considered.

*h.* Determination of the force shaking the engine bed.

*i.* Diagrams of shaking forces with different degrees of counterweighting.

*j.* Diagrams of pressures at crank- and at wrist-pin.

## A.

### The Friction Between a Pin and its Bearing.

The frictional resistances of motion and rest differ by quantities that are directly proportional to the coefficients of friction for motion and rest, respectively. By using the coefficient of friction for motion, we may treat the body as if it were at rest and yet

determine the direction and intensity of the forces as they exist under running conditions.

Let us suppose the pin to be at rest, without tendency to move in either direction, and that the resultant pressure $P_0$ between the pin and its bearing acts in the direction $c\ a\ e\ f$, Fig. 36. Then the pin can be brought to the eve of, say, left-handed rotation, in one of two ways, either by the action of an additional, single, force $Q$, with lever arm $c\ l$, or by the action of some couple $M$ having left-hand rotation.

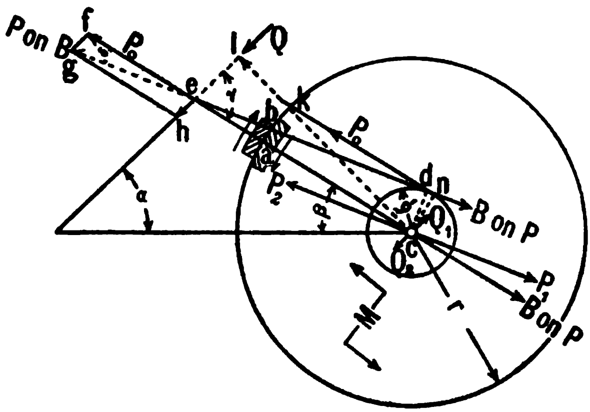


Fig. 36.

In the first case there will be an infinite number of solutions depending upon the location and direction of the turning force $Q$. The moment $F'r$ of the friction of the bearing will balance the moment $Q \times \overline{cl}$ of the new force and the point of application of the resultant pressure between pin and bearing will be shifted from $a$ to $b$, sufficient play being supposed to exist between the pin and its bearing to permit this to take place. The point $b$ is where the resultant $P(=eg)$ of the forces $Q(=eh)$ and $P_0(=ef)$ cuts the area of contact. Evidently the new resultant $P$ is different in intensity and direction from the original pressure $P_0$. The pin is kept stationary by $+P$ and $-P$; turning of the pin is prevented because

$$Q \times \overline{cl} = P \times \overline{cd} = P\rho' = F'r = \varphi Nr.$$

The resultant pressure $P$ between bearing and pin is therefore necessarily a tangent to the friction circle whose radius $\rho' = \varphi r$ when $P$ is taken to equal $N$. This is permissible because the angle of friction is so small, that we may regard its sine and tangent as equal.

The force $P$ for this first case can also be constructed by placing at center $c$ two equal and opposite forces $Q_2 = Q_1 = Q$. Then $Q$ and $Q_1$ will form a couple which, combined with the original force $P_0$, will shift the latter parallel to itself through a distance $\frac{P}{P_0}\rho'$; now combining $P_0$ in this new position with the force $Q_2$ remaining at the center we get the same force $P$ as before.

In the second case the pin is brought to the eve of left-hand rotation by the action of some left-hand couple $M$ which may act anywhere in the plane. This case does not often arise in practice, and is chiefly useful in demonstrations involving equivalence of forces. The effect of this couple is to shift the force $P_0$ parallel to itself to a position in which it is tangent to the so-called "friction circle" having radius $\rho' = \varphi r$. The point of contact for this case and the direction and intensity of its force $P$ are all different from those obtained for the first case. As the couple $M$ may act anywhere in the plane, let us suppose it replaced by an equivalent couple $Q_0 \times \overline{cl}$ whose two equal and opposite forces $Q_0$ act respectively at the center $C$ and along the line of turning force $Q$ mentioned in the first case. Then it is easy to show that this force $Q_0$ is different from $Q$. In Fig. 36, we have given, the angle $\gamma$, the radii $r$ and $\rho'$ and the lever arm $cl$. Then

$$\sin\delta = \frac{\overline{cd}}{\overline{ce}} = \frac{cd\sin\gamma}{\overline{cl}} = \frac{\rho'\sin\gamma}{\overline{cl}}, \text{ and } \frac{P}{P_0} = \frac{\sin\gamma}{\sin(\gamma-\delta)};$$

but
$$Q \times \overline{cl} = P \times \rho'$$

$$\text{and } Q_0 \times \overline{cl} = P_0 \times \rho' \text{ then } \frac{Q}{Q_0} = \frac{P}{P_0} = \frac{\sin(\gamma-\delta)}{\sin\gamma}, \quad (64)$$

$$\frac{Q_0}{P_0} = \frac{Q}{P} = \frac{\rho'}{\overline{cl}} = \frac{\varphi r}{\overline{cl}}. \tag{65}$$

As each piece is subjected to at least two forces, the driving force and the resistance, there will always be a force $Q$ available for bringing the pin or shaft to the eve of turning. We will therefore assume the first case as the common one. It should be noticed that the resultant action of bearing on pin is tangent to that side of friction circle which will give this resultant a component opposed to the rotation of the pin or shaft. In like manner the action of pin on bearing is such a tangent to the friction circle, that it has a component opposed to the bearing's (relative) rotation. This is because friction is a hindrance to motion. In Fig. 36, these equal and opposite actions are indicated by arrows and the letters $P$ on $B$ and $B$ on $P$.

## B.

## The Direction of Internal Stress in a Rod acted upon by Two Pins.

There are four cases under this head, which are due to the two kinds of relative motion possible between a pin and its bearing, and to the two kinds of stress, tension or compression, to which the rod may be subjected. The four figures* on next page illustrate these four cases.

We will suppose the link $AB$, Fig. 37, in all cases to carry the eyes or bearings into which fit the pins belonging to the cranks $MA$ and $NB$. The large arrows about the center $N$ or $M$ indicate which crank is the driver and the small arrows about the pin centers $A$ and $B$ indicate whether the rotation of rod $AB$ to crank $MA$ or $NB$ is right- or left-handed. The arrows on the rod $AB$ itself indicate whether it is in tension or compression.

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* These figures were taken from Hermann-Smith's Graphical Statics of Mechanisms.

In cases I and II the relative motions of the pins $A$ and $B$ to their bearings in the rod $AB$ are alike, being both right-hand rotations, but the internal stress is different in the two cases, being tension in case I and compression in case II.

In cases III and IV the relative motions of pins $B$ to their bearings in the rod are still right-handed, but those of the pins $A$ to their bearings are now left-handed. The inner stresses are also different in these two cases, tension in case III and compression in case IV.

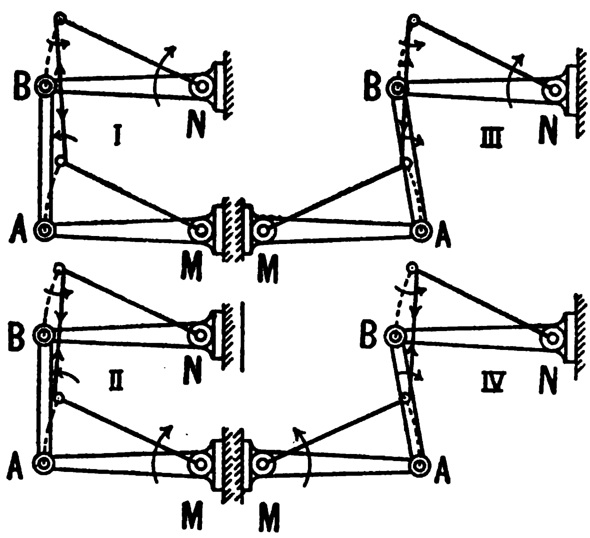


Fig. 37.

When there is no friction the inner stress acts along the center line of rod in each case.

When there is friction, the direction of the inner stress is different in each of these four cases.

In Figs. 38–41 sections are taken of both pin and its bearing near the point of contact and arrows added to indicate the relative motion of rubbing surfaces.

Now that there is friction, we do not at first know the point of application of the resultant of the forces acting at each surface of

contact; but we do know that the action *ap*, Fig. 38, of pin $A$ on bearing must have a component opposed to the (relative) motion of the bearing. This action *ap* prolonged backward will therefore be tangent to lower portion of friction circle *cde*. In

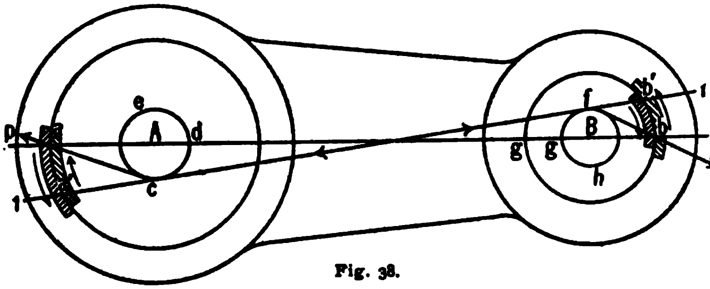


Fig. 38.

like manner the action of pin $B$ on its bearing when prolonged backward will also be tangent to its friction circle *fgh*, but at the upper portion of the circle. Neither *ap* nor *bp* represent the two directions of the forces exerted by the pins, for friction has changed the intensities of these forces and shifted their points of application to $a'$ and $b'$. As there are but two forces acting on the rod, it can only be in equilibrium when the two forces are directly opposite and equal. This condition determines the

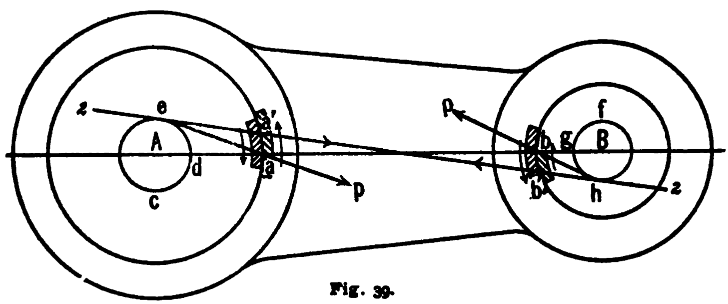


Fig. 39.

direction of the inner stress; it must be tangent to each of the friction circles of $A$ and $B$ as in the figure, the pin $A$ acting at $a'$ and pin $B$ at point of application $b'$.

In the case shown in Fig. 39 the rotation of the pins is still right-handed, but the rod is now subject to compression and con-

tact takes places at *a* and *b* on sides of the pins opposite to those of Fig. 38. Similar reasoning shows that in this case the direction of the inner stress must be along the line *2eh2*.

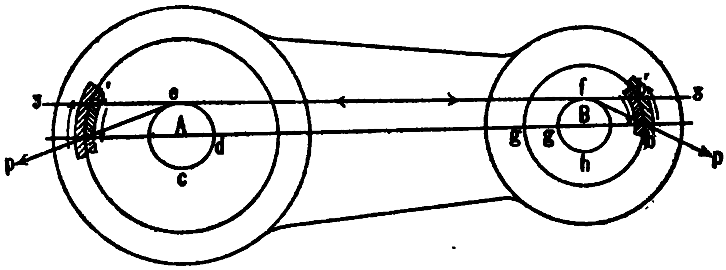


Fig. 40.

In Fig. 40 pin *B* keeps its right-hand rotation, but pin *A* has left-hand rotation, the rod being in tension. The figure indicates the condition of equilibrium and direction *3ef3* of the inner stress.

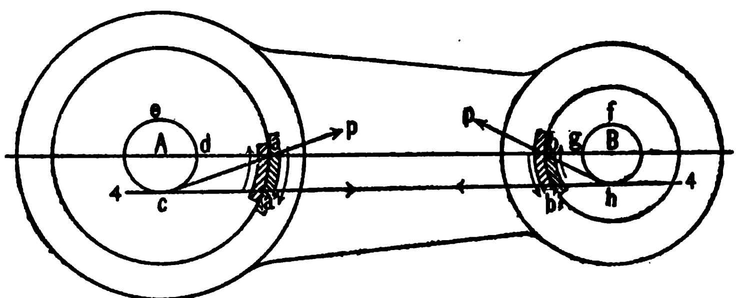


Fig. 41.

In this last case, Fig. 41, pin *B* has still right-hand rotation and *A* left-hand rotation. Compression however now exists in the rod and this brings the contacts *a* and *b* different from the preceding case, namely, on the opposite sides of the pins *A* and *B*. The tangent *ch* on line 44 is the direction of the inner stress.

These four cases have furnished four lines of stress coinciding with the four possible tangents, 11, 22, 33 and 44 to the friction circles. This would also have been the case, if the rod had carried the pins instead of the eyes or bearings.

## C.

## DIRECTION OF INTERNAL STRESS IN ROD WHEN ITS ACCELERATING FORCE IS KNOWN.

The known accelerating force *KG*, Fig. 42, is the resultant of the unknown forces acting at the crank-pin *C* and wrist-pin *W*. When there is no friction we will designate these forces by $P_c$ and $P_w$ respectively and their directions must be assumed to pass through their respective centers *C* and *W*. When there is friction the forces will be designated by $P_{cf}$ and $P_{wf}$ and they must then be tangent to their respective friction circles. In either case, unless some additional condition is given with reference to one or the other of these forces, the value of $P_c$ or $P_{cf}$, $P_w$ or $P_{wf}$ is indeterminate for there are evidently in each of these cases an infinite number of pairs of forces which can produce the given accelerating force *KG*.

The forces $P_{wf}$ and $P_{cf}$ (or $P_c$ and $P_w$) evidently produce both acceleration *PH* (= *KG*) and whatever internal stress may exist. This latter will vary according to the pair of forces assumed to produce the acceleration and also according to the points of application assumed for the members of this pair. For instance, let us assume that *PF* and *PE* represent the directions of the pair of forces $P_{cf}$ and $P_{wf}$, then any points *F'*, *F*, *S* and *E'*, *E*, *T* in the direction of these forces may be assumed as the points of application of the forces, and the corresponding directions of the internal stress will be *ST*, *FE*, *F'E'*. The intensities of these internal stresses along either of these directions will depend entirely upon the acceleration components of the forces $P_{wf}$ and $P_{cf}$. These components acting at the assumed points of application *S*, *T* or *F*, *E* or *F'*, *E'* have only one condition to fulfill, namely, to produce the acceleration *PH*. As this can be accomplished in an infinite number of ways, at the same pair of points of application, it is evident that those other components of $P_{wf}$ and $P_{cf}$, the internal stresses, have also an infinite number of

values. But when the directions of these acceleration components have once been decided upon, their intensities and those of their fellow components (the internal stresses), can be found by resolving each of the forces $P_{wf}$ and $P_{cf}$ along the directions assumed for their components. According to D'Alembert's principle* the internal stresses thus found must balance each other.

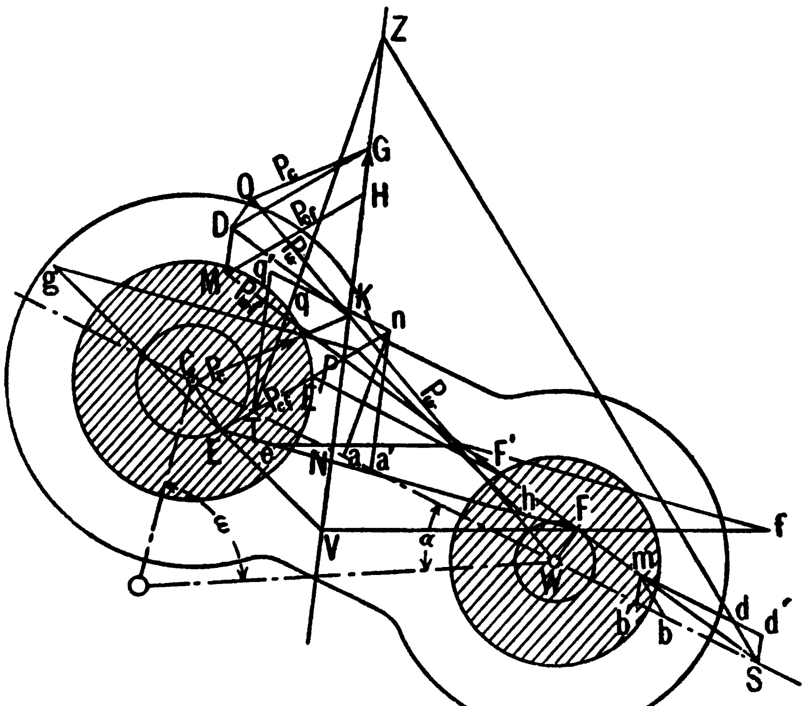


Fig. 42.

To illustrate what has been said, let us suppose the points of application chosen to be $S$ and $T$, then $TS$ will be the line of internal stress; now assuming the directions of the acceleration components of $P_{wf}$ and $P_{cf}$ to be $SZ$ and and $TZ$, then making

* According to D'Alembert's principle those components of the external forces which are not engaged in producing acceleration are in equilibrium.

$Sm = P\dot{M} = P_{wf}$ and $Tn = MH = P_{cf}$, we get $Sd$ and $Tq$ as the acceleration components and $Sb$, $Ta$ the components producing the internal stresses in the rod along the line $TS$. Measurement will show that $Sb = Ta$. In like manner we might have assumed the acceleration components $Sd'$ and $Tq'$ both parallel to the total acceleration $KG$; then resolving $P_{wf}$ and $P_{cf}$ we get $Sd' + Tq' = KG$ and internal stress $Sb' =$ internal stress $Ta'$. If $F$ and $E$ had been chosen as the points of application, $FE$ would have been line of internal stress. Then if $FV$ and $EV$ had been chosen as direction of acceleration components, $Fe = Eh$ would represent the equal internal stresses. It is evident that there is great freedom in the choice of the direction of the internal forces and later on we will indicate how this may be so chosen as to lead very directly to the determination of the actual pressures exerted by the pins upon their bearings in the connecting rod. In the engine problem these pressures are not indeterminate as they are here, for there the known effective steam pressure and the direction of the reaction of the guides, together furnish another condition, which is all that is necessary to completeley determine $P_{wf}$.

## D.

## Exact Determination of Force Needed to Accelerate Rod.

Any motion of a body in a plane is equivalent to the rotation of the body about its instantaneous axis. The rotation about this axis is, by a principle of Kinematics, equivalent to an equal rotation about any other, second, axis *plus* a translation, the latter motion being equal to that possessed by the second axis when rotating around the first, or instantaneous, axis. This equivalence of motion extends not only to the displacements, but also to the velocities and accelerations. The axis passing through the center of gravity will be taken as this second axis because it is the most convenient one, the centripetal accelerations about this axis completely neutralizing each other.

7

The proposition may also be stated in this better known form:

The total accelerating force of a body, whose points move in parallel planes, is compounded of the force needed to give the whole mass an acceleration of translation equal to that of its center of gravity and of a moment (or couple) capable of imparting to the body an angular velocity and acceleration about its own center of gravity equal to that possessed by the body when rotating about the instantaneous axis.

The combination of this force of translation with the moment or couple, gives a single resultant force that is equal and parallel to its component force of translation but has a location that is different from this component, the effect of the couple being merely to shift the location of the force producing the translation, a couple with $\left\{\begin{array}{c}\text{left-hand}\\ \text{right-hand}\end{array}\right\}$ rotation shifting the force to the $\left\{\begin{array}{c}\text{left}\\ \text{right}\end{array}\right\}$; in applying this rule the plane of forces must be viewed so that the force of translation is downward.

The moment of the force or couple capable of producing the desired angular acceleration $\frac{d\Omega}{dt}$, is, according to Mechanics,

$$\mathbf{I}\,\frac{d\Omega}{dt} = \mathfrak{m}k^2\,\frac{d\Omega}{dt} \tag{66}$$

where $\mathbf{I}$ is the principal, polar, moment of inertia, $\mathfrak{m}$ the mass of the connecting rod and $k$ the corresponding radius of gyration.

This expression is perfectly general so far as the distribution of the mass of the rod is concerned. Any arrangement of this mass which will not alter its center of gravity and which preserves the same total moment of inertia $\mathbf{I}$ unchanged will require the same total accelerating force as the rod itself. The problem may consequently be greatly simplified for graphical purposes by supposing the mass of the rod concentrated at two points that are on opposite sides of the center of gravity and in line with it.

In order that the equivalence of total accelerating force may be maintained it is necessary that the following three conditions be fulfilled,

$$\mathfrak{m} = \mathfrak{m}_1 + \mathfrak{m}_2, \tag{67}$$

$$\mathfrak{m}_1 h_1 = \mathfrak{m}_2 h_2 \tag{68}$$

$$\mathfrak{m} k^2 = \mathfrak{m}_1 h_1^2 + \mathfrak{m}_2 h_2^2. \tag{69}$$

and by combining these we get

$$k^2 = h_1 h_2, \tag{70}$$

where $h_1$ and $h_2$ are the distances from the center of gravity of the two points at which the mass is supposed to be concentrated and $\mathfrak{m}_1$, $\mathfrak{m}_2$ the masses at these points. Equations (67) and (68) keep the total mass and location of the center of gravity the same as before and therefore keep unchanged the accelerating, component, force due to the translation of the center of gravity. The fulfillment of equation (69) or (70) makes the turning moment of the rearranged, two-point, rod the same as that of the original rod with distributed mass. As the force and energy of this two-point rod are the same as in the original one, we can confine our determination of the total accelerating force to the simplified, two-point rod.

The problem is now reduced to finding the acceleration of the center of gravity of the rod and that of each of its two points of concentration. The problem is therefore now a purely kinematic one and is a special case of the general problem of finding the acceleration of any point on the connecting rod of the slider-crank mechanism.

In this general case of the mechanism and its motion, the acceleration $CO$, Fig. 43, of the crank-pin center $C$ is known and the acceleration $Ww''$ ($= Ow$) of the wrist-pin center or slide can be obtained as in Fig. 16 and p. 43. The accelerations of two points of the rod will then be known and the center of acceleration $G$ of the rod can be found.* The acceleration of any point of the

* See Weisbach-Hermann's Machinery of Transmission, Vol. III, Section I, § 21.

rod is directly proportional to the distance of this point from the center of acceleration $G$, and, for the instant in question, the acceleration of each point makes the same angle with its own

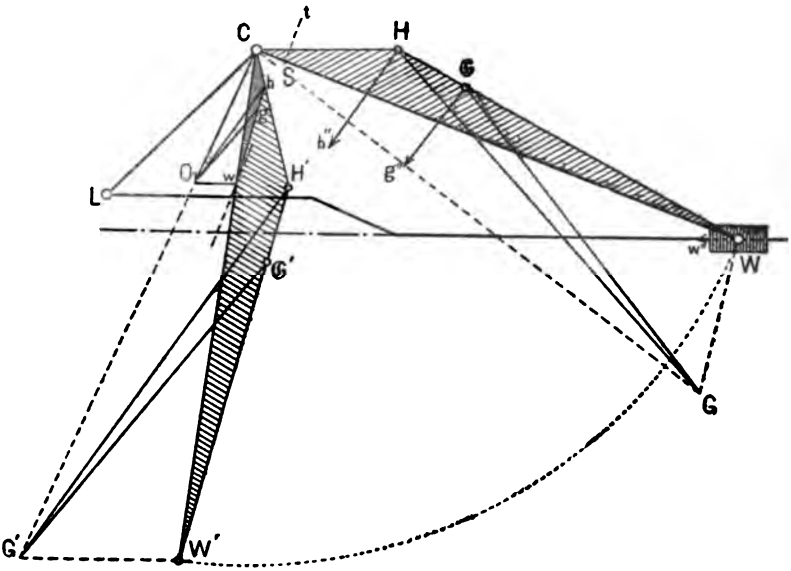


Fig. 43.

instantaneous radius of acceleration.* This solves the problem but not in a convenient way because the center of acceleration generally falls beyond the limits of the drawing. A method that is free from this objection and also simpler in other respects is found as follows:

In Fig. 43 revolve the triangle $CWG$ (having the rod $CW$ as a base and acceleration-center $G$ as a vertex) through an angle $GCG'$, equal to that made by the direction of acceleration of point $C$ with its instantaneous radius of acceleration $CG$. Then will the revolved radii $G'C$, $G'\mathfrak{G}'$, $G'H'$ and $G'W'$ be parallel to the directions of the accelerations of points $C$, $\mathfrak{G}$, $H$ and $W$, as well as proportional to the magnitude $CO$, $\mathfrak{G}g''$ $Hh''$ and $Ww''$ of

* See last paragraph on p. 45 and first one on p. 46 for one method of finding acceleration of any point of rod.

these accelerations. As the triangle *COw* is similar to the triangle *CGW* and *Chgw* to the rod *CH*$\mathfrak{G}$*W*, the vectors *OC*, *Oh*, *Og*, and *Ow* drawn from *O* as a pole will represent in direction and magnitude the accelerations of the points *C*, *H*, $\mathfrak{G}$ and *W* of the rod.

The pole *O* is the end of the acceleration *CO* of crank-pin and when *Ow* is found by method given in Fig. 16 and p. 43, the figure *Cwgh* can easily be made similar to the rod *CW*$\mathfrak{G}$*H*. Since *H* may be any point in the rod we have a simple method of finding its acceleration (without first finding the center of acceleration) provided the direction and intensity of the acceleration of two points of the rod are known.*

We can now give two simple constructions for the total force accelerating the rod of the ordinary slider-crank, when crank has uniform rotation.

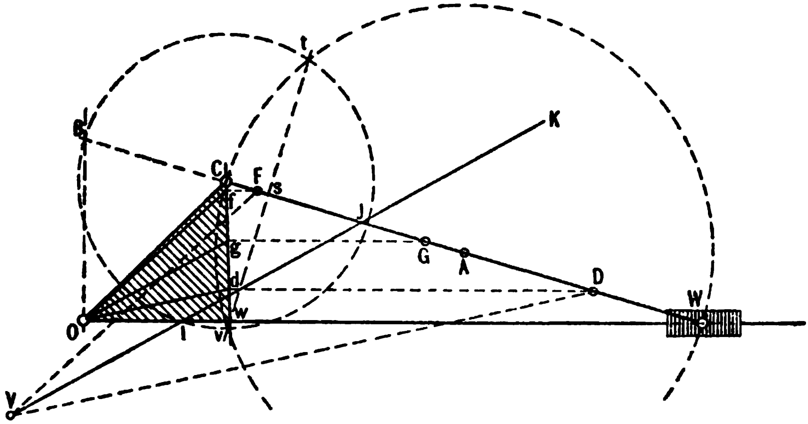


Fig. 44.—First Construction.

The mechanism *OCW*, Fig. 44, is the ordinary slider crank in which the stroke of the slide *W* passes through the center *O* of the crank *OC*. The center of the rod is at *A*, the center of gravity at *G* and *GF* = *GD* is the principal polar radius of gyration.

* This same simple method may also be deduced from a more general case. See Journal Franklin Institute for September and October, 1891.

We first construct the acceleration of the slide *W* by the method given on p. 43 and Fig. 16.

Prolong the rod *WC* till it intersects at *B* the perpendicular *OB* to the stroke *OW*. With *C* as a center and *CB* as a radius describe a circle *Btv*. On the rod *CW* as a diameter describe another circle. Join the intersections of these circles by the chord *tsv* and prolong it, if necessary, till it cuts a line drawn through crank center *O* and parallel to stroke. In Figs. 44 and 45 this intersection is at *w*, and *Ow* is the desired acceleration of the slide, provided crank length *CO* represents the centripetal acceleration of the uniformly rotating crank-pin.

(The construction would be exactly the same for the crossed slider crank of Fig. 46 if its crank had uniform rotation. This construction of the slide acceleration has no failing case even at the dead point.)

Now join *C* and *w*. *Cw* is a reduced image of the rod, making with it an angle equal to that made by each point's acceleration with its own radius of acceleration and point *O* has the same positions relatively to the image *Cw* of the rod that the center of acceleration has relatively to the rod itself. It follows from this that if we draw through points *F*, *G* and *D* of the rod, parallels *Ff*, *Gg* and *Dd* to the stroke, that the distances or vectors *Of*, *Og* and *Od* will represent in direction and intensity the accelerations of *F*, *G* and *D*, respectively. The actual locations of their accelerations will, of course, be parallels through these points to those vectors. One-half of the mass of the rod is supposed to be concentrated at *F* and the other half at *D*.

Draw *FV* parallel to *Of* and *DV* parallel to *Od*. Through their intersection *V* draw, parallel to *Og*, the line *VK*; it will be the actual location of the resultant force of inertia, and its intensity will equal the product of the whole mass of the rod by acceleration $\overline{Og}$ of the center of gravity.

In Fig. 45 the mass $\mathfrak{m}$ of the rod is divided into two unequal parts, one of which $\mathfrak{m}_1$ is concentrated at *W* and the other $\mathfrak{m}_2$ at

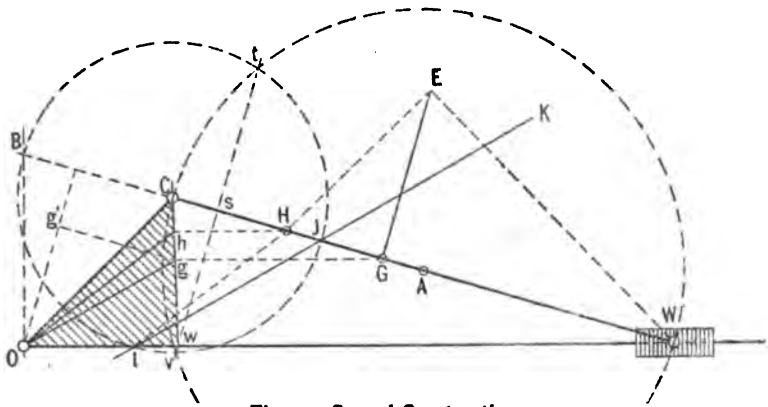


Fig. 45.—Second Construction.

$H$, the relation between the points $W$, $G$ and $H$ being $\overline{GW} \times \overline{GH} = k^2$, where $k$ is the principal polar radius of gyration. The two masses must be to each other as

$$\mathfrak{m}_1 : \mathfrak{m}_2 = \overline{GH} : \overline{GW}.$$

Construct as before the acceleration $Ow$ of the slide and $Cw$ the image of the rod. At $G$ erect the perpendicular $GE = k$. Join $E$ with $W$ and draw $EH$ at right angles to $EW$. This will evidently satisfy the condition $\overline{GW} \times \overline{GH} = k^2$.

Through $H$ and $G$ draw $Hh$ and $Gg$ parallel to the stroke, then will the vectors $Oh$ and $Og$ represent in direction and intensity the accelerations of points $H$ and $G$, respectively.

Through $H$ draw $HI$ parallel to $Oh$; it will cut at $I$ the direction $WI$ of the acceleration of the other mass at $W$. Through $I$ draw, parallel to $Og$, the line $IK$; it will be the actnal location of the resultant force of inertia of the rod and its intensity = whole mass of rod $\times$ acceleration $Og$ of center of gravity.

These two constructions give, of course, exactly the same result, the first being a little the easiest when the intersection $V$ falls within the limits of the paper. The great advantage of the second is that the direction of $W$ is given by the line of stroke, and that the intersection $I$ will always fall within the limits of the

drawing. Both constructions of the total force of inertia fail at the dead point, but this is of no consequence, because then the direction and location of the total acceleration of the rod is known, for it coincides with the stroke, and, as before, its intensity = the whole mass $\times$ acceleration $Og$ of the center of gravity.*

This construction is only one of an infinite number of possible cases, each of which satisfies the three conditions involved in equations (67), (68) and (69) or (70).

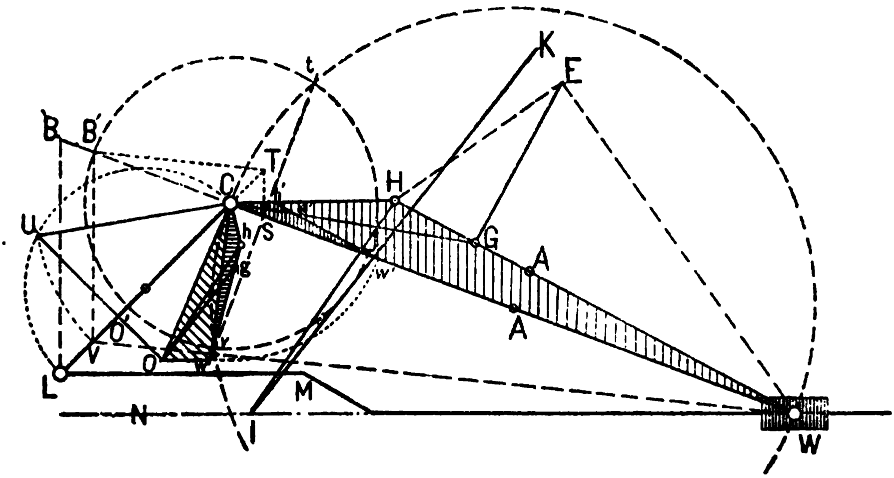


Fig. 46.

It is evident that $h_1 = h_2$ is a particular case of the general one just given, and is illustrated in Fig. 44.

Fig. 46 is meant to be the most general case of slider-crank, with

* The nearer point $H$, Fig. 45, is to crank-pin $C$ the more nearly does the point of intersection $I$ preserve a constant position. In the rods of good high-speed-engines the locus of point $I$ varies but little, the average value of $\overline{OI}$ being nearly equal to $CH = L - \dfrac{h_1^2 + k^2}{h_1}$; where $L = CW$ is length of connecting rod and $h_1 = GW$, Fig. 45, is distance of center of gravity $G$ from wrist-pin $W$. This could be made the basis of a close approximation, in which the point $I$ is treated as a fixed point and $IK$ is drawn at once parallel to $\overline{Og}$, thus omitting the drawing of the lines $Hh$, $hO$ and $HI$.

respect to the mechanism itself, to its motion and to the distribution of the mass of the rod. The mechanism $LCW$ is of the crossed slider-crank type in which the stroke of slide $W$ does not pass through center $L$ of crank. The rotation of crank $CL$ is not uniform as in the preceding cases, the acceleration of crank, pin $C$, taking (say) the direction and intensity $CO$ instead of $CL$ as before. The center of gravity of the mass is no longer in the center line $CW$ of the rod, but outside of it, at $G$. The point $H$ is on the line $WG$ as before, and again satisfies the condition $GW \times GH = k^2$, where $k$ is the principal polar radius of gyration. $H$ is found by the same construction, namely, we erect the perpendicular $GE = k$, join $EW$, and draw $EH$ at right angles to $EW$.

The velocity of the crank-pin is no longer represented by $CL$, as in Figs. 44 and 45, but by $CV$. It is found by construction, as follows: On crank $CL$ as a diameter construct a circle $LUC$; from end $O$ of acceleration draw a perpendicular $OO'U$ to $CL$; then will $CU = CV$ be the intensity of the velocity of the crank-pin $C$. To find the acceleration $Ow$ of slide, we proceed as in Figs. 44 and 45, except that a circle is described with $CB'$ as a radius instead of $CB$.*

In Fig. 46 the distance $OC$ and $Ow$, respectively represent the

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* There is still another way of finding point $S$ (and consequently point $w$) that is always available.

Draw $VB'$ perpendicular to stroke $WN$ and prolong it till it cuts the direction $WC$ of the rod at $B'$. Join $W$ and $V$, make $B'T$ parallel to $WV$ and draw $TS$ perpendicular to $WN$.

For the similar triangles $B'CV$ and $SCT$ give

$$CS : CT :: CB' : CV \text{ or } CB' \times CT = CS \times CV,$$

and the similar triangles $B'CT$ and $WCV$ give

$$CB' \times CV = CT \times CW.$$

Multiplying these two equations and canceling, we get,

$$CS = \frac{\overline{CB'}^2}{CW}$$

and this is similar to the result found for $BF'$, Fig. 16, pp. 41-43.

accelerations of crank-pin $C$ and wrist-pin $W$. The line $Cw$ is the reduced image of the center line $CW$ of the rod. We may consider this rod to be an exaggerated specimen of the Westinghouse type in that its center of gravity $G$ is off the center line $CW$ of the rod. We may regard the triangle $CHGW$ as the representation of the rod and proceed to construct a reduced image of it on $Cw$ as a base. To do this lay off $Cw' = Cw$, draw $w'h'$ parallel to $WH$ and connect $C$ with $G$. The triangle $Ch'g'w'$ is evidently similar to $CHGW$. Revolving it back to $Cw$ as a base we get triangle $Chgw$ as the desired image of the rod, and $Oh$, $Og$ will be, in direction and intensity, the accelerations of points $H$, $G$, respectively. Through $H$ draw $HI$ parallel to $Oh$; it will cut at $I$ the direction $WI$ of the acceleration of the other mass at $W$. Through $I$ draw, parallel to $Og$, the line $IK$; it will be the actual location of the resultant force of inertia of the rod, and its intensity, as before, is equal to the product of the whole mass of rod $\times$ acceleration $Og$ of the center of gravity.

## Combination of Weight of the Rod with its Force of Inertia.

An exact determination of the pressure exerted by rod against the crank- and wrist-pins must include its weight.

We will, therefore, find the resultant of the weight and of the resistance due to inertia. As the inertia is measured by mass times acceleration, the expression of the force of inertia becomes:

$$\text{Inertia of rod} = \frac{W}{g} \times \frac{Og}{OC} \times \frac{v^2}{R} = F_2 \qquad (71)$$

$W =$ weight of rod in pounds;

$v =$ velocity of crank-pin in feet per second;

$R =$ radius of crank in feet; and

$g =$ acceleration due to gravity $= 32.16$ feet.

$Og$, $OC =$ accelerations of center of gravity and of crank-pin as given by Figs. 44 and 45. In the case of Fig. 46 we must

substitute in the above expression for $\overline{OC}$ the value $\overline{O'C}$. As the forces connected with engine are usually reduced to pressure per square inch of piston, the total inertia above given and the weight $W$ should be divided by the area $A$.

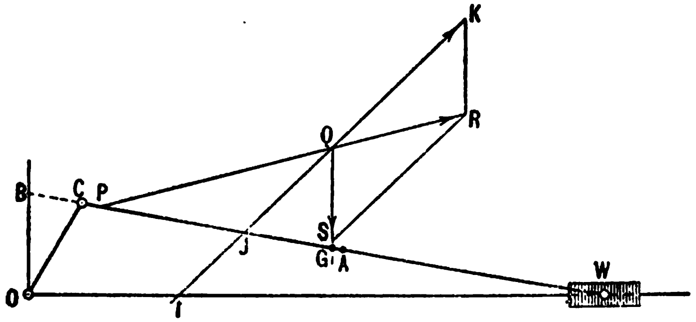



Fig. 47.

In Fig. 47 pass a vertical $GQ$ through the center of gravity $G$ till it cuts the direction $IK$ of the force of inertia of the rod. Make $QS$ = weight of rod per square inch of piston, and $QK$ = inertia-resistance of rod per square inch of piston. Complete the parallelogram; we get the diagonal $PQK$ as the desired resultant of weight and inertia. This resultant must be overcome by the combined action of the pressures of crank- and wrist-pin.

Example: Harris-Corliss engine, $26\frac{1}{8} \times 60$. R.p.m. = 60. Length of rod = 150 inches. Weight of rod = 1,200 pounds. Weight of other reciprocating parts = 1,300 pounds. Radius of crank = 2.5 feet. $v = 15.71$ feet per second. $v^2 = 246.74$. Principal polar radius of gyration = $k$ = 48 inches. Distance of center of gravity from wrist-pin = 78 inches. Distance of center of oscillation from center of gravity = 29.57. Assuming crank at 60° and drawing the mechanism to a scale of an inch to the foot, we get $\overline{Og} = 1.595$ inches and $\overline{OC} = 2.5$ inches. Substituting in the above formula and dividing by $A$ we have

$$\frac{F_2}{A} = 4.38 = \overline{QK} \text{ and } \frac{W''}{A} = 2.24 = \overline{QS}.$$

Exact computation* gives

$$\frac{F_2}{A} = 4.37,$$

which differs but 0·01 pounds per square inch from the result obtained by the graphical method. Considering the small scale, one-twelfth, employed, this shows the practical excellence of the graphical method.

In the above example the weight is one-half the inertia-resistance, but that is because the engine is large and of comparatively slow speed. In small, high-speed engines, the proportion is much less. Thus, in a *N. Y. S. S. P.* engine, 10 x 12, with rod of seventy pounds and 300 r. p. m. (*i. e.*, same crank-pin speed as in the large engine) we found

$$\frac{W''}{A} = 0.89 \text{ and } \frac{F_2}{A} = 9.21$$

or weight = one-tenth inertia. In the latter case the resultant will differ but little from the inertia in direction and intensity.

## F.

### Pressures at Crank- and Wrist-pin when there is no Friction at the Pins.

When there is no friction at the pins, let $P_c$ and $P_w$ represent respectively the crank-pin and wrist-pin pressure. Let line $lq$, Fig. 48, represent the direction of the resultant of the weight and inertia of rod, and let the intensity of this be assumed equal to $F_3 = lp = kh$. It is evident $P_c$ and $P_w$ combined must equal this resultant, and if one of these forces (say $P_w$) is known, the other $P_c$ can be found by the parallelogram of forces. To obtain $P_w$ we must satisfy two conditions, one of which is that $P_w$ must do its

* The computation was made by the help of formulas developed by Profs. Jacobus and Webb, in a paper published in *Trans. Am. Soc. Mech'l. Eng.*, vol. xi. The data for the above example was obtained from the same source (see p. 498).

part in producing the resultant $F_3$ acting along $lq$, and the other that $P_w$ must balance the resultant of the guide reaction $G$ and the driving force $K$ of the piston. This force $K$ is the difference between the effective steam-pressure $P$ and the inertia $F$ of piston, piston-rod and cross-head.

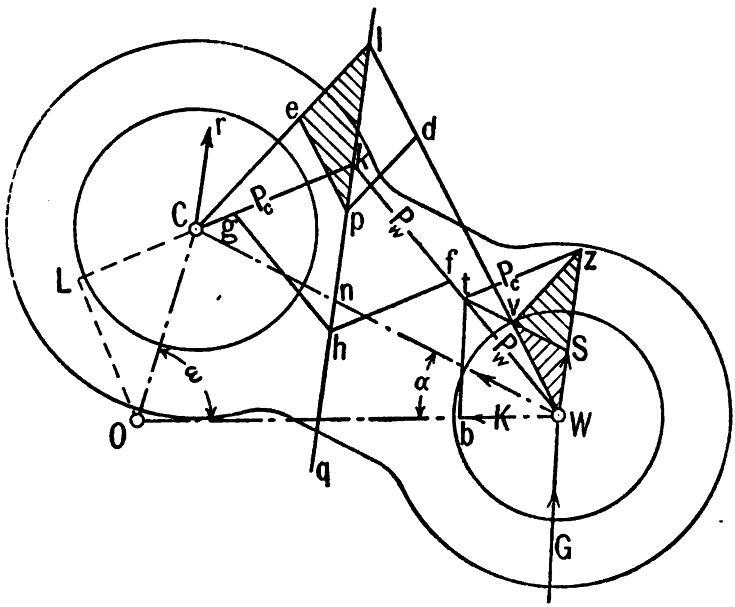



Fig. 48.

Through $W$ draw $Wz$ equal and parallel to $F_3 = lp = kh$. Join any point $l$ on $lq$ with $C$ and $W$. Draw $zv$ and $Wv$, respectively parallel to $lC$ and $lW$. Through the intersection $v$ draw $vS$ parallel to center line $CW$ of rod, which line also represents direction of internal stress, as $C$ and $W$ are the points of application of the forces. We will show later that this parallel $tS$ is the locus of the ends of all forces applied at $W$ that satisfy the first condition mentioned above. Now lay off $Wb = K = P - F_1$ $=$ driving force and draw $bt$ parallel to guide reaction $G$. The line $bt$ is the locus of the ends of all forces applied at $W$ that satisfy the second condition. The intersection $t$ of these two loci

$St$ and $bt$, gives $Wt = P_w$, the force desired. Prolong $Wt$ to point $k$ on line of resultant, then making $kh = F_3$ and $kf = Wt = P_w$ and completing the parallelogram, we get the crank-pin pressure $P_c = kg$ and the rotative effect $= P_c \times OL$.

For convenience of expression we will speak of the two components of each pin pressure as internal-stress component and inertia-component. *The latter expression is not strictly correct as the second component not only overcomes the inertia but also sustains the weight of the rod*, but this will not at all affect the accuracy of our results.

Since the force $P_c$ passes through crank-pin center $C$ and may have its point of application anywhere on its line of direction without affecting its accelerating capacity, we will assume this point to be at $C$, and since force $P_w$ passes through wrist-pin center $W$, we will, for the same reason, take its point of application at $W$. This will make the center line $CW$ of the rod the direction of the internal stress caused by either $P_c$ or $P_w$. Although the intensity of this internal stress varies as the inertia components $P_c$ and $P_w$ are varied, nevertheless there is but one pair of forces $P_c$, $P_w$ that will satisfy the conditions of this problem. To prove this it is only necessary to show that there is only one force $P_w$ that will satisfy the conditions at the wrist-pin. One condition is that it shall be in equilibrium with the known force $K = P - F_1 = Wb$ and the partly known reaction $G$ of the guides. By drawing through $b$ a parallel $bt$ to the known direction of $G$, we get one locus for the end of the force $P_w$. Another condition that $P_w$ must satisfy is, that it shall be the resultant of the internal stress acting along $CW$ and the corresponding inertia component acting in any arbitrarily chosen direction. Let $Wl$ be any such direction, then will $Cl$ be the direction of the corresponding inertia component that acts at the crank-pin $C$. If $kh = lp = Wz = F_3$ is the known resultant of the weight and inertia of the rod, then $ld = Wv$ is the intensity of the inertia component acting along the assumed direction $Wl$, and $le = vz$

is the inertia component along $Cl$. Now draw through $v$ a parallel $Svt$ to the inner stress $CW$; it will divide the triangle $Wvz$ into two triangles $WSv$ and $zvS$, respectively similar to triangles $Wnl$ and $lnC$. Hence

$$\overline{nW} \times \overline{SW} = \overline{vS} \times \overline{ln} = \overline{nC} \times \overline{Sz}$$

and

$$\frac{WS}{Sz} = \frac{nC}{nW} = \text{constant} \tag{72}$$

for this position and motion of the rod. Since $Wz$ is also constant for this position of rod, the parallel $Svt$ will pass through the same point $S$ whatever the direction and intensity $Wv$ of the inertia component. Therefore, the intersection of $Svt$ with the locus $bt$ will give $Wt$ as the only possible value of $P_w$ for this position and motion of the rod.

Prolonging $P_w$ to $k$ and drawing $kC$ and also $tz$, we get $P_c = kg = tz$. It is also evident from the figure that the reaction $G = bt$ of the guide is independent of the direction and intensity of the inertia-component.

## G.

### Pressures at Crank- and Wrist-pins when there is Friction at the Pins.

When there is friction at the pins, let $P_{cf}$ and $P_{wf}$ represent, respectively, the crank-pin and wrist-pin pressures, and let $GKL$, Fig. 49, represent as before the location of the resultant $F_3$ of weight and inertia of rod. We will again suppose $F_3 = GN = KQ = PO = TL$ to be known. In this, as in the former case, $P_{cf}$ and $P_{wf}$ combined must be able to sustain the weight and overcome the inertia of the rod.

Moreover, $P_{cf}$ must be tangent to a circle described from $C$ as a center with $\varphi \mathfrak{r}_c$ as a radius, and $P_{wf}$ must be tangent to a similar circle about $W$ with $\varphi \mathfrak{r}_w$ as a radius when $\varphi$ is coefficient of journal friction, and $\mathfrak{r}_c$, $\mathfrak{r}_w$ the radii, respectively, of crank-pin

and wrist-pin. These circles are called friction circles and each force is drawn on that side of the friction circle, which will give a component opposed to the motion of the eye or pin according as the action of the pin or action of the eye is the force under consideration.

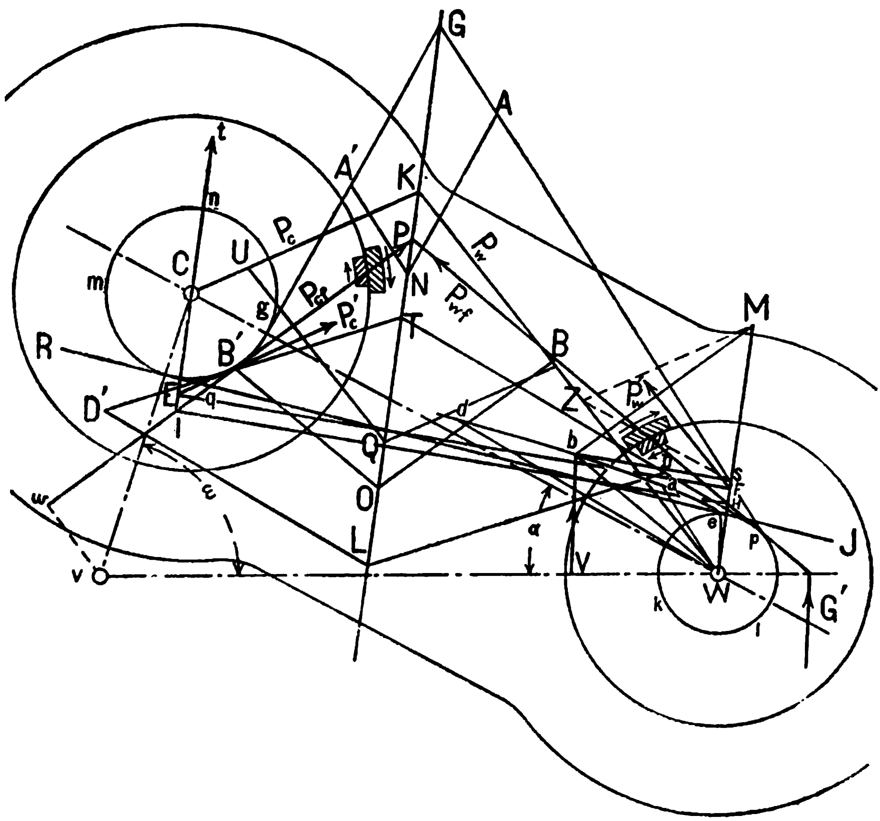



Fig. 49.

In Fig. 49 the friction circles have been greatly exaggerated, and the rod given unusual proportions for the purpose of better illustration.

With this modification the force $P_{wf}$ must satisfy the same two conditions as in the preceding case. But in that case it was

known that $P_w$ passed through the center $W$ of wrist-pin, and that point could, therefore, be assumed as its point of application. In the present case the possible values of $P_{wf}$ do not pass through such a common point, and we must, therefore, make use of trial methods to find $P_{wf}$.

We will first give a method which is, theoretically, the most exact, and then another, which is simpler, and, at the same time, accurate beyond the needs of practice.

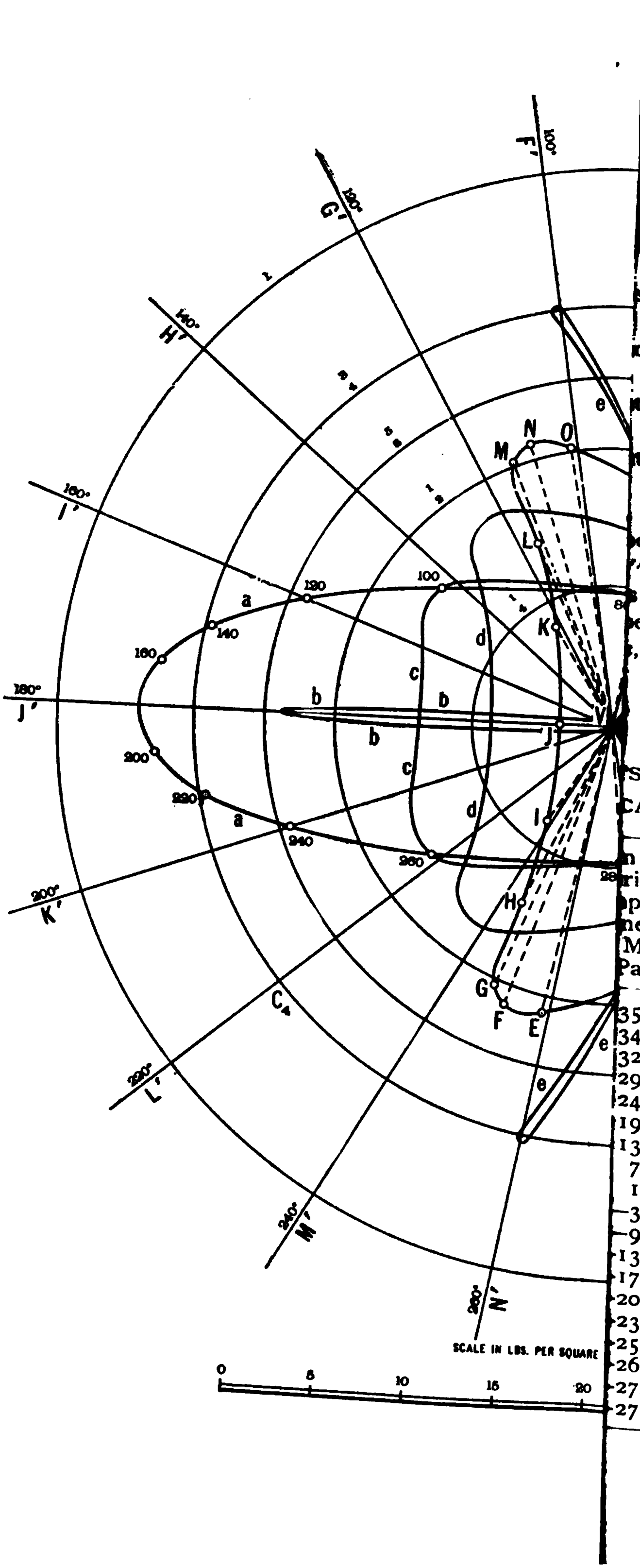
From each of the series of points $G, P, T$, on the line $GL$ draw a tangent to each of the two friction circles *mng* and *klp*. Make distances $GN = PQ = TL$ equal to the known force $F_3$ and resolve each of these distances into components along the respective tangents. Then with $W$ as a pole draw $Wa$ equal and parallel to $GA$, $Wb$ equal to $PB$, also $Wd$ equal and parallel to $TD$, and so on, thus getting a polar curve *eabd*, whose vectors satisfy the condition that the pin forces shall be capable of balancing the resultant $F_3$. The second condition is the same in this as in the preceding case, namely, that the wrist-pin pressure $P_{wf}$ shall be in equilibrium with the driving force $K = \overline{WV} = P - F_1$ and the given reaction $G$ (known in direction only). Now laying off $\overline{WV} = K$, drawing $Vb$ parallel to the guide reaction $G'$, and joining its intersection $b$ (with the polar curve *eabd*) to the pole $W$, we get in $Wb$ the exact intensity and direction of the wrist-pin pressure $P_{wf}$, but not its location. The latter is found by drawing $HPB$ parallel to $Wb$ and tangent to the friction circle *plk*. Finally, drawing $PB'I$ tangent to friction circle *mng* of pin $C$, and constructing the parallelogram of forces, we have $PB = Wb = P_{wf}$ and $PB' = P_{cf}$, the exact pin forces desired. The rotative force is measured by $P_{cf} \times \overline{vw}$.

In the simpler and only theoretically less exact method, the principal steps are like those taken in finding $P_c$ and $P_w$, in Fig. 48. The direction of the internal stress is, however, no longer $CW$, and the determination of a convenient direction for it, constitutes the main portion of the problem.

8

To illustrate the method, suppose the problem solved and the forces $P_{cf}$ and $P_{wf}$ just found to be each resolved into two components, one parallel to $GL$ and passing through $C$ and $W$, respectively, and the other in the direction of the internal stress. The two components parallel to $GL$ must together just equal $F_3$ and, therefore, be capable of sustaining the weight and overcoming the inertia of the rod. The direction of each of the two internal stress components will evidently be $HI$, and according to d'Alembert's principle they will just balance each other. Inspection of the figure will show that for friction-circle tangents differing but slightly in direction from $P_{cf}$ and $P_{wf}$, the direction of the internal stress will also differ but slightly from $HI$. This is the case even in the present figure, where the rod is very short and the friction circles excessively large. Now draw $P'_w$ parallel to $P_w$ and tangent to the friction circle $plk$, the location of this tangent will differ but little from the actual force $P_{wf}$. For the same reason $P'_c$ parallel to $P_c$ and tangent to $qmn$, will differ but little from $P_{cf}$. Joining the intersections $F$ and $E$ (of $P_w'$ and $P_c'$ with the parallel components $WM$ and $Ct$, respectively), we get in $FE$ a line whose direction is nearly parallel to the exact direction of the internal stress $HI$.

Our procedure in finding $P_{wf}$ is, therefore, as follows: We get the component $WS$ as it was got in Fig. 48: we next draw through $S$ a parallel $Sb$ to the line $FE$, just described; then will $Sb$ be a locus of the end $b$ of the desired force $Wb$. But another locus is needed to completely determine $b$; this is given by the condition that $Wb = P_{wf}$ shall be the resultant of the driving force $K = P - F_1$ and guide reaction $G'$. Therefore, laying off $WV = K$ and drawing $Vb$ parallel to $G'$ we get a second locus whose intersection $b$ with the first locus $Sb$ gives the desired intensity $Wb$ of the force $P_{wf}$, for $Wb$ is common to each of the force triangles $WSb$ and $WVb$, thus satisfying both conditions imposed upon $P_{wf}$. The exact location of $P_{wf}$ may now be found by drawing $pBP$ parallel to $Wb$ and tangent to the friction circle $plk$. The accuracy of the work may be checked by completing the parallelogram $OBPB'$ and seeing if $PB = Wb$.




of the lines representing counterweight.

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d and reciprocating parts ″ of piston; hence cen- equivalent to ⅝ of the er □″.

, *a*, are given in the last

*b*, *b*, *d*, *d*, *d*, *d*, etc., see

## S OF RESULTANT OF CATING PARTS.

| n of the rizontal ponents nertia of Moving Parts. | Sum of the Vertical Components of Inertia of Connecting Rod. Lbs. per □″. |
|---|---|
| 35.17 | 0.00 |
| 34.46 | 1.33 |
| 32.37 | 2.62 |
| 29.02 | 3.83 |
| 24.63 | 4.92 |
| 19.42 | 5.86 |
| 13 66 | 6.63 |
| 7.64 | 7.19 |
| 1.70 | 7.53 |
| −3.99 | 7.65 |
| −9.18 | 7.53 |
| −13.72 | 7.19 |
| −17.57 | 6.63 |
| −20.74 | 5.86 |
| −23.21 | 4.92 |
| −25.06 | 3.83 |
| −26.34 | 2.62 |
| −27.06 | 1.33 |
| −27.29 | 0.00 |

The approximation just given can be still further simplified for all cases that may arrise in practice. In such cases the friction circles are very small and far apart, and the tangent $RJ$ to both friction circles is scarcely distinguishable from the direction $HI$ or $EF$. We may therefore treat this tangent $RJ$ as the direction of the internal stress, draw $Sb$ parallel and then proceed as above in the determination of $Wb$ and location of its equal $P_{wf}$. The inaccuracy involved in this last, approximate, construction is much less than that connected with the best determined coefficients of friction.

## H.

### Determination of the Forces Tending to Shake the Engine Bed.

The constant forces acting on the engine bed, like the weight of fly-wheel and pull of belt, tend only to shift the bed. It is the variable forces that tend to shake it, namely, the effective steam pressure $P$ against cylinder covers, the pressure of the cross-head against the guides and the pressure of the crank-shaft against its bearings. By constructing a polygon of forces, it can be shown that the force tending to shake the engine bed is the resultant of all the unbalanced accelerating forces acting on the different links of the machine. In other words, the shaking force is the resultant of the forces of inertia of the moving parts.

The same conclusion is reached from general considerations. Suppose the forces of inertia to be replaced by their resultants acting as external forces. Then the whole system of external forces will be in equilibrium, the moving pieces may be considered at rest for the instant, the whole machine acting as one rigid piece. As the steam then acts equally in opposite directions upon the machine, its shifting and shaking influence are both nil. The internal forces at the bearings occur in pairs of equal and

opposite forces, and their shaking influence is therefore likewise nil. The only external forces remaining are the resultants due to inertia, the action of gravity and the pull of the belt. The weights are constant forces, but their points of application change with the motion. In stationary engines we may neglect the shaking influence due to this cause. As regards the pull of the belt Mr. W. Willis has shown by his experiments that the sum of the two tensions is not a constant quantity, as is generally assumed, but increases with the load. If we suppose the load constant the belt pull will also be constant, and will only tend to shift the bed without shaking it. The only external forces remaining are the variable forces of inertia, and these do tend to shake the engine frame. In the case of the engine, therefore, the shaking forces are, the resistances due to the inertia of the purely reciprocating pieces (piston, piston-rod and cross-head), to the inertia of the connecting rod and to the inertia of the crank disc. The first and last are easily obtained by computation, and the second can be obtained by the construction shown in Fig. 44 or Fig. 45.

## I.

## Diagram of Forces Tending to Shake Engine Bed.

Fig. 50 is such a diagram prepared by Prof. D. S. Jacobus and published in the Trans. Amer. Soc. Mech'l Eng'rs, Vol. XI. It contains a series of polar curves, *aaa*, *bbb*, *ccc*, etc., whose vectors, drawn from the pole or center $V$, represent in direction and intensity the resulant forces of inertia of all moving pieces. The data for the construction of the diagram are given on the same page, and under Case I, Table XI. The table accompanying Fig. 50 contains the components or the force of inertia for this particular engine, and is in convenient shape for use. The last column contains the ordinates, like $C_3C_2$, of the polar curve *aaaa*, and the last column but one the abscissas, like $VC_3$, of this same polar curve. As this polar curve *aaaa*, corresponds to the case of no counterweight, its vectors represent in direction and inten

sity the resultant of the forces of all moving pieces, except the crank. The remaining curves *bbbb*, *cccc*, *dddd*, etc., can therefore be directly deduced from curve *aaaa*, by laying off from each point *a*, and parallel to the crank position corresponding to this point *a* the centrifugal force of counterweight on crank disc.

For instance, when crank is at 40°, the vector $VC_2$ will be the resultant of the inertia-forces of all the reciprocating parts. To get a point on curve *bbbb*, for the same crank position of 40°, we lay off $C_2b = \frac{1}{4} \times 31.25 = 7.81$ parallel, but opposite to $VC'$. To get a point on curve *dddd*, for this crank angle of 40°, we lay off on the same line $C_2bd = \frac{5}{8} \times 31.25 = 19.53$ lbs. per □″, and so on for the other curves.

As the vertical components of curve *bbbb* are very small, its small amount of counterweight would be suitable for cases in which the shake occurs more readily in a vertical than in a horizontal direction, as when a horizontal engine is placed on an upper floor of a building. On the other hand the horizontal components of curve *eeee*, are small and the corresponding, heavier, counterweight is preferable when the horizontal forces produce most shake, as in engines set on tall foundations.

The diagram just described is suitable for stationary engines, and can readily be constructed for such cases as soon as the acceleration of the center of gravity of rod has been found graphically, (see page 101, Eq. 71 and Fig. 44 or 45). We have then to multiply this acceleration by the mass of the rod per □″ of piston, and resolve this force into two components, one horizontal and the other vertical. Of these two components add that which is parallel to stroke to the accelerating force (per □″ of piston) of the piston, piston-rod and cross-head. This sum will be one of the coordinates of the curve *aaaa* (Fig. 50). The remaining component of the rod will be the other coordinate of curve *aaaa*. The influence of counterweight can then be added in the manner already described in connection with Fig. 50. The diagram represents the shaking forces acting on engine bed in the vertical and axial of the engine.

When the engine frame is suspended on springs, as in locomotives, the exact location of the total accelerating force of rod should be taken into account. The exact location of the weights is also of consequence in determining the oscillations of the locomotive. Indeed, in this case it would be well to combine separately the force of inertia and weight of each moving piece and then combine these resultants into a total resultant with its point of application on the line of stroke and lay off each of these resultants from its own point of application and with its own direction and intensity.

Construct the curve *aaaa* for the engine to be designed, determining the inertia of the connecting rod by the constructions given in Fig. 44 or 45. For method of ascertaining the radius of gyration experimentally or by computation, see Appendix.

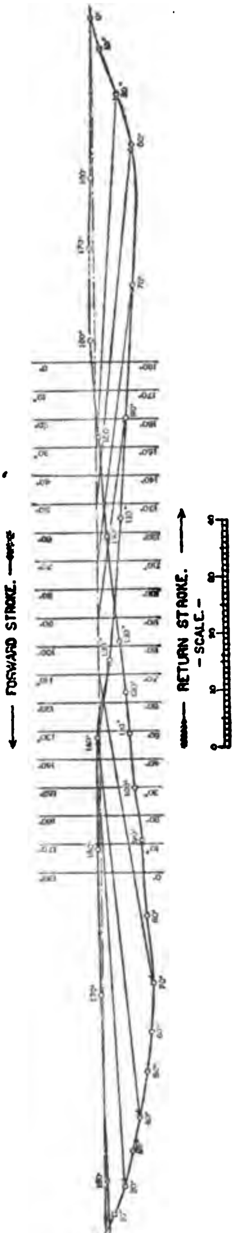


Fig. 51.

## J.

## Diagrams of Pressures at Crank- and Wrist-Pin.

Rapid changes in the directions of the forces acting at the pins are approximately indicated by such diagrams as Fig. 27. They do not necessarily indicate reversals, this term being understood to mean an instantaneous change of pressure from one side to the other of a pin. If the pin had been worn into an oval shape then it is possible that the changes shown in Fig. 27 might be accompanied by shocks.

In Fig. 51, the pressure of connecting-rod on wrist-pin is shown in direction and intensity for the engine whose data are given under Case I, Table XI, when friction, weight and inertia are all taken into account. These forces are laid off from what is equivalent to a time base, namely, the developed crank-pin circle. Rapid changes of direction of pressure occur between 140° and 150° for both forward and return strokes. It is not however instantaneous, continuous contact being preserved between the pin and its bearing.

But between slide and its guides the contact very suddenly changes from one side to the other in the neighborhood of 150° forward stroke and 160° return stroke, provided the upward,

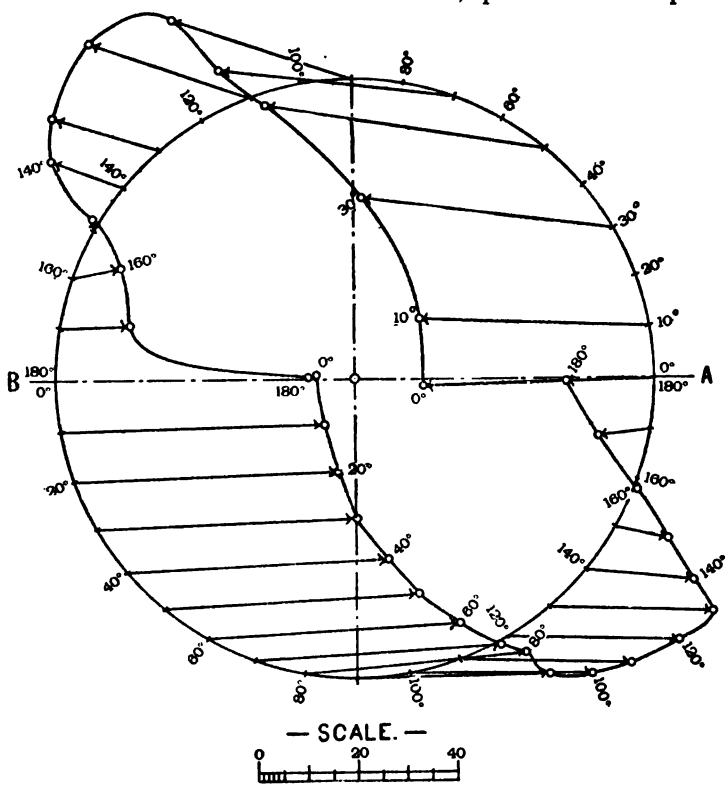


Fig. 52.

vertical, component or pressure of wrist-pin is sufficient to overcome the weight of cross-head and that part of piston-rod which is sustained by the slide. In such a case a pressure of slide against guide would suddenly change from a downward to an upward pressure and be accompanied by a more or less violent shock.

In Fig. 52 the pressure of connecting-rod on crank-pin is given in direction and intensity for the same engine and conditions assumed in Fig. 51. Between 140° and 160° of forward stroke and 150° and 170° of return stroke a rapid change of direction of force occurs, but continuous contact between pin and eye is maintained even then and no shock will occur at crank-pin under these particular circumstances. At the inner, or front dead point *A* there is a sudden change of pressure on crank-pin, which may cause tremor in the rod but no shock.

## IX.

### Diameter and Width of Belt Pulley.

Generally in high speed engines the fly-wheel acts also as belt pulley. In such a case the diameter is determined by the data given on pp. 75 and 76. When the diameter is not fixed in this way or by some special condition and there is thus left some choice, it should be so made as to give the belt a high speed, even a mile a minute is permissible, for the belt becomes narrower and the efficiency increases with the speed, the percentage of slip and the journal friction being smaller at the higher speed.*

*See pp. 347–360, also pp. 568 of Vol. VII. Trans. Amer. Soc. M. E. Mr. W. Lewis in experiments made for Sellers & Co., found that with vertical belts, the sum of tensions while running might increase to $\frac{4}{3}$ of the sum existing when belt is stationary; that with horizontal belts the sum of the tensions might increase up to the breaking limit. In both cases this increment in the sum is accompanied by increase of slip. In one extreme case, with the permissible slip of 3 feet per minute, the increment of the sum was as high as 100%. This does not call for any modification of the results given above however, as the coefficient of friction is dependent upon the slip and the assumption of a 0.27 coefficient means a definite slip and a ratio of tensions, in tight and slack sides, of about 2.4.

When belt speed is known, its width can be ascertained from the following table, provided the belt is laced and connects two equal pulleys,

| | | | | | | |
|---|---|---|---|---|---|---|
| Belt speed | = 1000, | 2000, | 3000, | 4000, | 5000, | in ft. per min. |
| Horse-Powers transmitted | = 5.0 | 9.8 | 14.3 | 18.3 | 20.8 | per □″ of belt |

This table assumes a permissible working stress of 300 lbs. to the square inch at laced joint and a coefficient of friction = 0.27 for leather on cast iron. For a riveted joint the working stress would be larger say from 400 to 550, and the H. P. trasnmitted per □″ of belt from 1.4 to 2 times greater. Some authorities use a coefficient of friction of 0.40, which would make the H. P. per □″ in the above about ¼ larger, but it would be accompanied by a velocity of sliding of belt on pulley of much more than three feet per minute.

When pulleys connected by belt are unequal, account must be taken of the arc of contact on the *smaller* pulley, by multiplying the horse-power given above by the fractions in this next table:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Arc of contact | = 120° | 130° | 140° | 150° | 160° | 170° | 180° | degrees. |
| Multiplier | = 0.79 | 0.83 | 0.87 | 0.91 | 0.94 | 0.97 | 1.00 | |

The maximum horse-power that the engine is likely to develop must then be divided by the H. P. per □″ of the belt found by these tables and the result will be the area of the cross-section of the belt. The area is given because there is considerable variation in belt thickness, and the mere designations of single and double do not necessarily mean $\frac{7}{32}$″ and $\frac{7}{16}$″ thickness respectively. Of course to get the belt width the area of belt is divided by the thickness.

In this engine assume that a double belt is to be used on the fly-wheel as a belt pulley and that the arc of contact on the driven pulley is 165°.

Many high-speed engines, like the center-crank type, are provided with two fly-wheels or belt pulleys and when both are used

for the purposes of transmission there will be a corresponding reduction in the area or width of each belt.

It is usually in connection with Fly or Band Wheels that we must distinguish between right and left-hand engines, between those that "run over" and those that "run under." Standing at the cylinder end and looking towards the crank, a $\left\{\begin{matrix}\text{right-hand}\\ \text{left-hand}\end{matrix}\right\}$ will have the fly-wheel and main bearing to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of the axis of the cylinder, and still retaining the position at the cylinder, if the top of the fly-wheel moves $\left\{\begin{matrix}\text{away from}\\ \text{towards}\end{matrix}\right\}$ the observer, the engine is said to $\left\{\begin{matrix}\text{run over}\\ \text{run under}\end{matrix}\right\}$.

## X.

### Graphical Determination of Diameter of Crank Shaft.

The problem is one of compound stress, and has been treated by Reuleaux in his Konstrukteur. Both bending and twisting forces strain the shaft. The bending forces are: weight of the fly-wheel, pull of the belt, the centrifugal force of the counterweight on crank disc, the pressure of connecting-rod against crank-pin and the reaction of the bearings. The twisting force is the moment of connecting-rod against crank-pin.

As the bending forces are not parallel, and do not all act in the same plane, the first step is to combine these separate bending moments into one resultant bending moment. The next step is to combine this resultant bending moment with the twisting moment so as to form either an equivalent bending, or an equivalent twisting moment. In the present case we will reduce all the moments to one equivalent bending moment.

The first step may be simplified by certain preliminary reductions and approximations. The weight of fly-wheel and the pull of the belt act in the same plane, and can be reduced to a single

force. The belt pull is the sum of the tensions on the tight and slack side of the belt, and this sum is not constant, but increases, while running, with the load. But this does not call for special calculation at this place, because this increase in the sums of the tensions is limited by the slip, and this was assumed at 3 feet per min. as a maximum. We will therefore assume the sum of tensions to be 1½ times the tension on the tight side of the belt when transmitting the maximum power. This, under our assumptions (p. 17) will generally be somewhat in excess of the true sum. Assuming that shop shaft and cylinder are on opposite sides of crank shaft, and that the belting makes an angle of 30° with the horizon, we can now combine the belt pull with the weight of the fly-wheel by the parallelogram of forces and note the angle made by the diagonal with the horizontal. The centrifugal force of the counterweight and the pressure of the connecting rod against crank-pin do not act in the same plane and should be given separate diagrams, if great exactness is desired. But such a refinement is here entirely unnecessary, and we may approximate by assuming them both to act in some convenient, intermediate, plane which is nearest to the larger of these two forces. These two may then be combined by the parallelogram of forces, again noting the angle the diagonal makes with the horizontal.

The bending forces have thus been reduced to two forces and to the reactions of the bearings. We now construct by the methods of graphical statics a diagram of bending moments for each of these forces.

Suppose that in Fig. 53 the scale of distances is ¼, or 3″ to the foot, and the scale of forces is, say 200 lbs. to the inch. The polygon of forces due to the fly-wheel resultant found above, is $ABC$, the length $AB$ representing the reaction of the out-board bearing and $BC$ the reaction in the main bearing, the two reactions having the ratio $\overline{MN} \div \overline{NQ}$. Choosing $O$ in the polygon of forces on the horizontal through $B$, the closing line $AD$ of the equilibrium polygon $ADGF$ will also be horizontal, and thus

convenient for subsequent combinations. The distance $OB$ of the pole $O$ from $AC$ is chosen so as to give a convenient factor for multiplying the vertical chords or intercepts $HI$ of diagram of moments $ADGF$. We know from the principles of graphical statics that the product of the intercept $HI$ by the constant $OB$ measures that part of the bending moment at cross section $KL$ of shaft which is due to the forces on the fly-wheel. As $OB$ is constant, the intercepts themselves may be taken to represent the bending moments of the cross-sections of the shaft directly above them. The numerical value of any bending moment is found by multiplying the proper intercept by the distance $OB \times 200 \times 4$, the last two factors respectively representing the scale of forces and the scale of distances on drawing.

If we take $OB$ equal 4 inches, the product of these three factors $= 3200$, which is a convenient factor with which to multiply directly the intercepts $HI$ obtained from $ADGF$, the diagram of moments.

In like manner for the second resultant, due to counterweight and crank-pin pressures, we find the polygon of forces $A'B'C'O'$ and the equilibrium polygon $D'E'C'$.

This second resultant is constantly varying in direction and intensity with the rotation of the crank. It is greatest at the beginning of the forward stroke. But this does not correspond with the piston position at which the maximum twisting moment exists. As the greatest stress on the shaft probably occurs near this position, we shall suppose the diagram of moments due to this second resultant to be found for the crank position corresponding to the maximum twisting moment. The vertical intercepts of polygon $D'E'C'$ represent the bending moments, which are due to the action of this second resultant and its numerical value for any cross-section is found by multiplying 3200 the intercept of this diagram immediately below the cross-section in question.

These two diagrams of bending moments are both placed, for convenient combination, in the plane of the paper, though the forces calling them forth are neither parallel nor do they act in

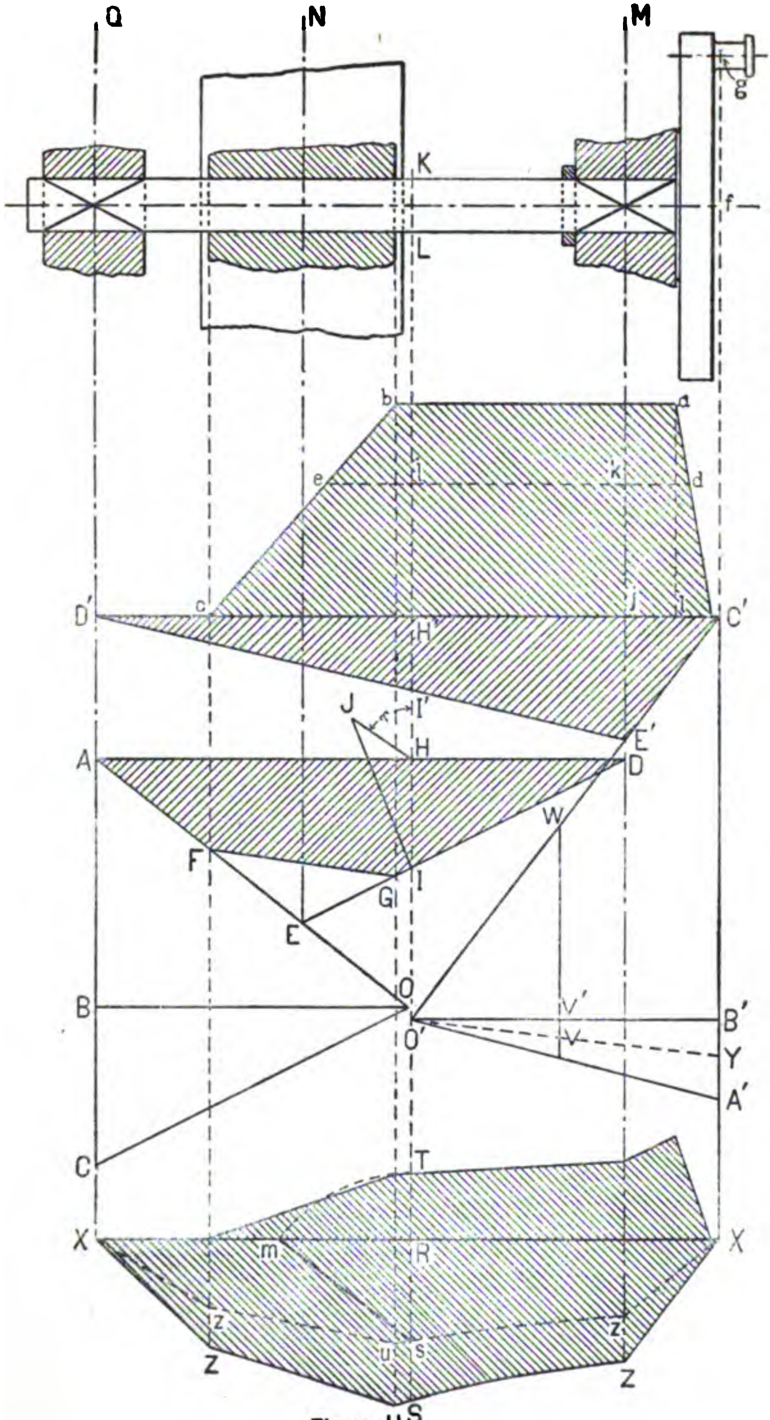

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Fig. 53.

the same plane. As the intensities of the bending moments are represented by linear quantities and their directions are those of the forces producing them, they may be combined like forces, by means of the parallelogram of forces. This we will now proceed to do, being careful to take into account the angle between the forces. It will probably often happen that this angle is a small one, in which case the intercepts of the two diagrams may, with little error for the present purpose, be directly $\left\{\begin{matrix}\text{added}\\ \text{subtracted}\end{matrix}\right\}$ when the directions of the bending moments developed at any cross-section by the two resultants are $\left\{\begin{matrix}\text{alike}\\ \text{unlike}\end{matrix}\right\}$.

When the angle $I'HJ = \Upsilon$, included between the arrowheads of the forces is large they should be combined on the principle of the triangle of forces. In the figure this is illustrated by laying off the length $H'I'$ of one diagram on $HJ$ and joining $J$ and $I$. We now take this line $IJ$ of the second diagram and lay it off below the horizontal $XX$ making it equal to $RS$, which is located on the vertical $SRIHH'$ passing through the assumed cross-section $KL$. Repeating this process for every other cross-section we get a third (the lowest) diagram $XRXZSZ$ whose intercepts represent the resultants of the purely *bending* moments attacking each cross-section.

The next step is to combine this resultant bending moment $M_b$ with the twisting moment $M_t$, so as to get an equivalent or ideal bending moment $M_{bi}$. The formula connecting these three quantities is:

$$M_{bi} = \tfrac{3}{8} M_b + \tfrac{5}{8} \sqrt{M_b^2 + M_t^2}. \qquad (73)$$

Before undertaking the graphical construction of this formula we will construct the diagram of twisting moments to the same scale as the bending moments. The twisting moment varies for the different crank-positions, but we will at once assume its maximum value $P_{cf}R$. To have the intercepts of the diagram of twisting moments on the same scale we must make $P_{cf} \times R = \overline{O'B'} \times y$. Making, in one of the polygon of forces,

$C'Y = P_{cf}$ and $O'V' = R = fg$, we get, by similar triangles, the distance $VW = \frac{P_{cf} \times R}{O'B'} = y$. Now lay off $la = y$ above the horizontal $C'D'$ and through $a$ draw $ab$ horizontal. The inclined line $bec$ can be taken to represent the varying twisting moment of crank-shaft within the fly-wheel hub. The vertical intercepts of diagram $abcC'$ represent the twisting moment on the same scale as the other moments.

We can now return to the graphical construction of the last formula. The last term containing the radical can also be written

$$\sqrt{(\tfrac{5}{8} M_b)^2 + (\tfrac{5}{8} M_t)^2}.$$

The first term $\frac{5}{8} M_b$, under the radical is found by taking $\frac{5}{8}$ of each vertical intercept of the diagram $XRXZ\,USZ$. For instance, $Rs = \frac{5}{8} RS$. The broken line in this diagram represents this division of $RS = M_b$ into $\frac{5}{8}$ and $\frac{3}{8}$ segments. In like manner $\frac{5}{8} M_t = kj = \frac{5}{8} la$ can be found. Making $Rm = \frac{5}{8} M_t = H'i$, the hypothenuse $ms$ ($= sT$) of the right-angled triangle $mRs$ is the graphical equivalent of the last, radical, term of our formula. Adding it to $Ss = \frac{3}{8} M_b$ we get $ST = M_{bi}$. Its numerical equivalent, with the scales assumed above, is $M_{bi}$ = length of $ST \times OB \times \frac{1}{4} \times 200$ inch lbs. This value of $M_{bi}$ is to be substituted in the general formula for shafts subjected to bending stress

$$d = 2.17 \sqrt[3]{\frac{M_{bi}}{f_b}} \text{ inches,} \tag{74}$$

where $f_b$ is the permissible working stress in lbs. per $\square''$ when shaft is subjected to bending stress. The result may be checked by the following empirical formula representing high-speed engine practice, for diameter of main, crank-shaft, journal

$$d = 0.44\, D + \tfrac{1}{2}; \tag{75}$$

here $D$ represents the diameter of the cylinder.

Having indicated the general method of procedure it will now be easy to apply the graphical solution to cases in which the fly-wheel overhangs the bed.

## XI.

## Determination of Plane of Division of the Brasses and Length of the Main Bearing.

The plane of division of the brasses should be at right angles to the resultant of all those pressures which exist when the engine is running with its average load. These pressures are components of the weight of shaft and fly-wheel, pull of belt, centrifugal force of counterweight and the thrust or pull of connecting-rod. For each crank position these four components may be combined by the polygon of forces so as to give the resultant force on bearing.

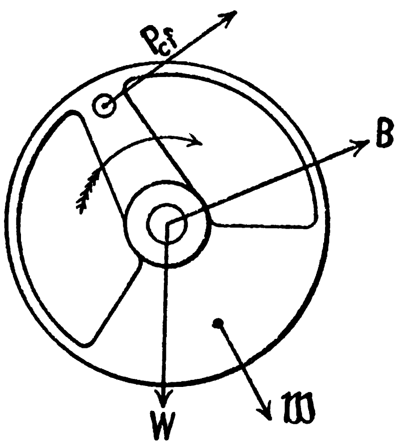



Fig. 54.

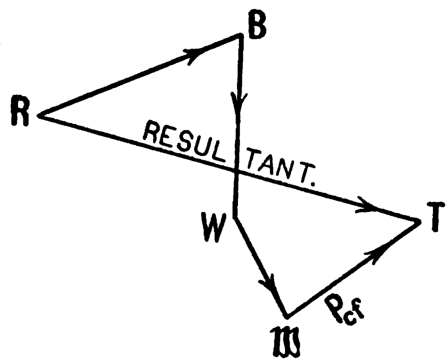



Fig. 55.

In Figs. 54, 55 and 56 these forces are shown and their force polygon. For a given load, speed and engine, $RB$ and $BW$ may be regarded as constant in direction and intensity, consequently, $RW$ may have an invariable position on the diagram Fig. 56. As the action of the counter-weight is constant in intensity and opposite to the crank in direction, we may take $W$ as the center of a circle described with $W\mathfrak{w}$ as a radius. From the points of this circumference we draw $\mathfrak{w}T$ equal and parallel to the (reduced)

values of $P_{cf}$ corresponding to the different crank positions, then will $RT$ represent the resultant of the two pressures exerted by the shaft against its main bearing. Fig. 56 shows two determinations, $RT$ and $RT'$, of these resultant pressures against the bearings for the 30° and 60° crank positions.

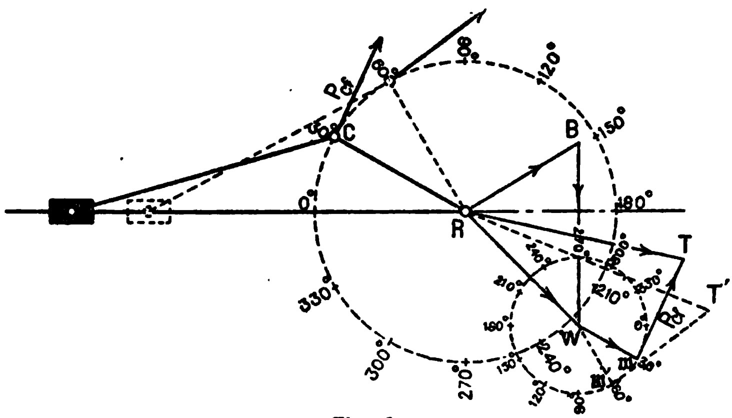



Fig. 56.

The forces in this figure (56) have been chosen at random and do not represent any special case. When a graphical determination, Fig. 53, of the diameter of the crank shaft has been made, the components of these pressures at the main bearing can be found from the force polygons there drawn. A resultant should be drawn for each of a series of equidistant crank positions and the plane of division of the brasses placed at right angles to the resultant of these resultants. Great accuracy is not necessary, in most cases inspection will determine best position of plane.

For the determination of the length $l$ of crank-shaft journal of a high-speed engine the following empirical formula, representing good practice, may be used

$$l = 1\tfrac{1}{2}\, d + 3''.0 \qquad (76)$$

where $d$ is the diameter of same journal determined according to another empirical formula, for same class of engines,

$$d = 0.44\ D + \tfrac{1}{2}'',$$

$D$ representing the diameter of piston.

## XII.

### Determination of the Dimensions of Steam Passages.

The first dimension to be settled in valve-gear problems is the requisite area of the ports, for upon this depends the maintenance of the desired steam pressure. The principal factor in its determination is the velocity of the steam current and as this varies with the piston's motion in different parts of the stroke, we must see to it that the minimum cross-section in the passage is adequate for every part of the stroke. This part of the problem divides itself into two parts: (*a*) the ascertainment of the minimum area needed at the given piston speed and with given cylinder area, and (*b*) an examination of the cross-sections of the steam passages that are really available or effective with the special type used, taking into account the contraction caused by the flow past edges and the narrowing of ports caused by the motions of the valves on each other or on their valve seats.

This second part presents no theoretical difficulties and we shall therefore not examine any special case here, but this should be done in the draughting-room when the type of valve has been chosen. It is practically important in case there are supplementary, or multiple, ports in the main or distribution valve. Allowance should also be made for the contraction of the steam current when the ports are very narrow, for this effects a very notable reduction of the port area geometrically available. We will now consider the minimum port area necessary for a given area of cylinder and a given piston speed.

Radinger's criterion of sufficient passage area was whether or not the admission line of indicator card remained horizontal up to cut-off.

According to Radinger's experiments,

$$\frac{\text{Area of port}}{\text{Area of cylinder}} = \frac{bl}{A} = \frac{\text{average speed of piston in feet per sec.}}{100} \qquad (77)$$

This rule evidently corresponds to an average velocity of 100 feet per second of the current of steam in the port; but the experiments seem to have been made on engines in which the maximum cut-off was equal or greater than 0.5, consequently if we assume that in the engines experimented upon the ratio of connecting rod to crank varied from 4 to 6 we can easily determine the maximum permissible velocity of the steam in the ports.

For when $\dfrac{\text{connecting-rod}}{\text{crank}} = 4$ to 6 we have the ratio of

$$\frac{\text{Max. velocity of steam in ports}}{\text{Ave. velocity of steam in ports}} = 1.62 \text{ to } 1.59 \text{ respectively.} \qquad (78)$$

This gives about 160 feet per second as the maximum permissible velocity of the current of steam in the ports when the steam passages are long and narrow. Radinger says that when the steam passages are short and the cut-off short (for instance, when there are separate admission valves for each end of the cylinder) somewhat smaller port areas may be employed than would result from above rule; in other words, in such cases the maximum permissible velocity of the current of steam may be somewhat greater than 160 feet per second.*

Mr. Charles T. Porter gives the rule that the velocity of the current of steam in the short ports should not exceed 200 feet per

* Some American engineers prefer the rule that the *average* speed in short steam ports should not exceed 150 feet per second, and in the exhaust ports should $\leqq$ 125 feet per second. When the steam is admitted and discharged through the same passage, some engineers recommend that its cross-section be proportioned for the exhaust, preferring an increase of clearance to an increase of back pressure.

# TABLE XIII.

## Velocity of Piston for a Unit of Velocity of Crank-Pin.

To obtain actual velocity of piston multiply tabular quantity by actual velocity of crank-pin.

Forward stroke is towards, and return stroke, away from, crank shaft.

| Crank Angles. | | Connecting Rod ÷ Crank = | | | | | |
|---|---|---|---|---|---|---|---|
| Forw'd. | Return. | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 | ∞ |
| 5 | 175 | .1089 | .1064 | .1045 | .1030 | .1016 | .0832 |
| 10 | 170 | .2164 | .2117 | .2079 | .2047 | .2022 | .1737 |
| 15 | 165 | .3215 | .3145 | .3089 | .3054 | .3005 | .2588 |
| 20 | 160 | .4227 | .4136 | .4065 | .4019 | .3957 | .3420 |
| 25 | 155 | .5189 | .5081 | .4995 | .4925 | .4866 | .4226 |
| 30 | 150 | .6091 | .5968 | .5870 | .5791 | .5724 | .5000 |
| 35 | 145 | .6923 | .6788 | .6682 | .6596 | .6523 | .5736 |
| 40 | 140 | .7675 | .7533 | .7421 | .7329 | .7253 | .6428 |
| 45 | 135 | .8341 | .8195 | .8081 | .7988 | .7910 | .7071 |
| 50 | 130 | .8914 | .8771 | .8657 | .8564 | .8488 | .7660 |
| 55 | 125 | .9392 | .9253 | .9144 | .9055 | .8982 | .8192 |
| 60 | 120 | .9769 | .9641 | .9540 | .9458 | .9390 | .8660 |
| 65 | 115 | 1.0046 | .9932 | .9842 | .9769 | .9709 | .9063 |
| 70 | 110 | 1.0224 | 1.0127 | 1.0052 | .9990 | .9939 | .9397 |
| 75 | 105 | 1.0304 | 1.0228 | 1.0169 | 1.0121 | 1.0082 | .9659 |
| 80 | 100 | 1.0289 | 1.0237 | 1.0199 | 1.0164 | 1.0137 | .9848 |
| 85 | 95 | 1.0186 | 1.0160 | 1.0139 | 1.0127 | 1.0109 | .9962 |
| 90 | 90 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 95 | 85 | .9738 | .9764 | .9785 | .9797 | .9816 | .9962 |
| 100 | 80 | .9407 | .9460 | .9500 | .9532 | .9559 | .9848 |
| 105 | 75 | .9016 | .9091 | .9150 | .9198 | .9237 | .9659 |
| 110 | 70 | .8571 | .8667 | .8743 | .8804 | .8855 | .9397 |
| 115 | 65 | .8080 | .8194 | .8285 | .8357 | .8418 | .9063 |
| 120 | 60 | .7552 | .7680 | .7781 | .7863 | .7931 | .8660 |
| 125 | 55 | .6992 | .7130 | .7239 | .7328 | .7401 | .8192 |
| 130 | 50 | .6407 | .6550 | .6664 | .6756 | .6833 | .7660 |
| 135 | 45 | .5801 | .5946 | .6061 | .6155 | .6232 | .7071 |
| 140 | 40 | .5181 | .5323 | .5435 | .5527 | .5603 | .6428 |
| 145 | 35 | .4549 | .4683 | .4790 | .4876 | .4949 | .5736 |
| 150 | 30 | .3909 | .4032 | .4130 | .4209 | .4276 | .5000 |
| 155 | 25 | .3264 | .3371 | .3458 | .3528 | .3586 | .4226 |
| 160 | 20 | .2614 | .2704 | .2776 | .2821 | .2884 | .3420 |
| 165 | 15 | .1962 | .2032 | .2088 | .2123 | .2171 | .2588 |
| 170 | 10 | .1309 | .1356 | .1394 | .1426 | .1451 | .1737 |
| 175 | 5 | .0655 | .0679 | .0698 | .0714 | .0727 | .0872 |

second. This is what we shall assume as suitable for short passages. Representing the actual velocity of the steam or piston by $V_s$, and by $b'$ the minimum permissible opening of port when the crank makes an angle $\omega$ with the line of centers, we have the following formula when the steam passages are long and narrow:

$$\frac{b' \times l}{A} = \frac{V_s}{160} \text{ or } b' = \frac{A}{l}\,\frac{V_s}{160} \tag{79}$$

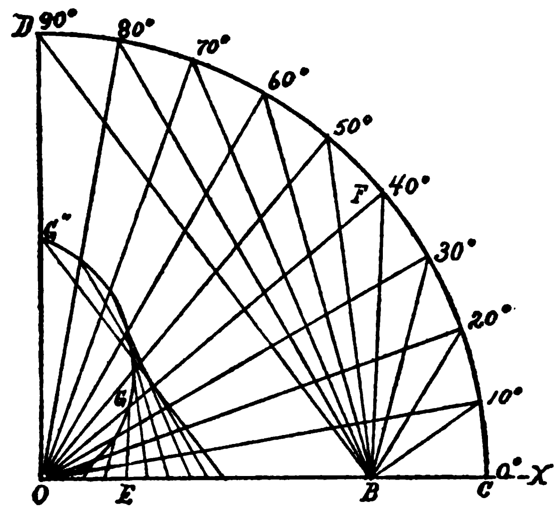
$$\text{or} \quad b' : \frac{A}{l} = V_s : 160$$

when the passages are short, we have

$$\frac{b' \times l}{A} = \frac{V_s}{200} \text{ or } b' = \frac{A}{l}\,\frac{V_s}{200} \tag{80}$$

$$\text{or} \quad b' : \frac{A}{l} = V_s : 200 \tag{81}$$

The opening of the port $b'$ corresponding to the different crank angles may now be graphically determined (Fig. 57) as follows: Assuming that the area of cylinder $A$ and length of port $l$ are




In this diagram $\overline{OC} = \frac{A}{l} = \frac{\text{area of cylinder}}{\text{length of port}}$.

Fig. 57.

given, and finding the velocity of the piston from Table XIII on page 128, we first draw a series of radii, making the angles 10°, 20°, 30°, etc., with the line $OX$. These angles correspond to the crank angles. We now describe a circle $CD$ with $O$ as a center, and $\frac{A}{l} = OC$ as a radius. We must lay off $OB$ on $OX$ and equal either to 160 or 200, according as the steam passages are long or short. To find the minimum port opening $b'$, corresponding to a particular position $OF$ of crank, we join the point $B$ with $F$ (the intersection of the circle $CD$ with the radial line $OF$, representing the crank angle in question) and then lay off $OE$ on $OX$, equal to the corresponding velocity $V_s$. We now draw through $E$ a line $EG$ parallel to $BF$, then because $OF = \frac{A}{l}$, $OE = V_s$, $OB = 160$ or $200$

$$OG : OF = OE : OB.$$

$$OG : \frac{A}{l} = V_s : 160 \text{ or } 200; \tag{82}$$

comparing this proportion with the preceding ones we see that $OG = b' =$ minimum opening of port for crank-position $OF$; in like manner all values of $b'$ can be determined and a curve $OGG''$ may be drawn through the extremities of $OG''$, $OG$, etc., which will represent the values of $b'$; the portion of the radius-vector included between the curve $OGG''$ and the pole $O$ will give the value of $b'$ for the corresponding crank angle.

When $b'$ and $\frac{A}{l}$* are drawn full size, $V_s$ and 160 should each be laid off to same scale, say 20 feet (per second) to the inch. If $\frac{A}{l}$ be drawn on an $\frac{1}{z}$ scale while $b'$ is drawn full size we must lay off 160 on a scale of $z \times 20$ feet to the inch while $V_s$ is laid off

* $\frac{A}{l}$, it should be noted, is a linear dimension and is expressed in inches.

as before, to a scale of 20 feet to the inch. When main and expansion valves are of the ordinary type the value of $b'$ obtained by the above diagram may be laid off directly upon the main and expansion valve diagrams and compared with the actual openings of port given by the latter. But when the valves are of the gridiron type having two admission ports intead of one, we must halve the values of $b'$ obtained from the above diagram and thus reduced lay them off on the valve diagram for comparison. If the expansion valve is of the piston type we must lay off the values of $b'$ on a scale equal to $\frac{l}{\pi d'_1}$ ($d_1'$ = diameter of piston valve), for the minimum opening $b''$ of piston-valve port (*i. e.* the minimum for a particular crank angle) must equal

$$b'' = \frac{l}{\pi d_1'} b' \qquad (83)$$

The *full* width $b'''$ of port for piston valve may of course be found by substituting for $b'$ its maximum value $b$ in the preceding formula. Formula (83) assumes that the steam passage within the main valve has everywhere a cross-section $\geqq$ than the maximum, annular, port opening effected by the piston valve.

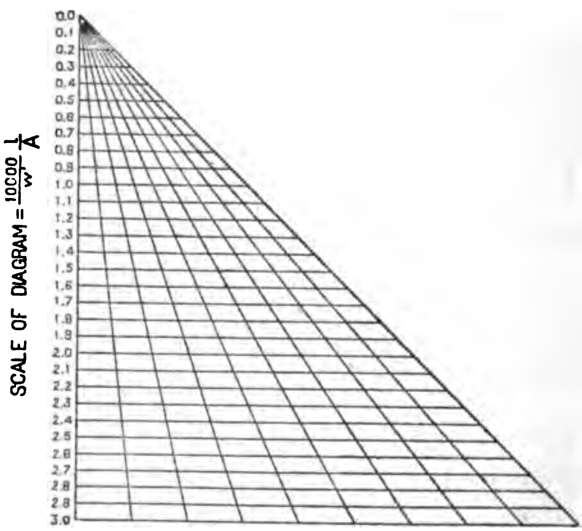
As regards the steam passage itself (not the port) if it is large enough for the exhaust it will be large enough for the admission. Its area should therefore be the quotient,

$$A \times \frac{\text{Average piston speed (ft. per sec.)}}{100 \text{ to } 125}. \qquad (84)$$

The port opening for exhaust should also be tested, taking 200 feet per second as the maximum velocity of the current that is permissible.

By plotting the port-openings as ordinates on the piston stroke as a base, and connecting them by a curve, the slope of this curve where it crosses the base will be a measure of the rapidity of cut-off.

Where many valve-gear problems must be solved, as in a college draughting-room, two diagrams like Fig. 57 may be drawn which will cover the whole range of practice. In both these general diagrams the ratio of connecting-rod to crank $= 6$, but the maximum permissible velocity of steam is 200 feet per second in one diagram and 160 feet per second in the other, corresponding to short and long steam passages respectively. For these general diagrams assume $\frac{A}{l} = 10$ and average speed of piston in feet per minute ($w'$) equal to 1000. The diagrams will then be applicable to any problem, giving the minimum port-openings $OG$ ($= b'$) to a scale of $1 \div \frac{Aw'}{10000l}$, that is $\overline{OG}$ on the diagram will be $\frac{10000}{w'} \frac{l}{A}$ the true size. In this way a diagram whose scale varies with the assumed data can be made to fit all cases. To reduce the port-openings given by these diagrams to their full-size values on the Zeuner, valve-circle, diagram a reduction




PORT OPENING GIVEN BY DIAGRAM.

Fig. 58.

arrangement, like that shown in Fig. 58, will be found very convenient. The full sizes are taken from horizontal line 1.0.

As regards the thickness of the cylinder walls, Prof. W. C. Unwin* says, "(*a*) that it should be strong enough to resist the internal steam pressure; (*b*) rigid enough to prevent any sensible alteration of form; (*c*) it must be thick enough to insure a sound casting; (*d*) thick enough to permit reboring once or twice when worn. Generally other considerations than strength are of so much importance, that the empirical rule agrees better with practice than a rule making the thickness $t$ depend upon steam pressure:" hence when $D$ is diameter of cylinder

$$t = 0.02\,D + 0.5 \text{ to } 0.02\,D + 0.75. \qquad (85)$$

## XIII.

### Valve Diagrams and Dimensions of Valve and Gear.

We shall assume that the reader is acquainted with the principal functions and common varieties of valves, and also with the terms designating the most important dimensions and parts of ordinary valve gear. This is usually given in elementary text-books on the steam engine. There is such variety in valves in the matter of arrangement of steam passages, in the method of balancing and in the division or assignment of functions, and all this involves so much special dimensioning that it would need a separate treatise to do justice to this part of the subject. We can here only attempt to point out the principal types and the methods of ascertaining the leading dimensions.

The ultimate object of all valve-gear discussion is to establish the relation existing between movement of piston and movement of valve. The piston's motion has already been fully discussed on pp. 35–39, and a table on pp. 26–27 gives its travel for different crank positions. After the travel of the valve has been ascertained for these same crank positions it will be easy to construct a diagram giving directly the desired relation.

* Elements of Machine Design. Eleventh Edition, Part II, p. 33.

The valve motion that has been most extensively used in practice is that which is known to mathematicians as "harmonic motion," and to engineers as, the motion of the slotted cross-head. Almost all valve-motion deviates a little from this, owing to the angular motion of the eccentric rod. Of late years a few valve-gears have been devised in which the valve's movement differs considerably from that corresponding to the "harmonic law;" but as yet these gears have not come into extensive use. They have usually complicated mechanisms that do not follow any simple law in their valve-movements and are therefore not readily amenable to scientific treatment. We shall confine ourselves almost wholly to the first, widely used, class. Link-motions of the common forms do however come within this class, but only the Porter-Allen (Fink) form is much used in stationary, high-speed, engine work. In this section only this latter form of link motion will receive a discussion. The Appendix will contain a general, graphical, method for obtaining the valve diagrams of the common forms of link-motions.

The valve-gears subject to the "harmonic law" can be divided into two groups:

*a*. Gears with but a single valve which slides on a stationary seat.

*b*. Gears with two valves, of which one slides on the other.

The first of these will be considered under the two heads of invariable steam distribution and variable steam distribution.

In second group the main part of our problem will be to show that the complex mechanism producing the desired relative motion of the valve on its seat, is equivalent to the action of one, single, eccentric capable of giving the valve in question an identical, absolute, motion, on a similar, but stationary, seat. The problem before us is therefore mainly a kinematic one. In order that we may simplify the usually difficult and complicated parts of this subject it will be necessary to review the elementary parts and present them in a new light.

## SINGLE-VALVE GEAR.—INVARIABLE STEAM DISTRIBUTION.

We have already stated that the main object of valve gear discussion is to find the relation between piston and valve movement, and that the piston's position for various crank angles has already been ascertained by suitable formulas and tables. As the eccentric is nothing but a short crank, a similar relation, of course, exists between the valve-slide positions and eccentric angles, but with this difference, that in the former case the ratio of crank to connecting rod is a comparatively large fraction, say $\frac{1}{5}$, while in the latter case the ratio of eccentric radius to eccentric rod is a very small fraction, often less than $\frac{1}{50}$. In the former case the piston's motion is unsymmetrical in the first and second halves of its stroke, while the valve's motion is practically symmetrical on each side of its middle position and for an infinitely long eccentric rod would be perfectly so, and this is the assumption which is always made in this sort of discussions and which will therefore underlie all our subsequent work. Moreover, it is practically most convenient to estimate piston travel from the beginning of its stroke, while valve travel is best estimated from its *center of motion* or *mid-position*, because of the symmetry of its motion and of the arrangement of the ports. For these two reasons the expressions for piston and valve travel are usually different. Just at present we will confine ourselves to the construction of a diagram representing the distance of the valve or slide from its mid-position at the different crank angles and leave the graphical representation of the piston's travel to the moment when we shall combine the two movements into one diagram and thus exhibit graphically their desired relation.

We shall first treat the slide-valve as a simple, rectangular, plate, moving on an unperforated seat and driven by an infinitely long eccentric rod. In Fig. 59, let $O$ be the center of the engine shaft, $OC_1$, $OC_2$, $OC_3$, $OC_4$ different crank positions and $OE_1$, $OE_2$, $OE_3$, $OE_4$, the corresponding eccentric positions, the eccentric

being set ahead of the crank by a constant angle $C_1OE_1$. Let $NZ$ be the dead center line of the eccentric $OE$ (which may or may not coincide with that of the crank $OC$), then will $OP$ be the mid-position of the eccentric. When the eccentric is in this position the valve will also be in its mid-position. When the eccentric is in position $OE_1$, the slide will be at the distance $E_1S_1$ from its mid-position and this distance we will call the travel

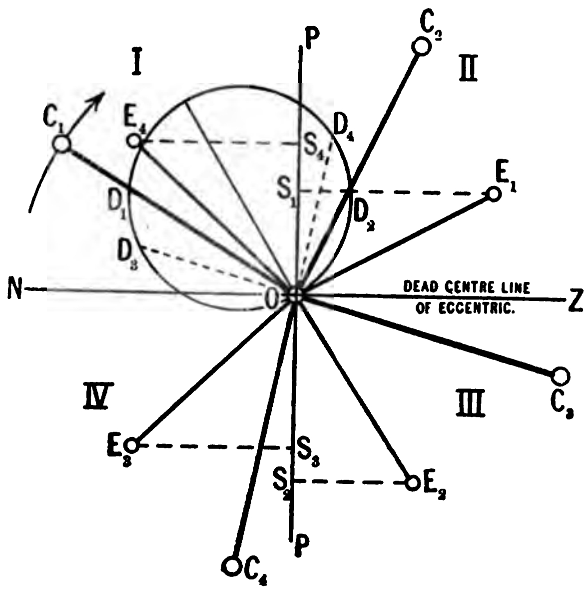



Fig. 59.

of the slide. For the other eccentric positions $OE_2$, $OE_3$, $OE_4$, the slide-travel is $E_2S_2$, $E_3S_3$, $E_4S_4$, respectively. If now we lay off this travel $ES$ on the corresponding crank positions while the eccentric is to the right of $OP$ and on the prolongations of the corresponding crank positions when the eccentric is to the left of $OP$, and making $OD_1 = E_1S_1$, $OD_2 = E_2S_2$, $OD_3 = E_3S_3$ and $OD_4 = E_4S_4$, we get the curve $OD_3D_1D_4D_2$, which is called the polar diagram of the slide motion. It is evident from the method of construction that the chord cut by this curve from the

crank or its prolongation is the travel of the slide from its middle position. It is also evident that the slide is respectively to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of its middle position when the curve cuts the $\left\{\begin{matrix}\text{actual}\\ \text{prolonged}\end{matrix}\right\}$ crank, and that the slide is going $\left\{\begin{matrix}\text{away from}\\ \text{towards}\end{matrix}\right\}$ its middle position when these chords $\left\{\begin{matrix}\text{increase}\\ \text{decrease}\end{matrix}\right\}$.

With an infinitely long eccentric rod, the curve $OD_3D_1D_4D_2$ becomes a circle. To prove this let us take a general case, Fig. 60, in which dead center line $OX$ of crank makes an angle $XOZ$ with the dead center line $OZ$ of the eccentric. From the right-hand half of $NOZ$, and in a direction *opposite* to the crank's rotation, measure the angle $ZOE = \sigma$ between this dead center $OZ$ and the eccentric position $OE$ that corresponds to any crank position $OC$. Then lay off from the crank $OC$ in the *same* direction as the rotation, this angle $\sigma = COD$. Make $OD = OE$ and on $OD$ as a diameter describe the circle $OF'D$. The right-angled triangles $OEF$ and $ODF'$ are evidently equal, hence $OF' = OF = ES =$ valve travel, that is, the circle $OF'D$ cuts a chord $OF'$ from any crank position $OC$ that is equal to the

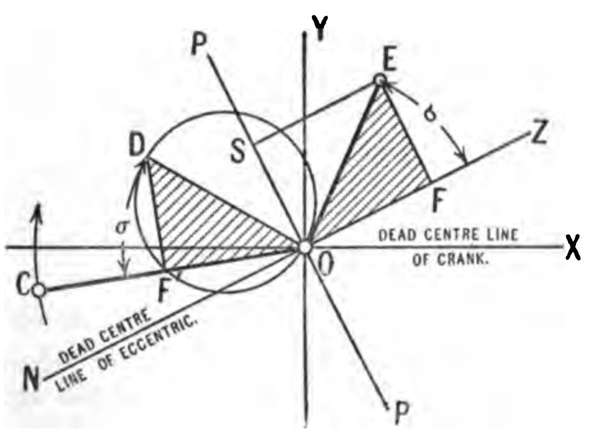



Fig. 60.

valve travel $ES$ for that instant. As the polar co-ordinates of circle $OF'D$ and of the polar curve $OD_3D_1D_4D_2$ (Fig. 59) are alike, the latter is also a circle, and may be called the slide- or valve-circle.

By reversing the steps of this construction we may evidently find the position of the eccentric from the crank position, when the valve circle is given. Our rule then is, to measure from the crank, in the direction of its rotation, the angle $\sigma$ included between it and the diameter of the valve-circle, and then lay off this angle $\sigma$ from the dead center line $OZ$, in a direction *opposite* to the rotation of the crank, aud this will give the desired position of the eccentric.

The angle $ZOE = \sigma$ is estimated from the portion of the line $OZ$ to the *right* of center $O$, and so long as the eccentric (or slide) is to the *right* of its middle position the angle made by the eccentric with the portion $OZ$ of the reference line will be less than 90°. When this is the case the diameter $OD$ of the slide-circle is always less than 90° from the crank position, which ensures that this circle will cut the $\left\{\begin{matrix}\text{actual}\\ \text{prolonged}\end{matrix}\right\}$ crank whenever the eccentric or slide is to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of its middle position. This is true for all values of $\sigma$; for all positions of the crank, for either right- or left-handed rotation, and whether slide-seat is to the right or left of shaft $O$, provided, of course, that there is no reversing lever between eccentric and slide.

The position of this valve-circle does not change while the crank rotates, its diameter making with the fixed reference line $OZ$ an angle $DOZ$ that is equal to the constant angle $COE$ between crank and eccentric. That this is true is evident from:

$$COE = COD + DOE = ZOE + DOE = ZOD.$$

The valve-circle's position is therefore dependent only on the relative position of eccentric to crank. Provided this eccentric setting remains exactly the same in direction and magnitude, the crank may have either right-handed or left-handed rotation,

may occupy any position in any one of its quadrants and yet give the self-same valve-circle whenever the construction described above is strictly followed.

For a given eccentric setting the valve-circle will have the same position, that is, the slide's distance, position and direction of motion relatively to its center of travel will always be the same for the same crank position. But when the eccentric setting is different, the motion of the slide relatively to the center of travel will be different, even though the crank position and rotation are exactly the same. This is illustrated in Fig. 61:

I. Slide is to the right of its middle and is going towards it.
II. Slide is to the left of its middle and is going towards it.
III. Slide is to the left of its middle and is going away from it.
IV. Slide is to the right of its middle and is going away from it.

The corresponding valve-circles are given in the second row of Fig. 61 when the crank position is $OC$ and the rotation right-handed, and in the third row when crank position is $OC'$ and the rotation left-handed. The angle between the crank and eccentric being the same in magnitude for both rotations, the laying off angle $\sigma$, used in finding location of diameter of valve-circle, will be the same for the corresponding figures of the second and third row.

In the last row of Fig. 61 we have the same four, possible, types of slide valves which are kinematically exactly like the corresponding Cases I, II, III, IV of the first row, and have the same valve-circles. The eccentric driving these slide valves is not shown because convenience of illustration requires the valve to be placed above the cylinder and because the setting of the eccentric relatively to crank is shown in the first row for both right- and left-handed rotation. Each valve is closing left port.

Each of these four types has its special, valve, characteristics.

A valve is said to have $\left\{\begin{matrix}\text{positive}\\ \text{negative}\end{matrix}\right\}$ lap when the port is $\left\{\begin{matrix}\text{closed}\\ \text{open}\end{matrix}\right\}$ in the valve's mid-position. We will call a valve

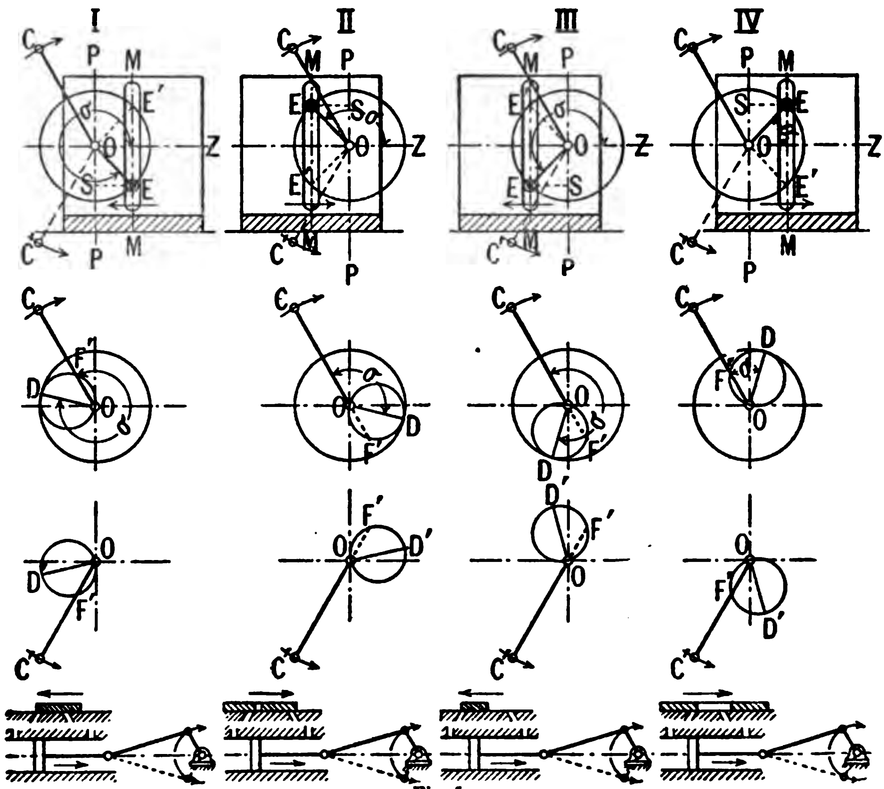



Fig. 61.

*edge* $\left\{\begin{smallmatrix}\text{direct}\\ \text{indirect}\end{smallmatrix}\right\}$ when it closes the *left* port (or passage leading to left end of cylinder) by moving to the $\left\{\begin{smallmatrix}\text{left}\\ \text{right}\end{smallmatrix}\right\}$. When *all* edges of the valve are $\left\{\begin{smallmatrix}\text{direct}\\ \text{indirect}\end{smallmatrix}\right\}$ we will call the valve itself $\left\{\begin{smallmatrix}\text{direct}\\ \text{indirect}\end{smallmatrix}\right\}$. When some of the edges are direct and others indirect (as in the ordinary D valve where the steam edges are direct and the exhaust edges indirect), we will call the valve itself direct or indirect according as the edges regulating *admission* of steam are direct or indirect.* These definitions hold whether the ports are

* English and American engineers, so far as the writer is aware, have never used these terms, *direct* and *indirect*. WIEBE makes use of them in his "Darstellung der Verhältnisse der Schieberbewegung." They are short, clear, terms and may profitably be employed in this extensive subject.

in the seat or the face of the valve. In Fig. 61, fourth row, we have

I. A valve with positive lap and direct cut-off.
II. A valve with positive lap and indirect cut-off.
III. A valve with negative lap and direct cut-off.
IV. A valve with negative lap and indirect cut-off.

We will designate them briefly as, I positive direct, II positive indirect, III negative direct and IV negative indirect valves.

The ordinary D valve in such extensive use is positive direct on its steam side and positive indirect on its exhaust side. The Meyer expansion valve with its halves placed so close together that in the middle position they fall within the outer edges of their own steam ports is an example of a negative direct valve, and the same expansion valve when its cut-off plates are placed so far apart that in the middle position of the valve its inner edges are outside of the inner edges of their own steam ports, is an example of the negative indirect valve.

Other positions of these valves between these extremes will give positive direct and indirect types. With the same eccentric setting therefore it is possible for a slide to represent any one of the four types of valves, according to the position of the pair of ports relatively to the middle position of the slide. (But with the same eccentric setting and same throw of valve the cut-off will be different for each type.) The terms positive and negative, direct and indirect, refer only to this relative position of ports to valve and are entirely *independent of the eccentric setting*. (When the valve controls only one port, instead of two, the terms direct and indirect disappear entirely, only the distinctions positive and negative then remaining.) The angle between crank and eccentric, measured from the former in the direction of rotation, for any one of these valve types may vary from 0° to 360°, although commonly each type has its eccentric set in the eccentric-setting-quadrant shown in Fig. 61.

In all of the engines of the fourth row of Fig. 61, for the same rotation, the same cut-off is taking place during the same stroke.

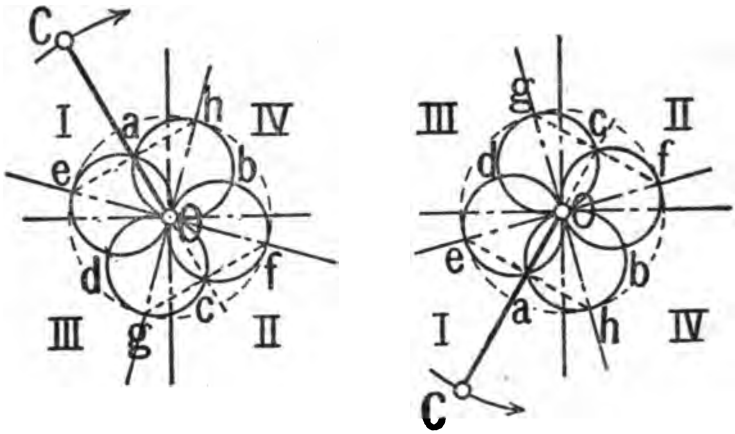



Fig. 62.

Hence four different types of valves can be arranged to accomplish the self-same object. The eccentrics which drive them will have four corresponding settings relatively to the crank. There will be four different valve-circles to represent the motion. They are grouped together in the two diagrams of Fig. 62, which differ only in the direction of crank rotation.

To find the valve-circle belonging to any type we draw the crank position $OC$ corresponding to the cut-off effected by the valve, and prolong it beyond center $O$. Then lay off the lap, $Oa = Oc$, and draw circle $ehfg$ with the radius of eccentric. At $a$ and $c$ erect the perpendiculars $hae$ and $gcf$ and thus determine the ends $e$, $h$, $f$ and $g$ of the diameters $Oe$, $Of$, $Og$, and $Oh$ of the four valve-circles. To find the one belonging to the type of valve under consideration, notice whether this valve, when at cut-off, is to the right or left of its middle position on its seat. If it is to the $\left\{\begin{smallmatrix}\text{right}\\ \text{left}\end{smallmatrix}\right\}$ one of the two circles $\left\{\begin{smallmatrix}\text{I, IV}\\ \text{II, III,}\end{smallmatrix}\right\}$ will be the desired one, because these cut the $\left\{\begin{smallmatrix}\text{actual}\\ \text{prolonged}\end{smallmatrix}\right\}$ crank. This now reduces the choice to one of two circles. Now notice whether the valve in its cut-off position is moving towards or away from its middle position. If it is moving $\left\{\begin{smallmatrix}\text{towards}\\ \text{away from}\end{smallmatrix}\right\}$ this mid-position the chord $Oa$ (or $Oc$) of the proper valve-circle will $\left\{\begin{smallmatrix}\text{diminish}\\ \text{increase}\end{smallmatrix}\right\}$ as the crank continues the rotation from its cut-off position. In Fig. 62 chord

*Ob* is accidentally equal to chord or lap *Oa*, but it is not so generally.

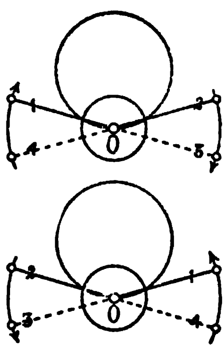
Hitherto in valve-gear discussions two diametrically opposite valve-circles were employed for they helped to show the main occurrences in the steam distribution at both ends of the cylinder. But in designing valve-gears we usually pass from the valve-circle to the actual gear and there is then danger of confusion as to the eccentric setting when two circles are used. There can be no objection to the use of two circles provided they can be distinguished by some means, say, by drawing the representative one in full lines. We shall however make use of but one valve-circle when there is but one crank. When there are two cranks, diametrically opposite, as in the Westinghouse engine, we shall use two valve-circles.

We will not here describe the occurrences of the steam distribution, as this is given in many of the elementary treatises on the steam engine and we have supposed the reader to be familiar with the elements of this subject. Besides, the occurences for the ordinary D valve are inscribed on the Zeuner valve diagram in Fig. 63. But the following table may be of assistance when some of the unfamiliar valve-types are under consideration.

## TABLE XIV.

PERIODS DURING WHICH PORT CONTROLLED BY VALVE IS OPEN OR CLOSED (GIVEN BY THE LIMITING AND INTERMEDIATE CRANK POSITIONS).

| Condition of Port. | | Type of Valve.* | | | |
|---|---|---|---|---|---|
| | | I | II | III | IV |
| | | Positive Direct | Positive Indirect | Negative Direct. | Negative Indirect. |
| Right Port. | Open | 3-4 | 1-2 | 2-3-4-1 | 4-1-2-3 |
| | Closed | 4-1-2-3 | 2-3-4-1 | 1-2 | 3-4 |
| Left Port. | Open | 1-2 | 3-4 | 4-1-2-3 | 2-3-4-1 |
| | Closed | 2-3-4-1 | 4-1-2-3 | 3-4 | 1-2 |



* *Left* port *closed* by positive direct valve when crank is at 2.
" " " " " indirect " " " " 4.
" " " " negative direct " " " " 3.
" " " " " indirect " " " " 1.

This table is applicable to both exhaust and admission by considering the lap circles of the adjacent figures to represent in the former case the inside, exhaust, lap, and in the latter case the steam lap, provided account is taken of the valve type of the edge in question. Intercepts of lap and valve-circle measure openings.

As the valve-circle gives us the valve travel to the right or left for any crank position and Tables, pp. 26–27, the corresponding piston-position, we can represent graphically by an oval sort of curve the desired relation between piston movement and valve-travel. The ordinates of this curve are the chords cut from the crank by the valve-circle and the abscissas are the distances of piston from end of stroke given by Tables V and VI. Lines drawn parallel to the base line, and above and below the latter a distance equal to the outside and inside laps, give the principal occurrences in the steam distribution. The diagram is easily drawn when the valve-circle has been found.

There is another diagram which requires still less work and gives readily the actual piston and valve travel for any crank position. Let $OA = OB$, Fig. 63, equal the crank radius to some reduced scale. For convenience we will take the length of the connecting-rod three times that of crank. Then with the length $\overline{CB} = 3\ \overline{OB}$ of this rod describe the arc $EBF$ and with the same radius strike off through $A$ an equal and parallel arc $GAH$. The horizontal lines included between these arcs will of course be equal to the stroke, and the horizontal intercepts between these arcs $EF$ and $GH$ and the crank-pin circle $AJLBU$ will represent exactly the distances of the piston from one or the other end of the stroke. For instance, when the crank-pin is at $J$, the intercept $IJ$ will represent the distance $s$ of the piston from the left end of stroke. This is evident from equation (20) p. 35, where

$$s = R\,(1 - \cos \omega) + L\,(1 - \cos \alpha)$$

Here the first term of the second member is equal $AZ = WJ$ and the second term of this member is equal to $VA = IW$.

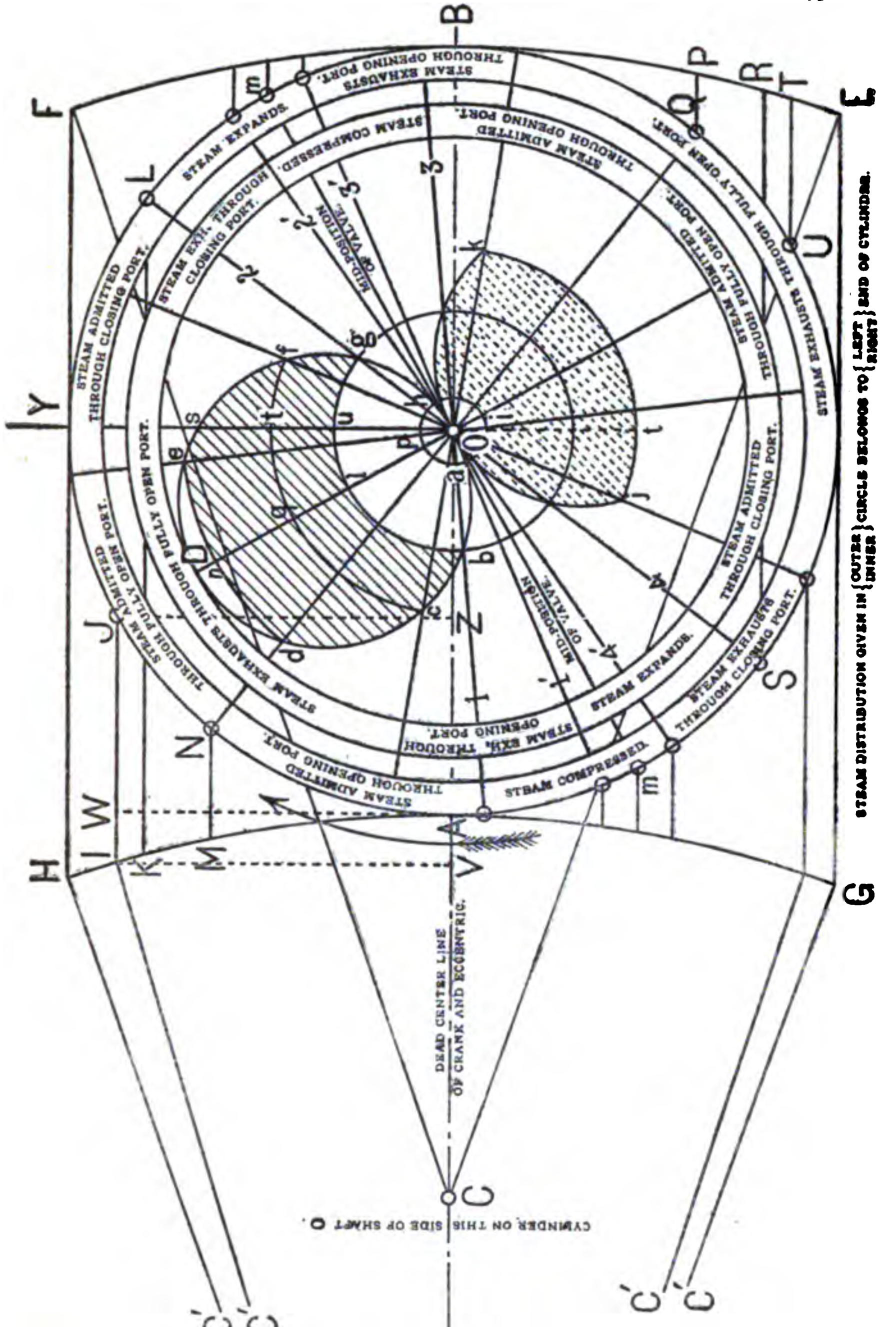

STEAM DISTRIBUTION GIVEN IN {OUTER / INNER} CIRCLE BELONGS TO {LEFT / RIGHT} END OF CYLINDER.
DEAD CENTER LINE OF CRANK AND ECCENTRIC.
CYLINDER ON THIS SIDE OF SHAFT O.
STEAM EXHAUSTS THROUGH OPENING PORT.
STEAM ADMITTED THROUGH OPENING PORT.
STEAM COMPRESSED.
STEAM EXPANDS.
STEAM EXH. THROUGH CLOSING PORT.
STEAM ADMITTED THROUGH CLOSING PORT.
STEAM EXHAUSTS THROUGH FULLY OPEN PORT.
STEAM ADMITTED THROUGH FULLY OPEN PORT.
STEAM EXHAUSTS THROUGH CLOSING PORT.
STEAM EXH. THROUGH OPENING PORT.
MID-POSITION OF VALVE.
STEAM EXHAUSTS.
F
B
P
Q
R
T
E
U
L
Y
J
N
W
I
H
K
M
V
A
Z
S
G
C
O

Fig. 63.

When the crank is at the cut-off positions $OL$ and $OS$, the piston travel is $KL$ and $RS$ for forward and return strokes respectively, and will be found to be unequal. The valve travels for the same instants are equal to $Og$; the scale for valve travel will usually be different from that for piston travel. At another crank position, $ON$, the piston travel is given by $MN$ and the valve travel by $Od$.

When the crank or its prolongation traverses the $\left\{\begin{matrix}\text{sectioned}\\ \text{dotted}\end{matrix}\right\}$ area, one or the other of the steam ports is open for $\left\{\begin{matrix}\text{admission}\\ \text{exhaust}\end{matrix}\right\}$. When crank is at $OY$, the distance of the piston from beginning of the stroke is $HY$, the valve travel is $Os$, the opening of the left steam port is $us$ and is closing, while at the other end of the cylinder the steam is exhausting through a fully open port, $pt$. In Fig. 63 the four crank positions 1, 2, 3, 4, are drawn which correspond to the opening or closing of the steam ports by the admission edges of the valve; the crank position $1^1$, $2^1$, $3^1$, $4^1$, correspond to the opening or closing of the steam ports by the exhaust edges of valve. This diagram is for a positive valve. It will be a profitable exercise to closely compare the steam distribution given by Table XIV with that given by Fig. 63.

An ordinary D valve is usually set by giving it equal leads at the dead points of the crank; but this makes every other occurrence of the steam distribution take place unequally for the two strokes. We have already pointed out that there is no necessity for equality of lead; that the lead may be unequal provided the minimum value is sufficient to maintain the steam pressure, and cushion properly; nor is a positive lead necessary, there are cases when a negative lead is justifiable and desirable.

Engine makers lay some stress upon having the indicator cards from the two cylinder ends look alike. Equality of cut-off has more influence on this than any other factor. For this reason and not because equality of cut-off is necessarily conducive

to smooth running, we will give a method* for finding the unequal laps and the angle between eccentric and crank when the cut-off is equalized for both strokes, say, made equal to 0.8.

In Fig. 64 take the distance $OO'$ equal to the sum $e + e'$ of the laps at the two ends, and on stroke $AB$ and $A'B'$ describe crank-pin circles. Strike off the equal and parallel arcs $BK$ and $A'R$ with a radius equal to the connecting-rod, as in Fig. 63. Make $KL = RS = 0.8 \times \overline{AB}$. In the neighborhood of $L$ draw a series of perpendiculars to $OO'$ which cut the lower part of crank-pin circle $BLA$ at points $u_1 \ldots u_4$. Then with $S$ as a center,

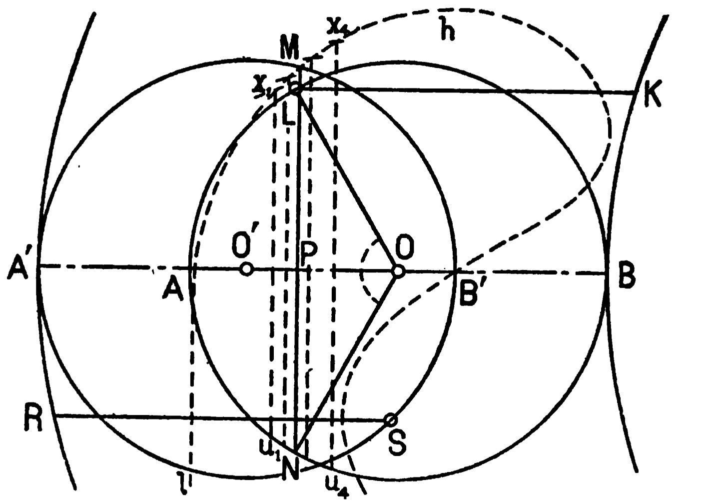



Fig. 64.

and the distances, $Lu_1$, $Lu_2$, $Lu_3$, $Lu_4$, etc., as radii, strike off the arcs that will cut the corresponding perpendiculars through $u_1$, $u_2$, $u_3$, $u_4$, etc., at the points $x_1$, $x_2$, $x_3$, $x'_4$. Join these intersections by a curve $h$. It will cut at $M$ the other circle on $A'B'$.

From this point $M$ drop another perpendicular $MN$ to $OO'$ cutting the latter in $P$ and the lower part of circle $ALB$ in $N$.

* See Burmester's "Lehrbuch der Kinematik," pp. 672-674, and Figs. 675 and 676.

Then will chord $LN = SM$ by construction and the angle $LON =$ angle $SO'M$. Each of these angles is equal to the angle between crank and eccentric and hence to the angle between diameter of valve-circle and dead point line of eccentric (see p. 137). For this last angle $DOB = DOL + LOB$, Fig. 63, and the first part $DOL$ has a cosine = lap ÷ eccentricity. The second part $LOB$ is represented by $LOP$ or $SO'P$ in Fig. 64 and the first part by $PON$ and $PO'M$. Hence $PON + LOP = LON = PO'M + SO'P = SO'M =$ angle between crank and eccentric. By giving the greater lap $OP$ to the end of the valve farthest from shaft and lap $O'P$ to the other end of the valve the two cut-offs will each equal 0.8 and the eccentric setting $= LON$. The other intersections of auxiliary curve $h$ with $A'MB'$ are of no use in this connection.

Dr. Burmester also solves another problem, namely, one in which, for a given setting of eccentric, the cut-off shall be equal. Construct circles on stroke $AB$ and $A'B'$ with same center $O$ and $O'$ as before. Then take any point $f$ in circle $ALB$ and lay off angle $fOe$ equal to given setting of eccentric. Through $e$ draw $ee'$ perpendicular to $OO'$, intersecting circle $A'MB'$ at $e'$. Lay

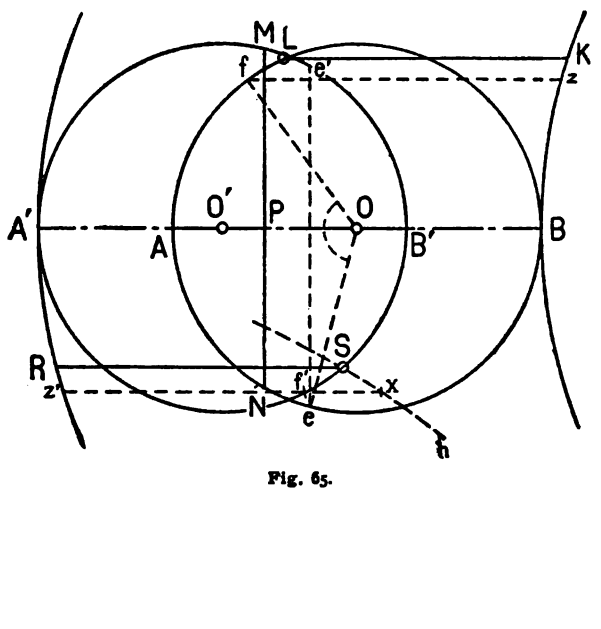



Fig. 65.

off $e'O'f'$ on this circle equal to the given setting and through $f'$ and $f$ draw parallels $z'f'$ and $zf$ to $OO'$ till they cut the arcs $A'R$ and $BK$. Then make $z'x = zf$; this will give one point $x$ of auxiliary curve $h$. By similar constructions other points of this curve $h$ can be found. The auxiliary curve $h$ cuts the circle $A'MB'$ in some point $S$. Finally make angle $SO'M = fOe$, drop the perpendicular $MPN$ on to $OO'$ and make angle $NOL = fOe$. Then will cut-off $\frac{RS}{A'B'} = \frac{KL}{AB}$, and $OP$ will be the outside lap of the valve at the end farthest from the shaft and $O'P$ the lap for the other end of the valve.

In the mathematical discussions of valve-gears, it is found more convenient to use the angle of advance $\delta$ instead of the eccentric setting or angle between crank and eccentric. It may be expressed by a formula or it may be measured in any valve-gear as follows: Start with eccentric arm half-way between its own dead points and then move the crank to its (the crank's) nearest dead center. During this motion the angle passed through by either crank or eccentric will be the angle of advance and it will be positive if this motion or rotation is in the same direction as the engine rotation, otherwise it will be negative. The formula for this angle at any crank position is:

$$\delta = \lambda - \omega' - 90^\circ * \qquad (86)$$

Letting $x$ represent angle between crank and eccentric, we have

$$x = 90^\circ + \delta \pm \chi = \lambda - \omega' \pm \chi, \qquad (87)$$

where $\chi$ represents the angle between the two dead point lines and

* According to Rankine: "By angular advance is to be understood the angle at which the eccentric arm stands in advance of that position, which would bring the slide-valve to mid-stroke when the crank is at its dead points." $\lambda$ represents the angle made by the eccentric with its own dead point, $\omega'$ the angle made by the crank with its dead point line; either of these angles may be greater than 180°.

$\left\{ \begin{matrix} \text{upper} \\ \text{lower} \end{matrix} \right\}$ sign is used when dead point line of eccentric $\left\{ \begin{matrix} \text{follows} \\ \text{precedes} \end{matrix} \right\}$ that of the crank in the direction of rotation. When there is a reversing lever between eccentric and valve the formula for the angle between crank and eccentric then becomes

$$x' = 180° - x = 90° - \delta \mp \chi. \tag{88}$$

Sometimes a reducing lever is placed between the valve and eccentric for the purpose of reducing the size of the latter and thus diminishing its tendency to heat under high speed. The only modification which this makes in our diagram is to make the valve-circle diameter correspondingly larger. For we assume that the valve is driven directly by the eccentric. Sometimes, as in Corliss valve-gear, a reducing lever is used to give a sort of differential motion, and the valve direction and chord of lever-pin are made to differ purposely.*

The introduction of a *reversing* lever between eccentric and valve has the same effect on the valve motion, as if the eccentric were shifted to a diametrically opposite position and the valve then driven directly (*i. e.* without reversal) by the new eccentric. Hence here also, unless the contrary is specified, it is to be understood that diagram is drawn as if valve were driven directly by the eccentric.

This finishes our discussion of the simplest case of valve gearing. Under the head of single-valve gearing, with invariable steam distribution, there still remains a case which is mainly of interest because of its bearing on link motions, namely, the case in which the valve stroke does not pass through the center of the shaft, but at a certain distance $c$ from the latter. In this mechanism the stroke of slide is more than twice the length of the eccentricity; moreover the two dead points of the crank are not diametrically opposite as in the ordinary slider-crank.

* See also levers $O'RP$ and $O'RQ'$ Fig. 82, Porter-Allen Engine.

In Fig. 66 $V$ is a valve, whose line of stroke $VY$ passes center of shaft at distance $OY = c$. It is driven by an eccentric $OE = r$ and rod $EV$ of length $l$. In valve-gears the point from which valve-travel is usually estimated is one near the middle of the valve-stroke; the point of valve whose travel is measured may be any point rigidly attached to valve; in this figure the right-hand end $V$ of eccentric-rod is the one chosen. If we find the position of $V$ corresponding to each of the crank's dead points and then bisect the distance between these two positions of $V$, the point of bisection will be the point from which valve-travel is measured.

Let $M$ be this center of valve-travel and $OE$ any position of the eccentric. $MV$ will then be the corresponding valve-travel or distance from middle position. At the point $E$ draw $EC$ equal

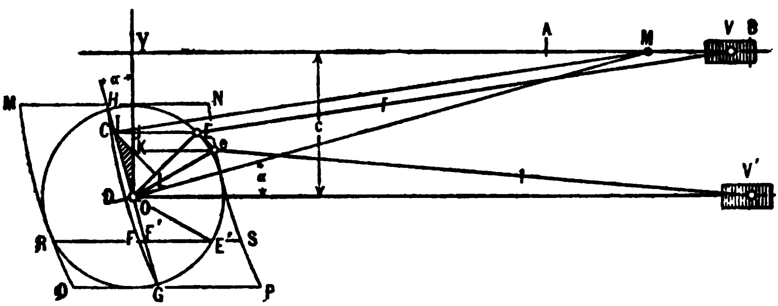



Fig. 66.

and parallel to $MV$, and join $MC$. The quadrilateral $CEVM$ is consequently a parallelogram and $CM = EV = l$. In other words, the locus of the point $C$ is a circle described from $M$ as a center, with radius $l$. The valve-travel for any other eccentric position $OE'$ can therefore be found by drawing the parallel $E'F$ up to the arc $GDH$.* Draw the chord $GOH$ of this arc per-

* The travel of the valve from the end of the stroke for any position $OE'$ of eccentric is evidently either $E'S$ or $E'R$, according as one or the other end of the stroke is meant. The arcs $MRO$ and $NSP$ are struck, from the ends $A$ and $B$ of valve-stroke, with radius $l$ of rod.

pendicular to line $OM$; in this figure arc and chord meet at points $G$ and $H$ that seem to be on eccentric circle $EE'R$; but they do not necessarily lie on this circle.

Even in this sort of valve gears the rod $l$ is long in comparison with the radius $r$, the rise $OD$ of the arc is very small and the arc may then be replaced by the chord $GH$ passing through center $O$. The distance $EI$, parallel to stroke, now represents the valve-travel, and for any other eccentric position $OE'$ the distance from $E'$ to the straight line $GOH$ measures the travel of the valve. When eccentric center $E$ and point $H$ are on the same horizontal, the valve-travel $EH$ will be the same wheather measured up to chord $GOH$ or arc $GDH$; the same may be said when $E$ and point $G$ are on the same parallel to stroke. These two points of $E$, if constructed, would be found to be diametrically opposite and equidistant from arc or line of reference; if from each of these $E$'s as a center and with radius $l$ we describe an arc, cutting stroke $AB$, and then bisect the distance between these arcs the point of bisection will be the center of reference $M$ assumed above.

But the travel may be more easily found and this case reduced to that of the ordinary eccentric, by finding an eccentric $Oe$ whose distance $eK$ from the vertical $OY$ is always equal to $EI$*. To find such an eccentric, we drop from point $I$ a perpendicular $IL$ upon the eccentric $OE$, and erect $Ee$ perpendicular to $OE$. Now from the point $K$, where $IL$ cuts the vertical $Y$, we draw $Ke$ parallel to $EI$ and join $O$ and $e$. It is evident that $eEIK$ is a parallelogram and $eK = EI$. To be an equivalent eccentric $Oe$ must not change its length or position relatively to $OE$, that is, $Ee = IK$ must be a constant for all positions of the original eccentric $OE$. The figure shows that triangle $OKL$ is similar to

* Up to this point the demonstration is like that given by Prof. A. Fliegner in his work "Umsteuerungen der Locomotiven."

each of the triangles $IJK$ and $OJE$. These triangles are therefore similar to each other, and we have

$$\frac{IK}{OE} = \frac{IJ}{OJ} = \text{tangent of angle } HOY = \frac{c}{l} \text{ nearly.} \qquad (89)$$

But this angle is constant, being equal to the angle $MOV'$, hence $IK = Ee$ is a constant, and $Oe$ has all the properties of a virtual or equivalent eccentric. If we connect $e$ by a rod $l$ with a slide $V'$, whose stroke passes through $O$, the travel of $V'$ from its middle position will be very nearly like that of the slide $V$ from $M$. Evidently stroke of $V'$ is greater than if driven by $E$.

The distance of $E$ from chord $GOH$, or of $e$ from vertical $\dot{O}Y$, does not exactly represent the travel of the valve, for it does not take account of the angularity of the eccentric rod (though it does consider the direction of line $OM$, and thus takes partial account of the length of this rod when distance $c$ is given). The exact valve-travel is only given by the distance of eccentric-center $E$ from arc $GDH$.

## SINGLE-VALVE GEARS, STEAM DISTRIBUTION VARIABLE.

The characteristic feature of a single-valve gear is that its valve slides on a fixed, stationary seat (p. 134). In this sense an engine may contain one or more single-valve gears which divide among themselves the functions of the steam distribution.

The variable elements in these gears exist mainly to vary the expansion. They may roughly be arranged into two groups, those in which the driving eccentrics themselves can be varied, and those in which the eccentrics are themselves non-adjustable, the variation in the valve motion being effected by mechanism between the eccentrics and the valves. To the second group belong the link-motions, and these we will take up in the Appendix, because they are more complex and less extensively used in high-speed engine work than the first group.

In the first group the eccentric is varied by simultaneously changing both its throw and angles of advance.* The principal means by which this change is accomplished is the "swinging eccentric" device. In this the center of the eccentric is moved across the shaft in an arc **LL** having the point $P$ as its center, as in Figs. 68 to 71 and Figs. 73 to 78. Ten of the "Single-Valve Automatics" represented use this device, the mechanisms for producing the swinging motion differing more or less; (see catalogues of engine builders). The object of this change is to alter the power of the engine by altering the amount of expansion, but other changes in the steam distribution also take place at the same time when there is but a single valve to effect them. For example, the compression begins earlier when the expansion begins earlier, great cushioning going hand in hand with great expansion. These two functions of the steam distribution have perhaps the most influence on the form of the indicator card and thus are the principal means of regulating the power per stroke and the uniformity of the driving force. These are important

* The only engine known to the writer that uses a single valve and varies the cut-off by varying only the angle of advance, is the oscillating engine built by the J. T. Case Co. Strictly speaking this engine has two valves, for the ports in the rocking cylinder control release, exhaust closure and admission, the cylinder itself performing the functions of the main valve.

On the other hand, the Buckeye engine has apparently a double-valve gear and yet in reality is made up of two single-valve gears the first being the one to which the main valve belongs and the second having the expansion valve. In this second gear, the expansion valve slides on a moving main valve, but the mechanism is such that the *relative* motion of expansion to main valve is scarcely affected by the *absolute* motion of main valve. The latter therefore acts as a stationary seat of the expansion valve which may be discussed and designed as if the ports it controlled were fixed and as if its rocker arm had a stationary pivot. We have however discussed the Buckeye's gear in Figs. 96-98, with the double-valve gears, which it so strongly resembles externally. When an engine has its distribution effected by several valves, each sliding on a fixed seat, the functions of one of them may well be varied by varying only the angle of advance of its eccentric or only the throw or lap.

factors in securing *steady* running of the engine. To effect *smooth* running there must be not only absence of great fluctuations of power per revolution, but also absence of vibrations and shocks. Shocks occur when there is "play" (or clearance) at a bearing accompanied by such sudden and complete changes of direction in the force acting on the bearing that continuous contact between the pin (or slide) and its bearing ceases while the space (or "play") between the two is traversed by one or the other of the two pieces. Tremors and vibrations of course accompany shocks, but they may also arise independently, from sudden changes in the intensity of the pressure between two pieces. Thus at the end of a piston-stroke the final cushion-pressure may be considerably less than the initial pressure on piston at beginning of next stroke, the sudden, additional load on piston sending a tremor through all the connections. Sudden reversals of pressure should therefore be avoided particularly at the dead point where they are most dangerous. As the extent to which cushioning is carried, the character of the lead (positive or negative) and its extent, strongly influence these dead-point-pressures, the variations effected by the valve gear in these quantities per revolution and at the different grades of expansion become a matter of consequence. In times past engineers laid great stress on having the steam lead equal at the two cylinder ends whatever the grade of expansion, particular care being taken to "set" the valve with equal lead. This desire for *equalization* at the two cylinder ends extended itself to the cut-off, the release and the cushioning as well as to the lead. But it was recognized, particularly in locomotive practice, that it was impossible with existing valve gear to *equalize* all the functions at the same time, equalization of any one function was at the expense of the others. Generally it was the lead (a positive one) that was thus favored. At the same time most engineers also held that a lead that was constant at all grades of expansion was a desideratum. It was regarded as the special merit of the Gooch link-motion that it

possessed this virtue. But since the advent of the high-speed steam engine a difference of opinion on this matter has developed itself among engineers.

In the pioneer engine of this type, the Porter-Allen, the leads were deliberately made unequal at the two ends (Figs. 86, 87 and Figs. 89, 90 of Porter-Allen valve diagrams) and the lead also varied from mid-gear to full gear, as in other link-motions. The Straight Line engine in its earliest forms had a lead that was equalized at the ends and was nearly constant at all grades of expansion; in the later forms the lead is now variable for different cut-offs; it is sometimes negative and there is some inequality at the ends. In other and excellent engines the old views on this subject are still carried out so that the matter cannot be regarded as finally settled either way, Prof. Sweet, the designer of the "Straight Line" engine, going so far as to determine experimentally, with the indicator, the conditions of smooth running for each engine. The collection of examples given shows the variation of American practice in this respect. We shall not attempt any comparisons of these engines, for the exact effect of the steam distribution on the steadiness and smoothness of running can only be determined by a knowledge of the clearances, the pressures, the weights of the reciprocating parts and the regulating capacity of the governor under variable loads.

We have thus far said nothing concerning the release, because it has usually less influence on smoothness of running than the other factors, though it too effects somewhat the pressure on piston at dead point. The release is of consequence to economical running because of its influence on the amount of back pressure, an early release and widely opened exhaust ports tending to keep this pressure small. On the other hand release begins too early when it curtails the period of expansion to any notable degree. We have given the beginning of the release for only a few cases; it may be easily found from the exact diagrams, by following the directions given with Fig. 67.

Returning now to the influence that the shape and location of the locus **LL** of the eccentric centers $E$ has on the steam distribution, we note first, that as the chord of the utilized portion of the arc **LL** is more nearly perpendicular to the dead point line of the eccentric, the lead is more nearly constant at the different grades of expansion, provided the curvature is the same, and secondly, that with this perpendicular location an arc **LL** of great radius (*i.e.*, of slight curvature) causes smaller variations in the lead than an arc described with a small radius.

Radii drawn from the center $O$ to the locus **LL** (or $E_0E_1E_2$) of centers, give the length and position of the eccentrics corresponding to the different grades of expansion, the locus moving with the crank to which it is rigidly attached. The broken line $D_0D_1D_2$ is the locus of the vertices of the diameters of the valve-circles corresponding to the eccentrics $E_0, E_1, E_2$. The position of these diameters were found from their eccentric by laying off the angle $\sigma$ as in Fig. 60. In Fig. 74 the locus **L′L′** or $E_{v2}E_{v1}E_{v0}$ was derived from **LL**, the locus actually described by the swinging eccentric, by constructing a series of offsets each equal to $\frac{1.5}{34}r_n$, according to the method given in connection with Fig. 66. The new eccentrics thus obtained are regarded as virtual eccentrics and their centers are indicated by $E_{v0}, E_{v1}, E_{v2}$. The valve circles were derived from the virtual eccentrics $OE_{v0}$, $OE_{v1}$, $OE_{v2}$.

The particular eccentrics chosen for representation are those corresponding to minimum, quarter and maximum cut-off, the first and last of these being taken from data furnished, in most cases, by the manufacturers themselves. The quarter cut-off is generally an average value of the cut-offs in the two cylinder ends when angularity of connecting-rod is taken into account. The intermediate cut-offs in the case of the Westinghouse engines are somewhat larger than 0.25 or about 0.30. Each of these valve-circles will give the steam distribution shown in Fig. 63, when the lap and port-width circles are drawn. The crank

positions and piston travel at cut-off, preadmission and beginning of compression are only drawn for the eccentric and valve-circle corresponding to ¼ cut-off; the shaded area shown in the diagram cuts, from the crank, intercepts that represent the port-openings at quarter cut-off. If there is more than one port each intercept must be multiplied by the number of ports to get the total opening. It must not be forgotten that each of these valve-circles cuts from the crank or its prolongation a chord that represents the distance of the eccentric center $E$ from the perpendicular $YY$, through $O$, to the dead center line of the eccentric. With the exception of Figs. 74, 77, this perpendicular is the vertical through $O$. The valve-circles entirely neglect the angularity of the eccentric rod. Whenever the length of the latter was known, a dotted, central, arc of reference was drawn from which the valve travel from the center of this arc can be found exactly. This arc center is obtained as follows: the crank is placed at one of the dead points and the center $E$ of the eccentric in question (usually the one for maximum cut-off) is placed in its corresponding position, then with $E$ as a center and length of eccentric-rod as a radius strike off an arc cutting the valve-stroke; repeat the operation for the other dead center of crank, the point bisecting the distance between these arcs will be the center of the arc of reference required. In the Rice, Ball, Southwark, Straight Line and Westinghouse compound engines, the central arc of reference differs so little from the vertical through $O$ that no effort was made to draw the polar curves representing exactly the valve-travel, the valve-circle being sufficiently accurate for all practical purposes. In the Westinghouse Standard there is also only a slight difference between the perpendicular $Y'Y'$ and the arc $Y''Y''$; still for the case of maximum eccentricity the exact polar curves were plotted on the diagram in dotted lines, the one outside the valve-circle corresponding to the positions of the valve to the left of the center of the reference arc and the one inside to the positions of the valve to the right of this center.

Comparison with the intermediate valve-circle shows that the

steam distribution is but slightly changed by substituting the exact polar curves for the less exact circle.

The cut-offs are apparent ones and are estimated from *beginning* of stroke in the same manner as in former sections. Pre-admission, beginning of compression and release are however measured from the *end* of the stroke and each is then divided by the stroke itself, so that the results are always expressed as fractions of the stroke. The release is not tabulated on the diagrams. The symbols used are

$a$ = width of port.
$l$ = length of port in cylinder.
$d$ = diameter of piston valve.
$e$ = steam lap.
$i$ = exhaust lap.
$\varepsilon$ = apparent cut-off.
$v$ = steam lead.
$r$ = eccentricity.
$R$ = radius of crank.
$L$ = length of connecting-rod.
$\varepsilon'$ = beginning of release, measured from end of stroke and divided by stroke.
$\eta$ = beginning of compression, measured from end of stroke and divided by stroke.
$\eta'$ = beginning of admission, measured from end of stroke and divided by stroke.

The period of compression would properly be represented by the difference $\eta - \eta'$.

The subscripts $_0$, $_1$ and $_2$ affixed to these symbols signify that they belong to minimum, quarter and maximum cut-offs respectively, as explained above.

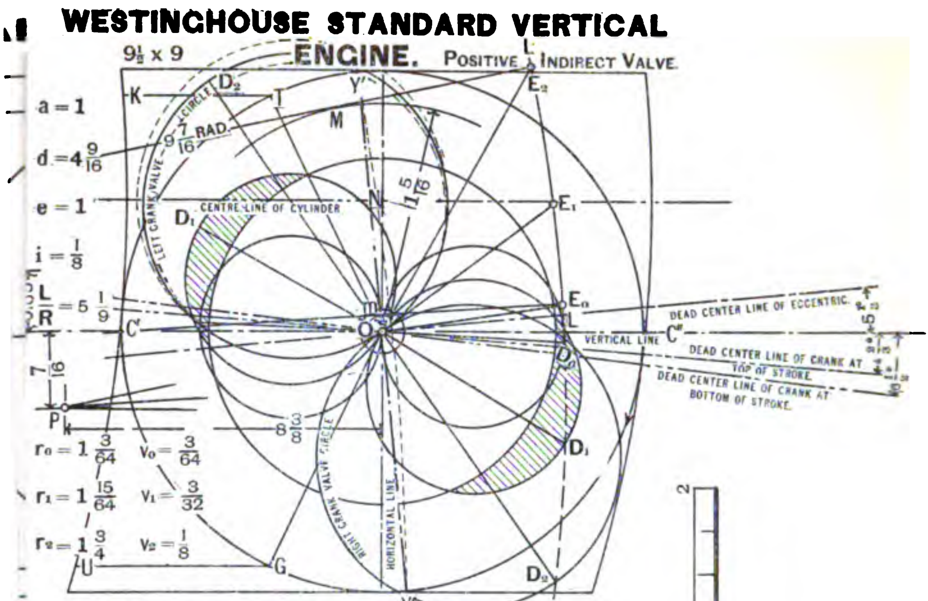
All the engines represented "run over" (see p. 118), *i. e.*, in these diagrams, the rotation is right-handed. Then in Fig. 67, for the distribution corresponding to quarter cut-off, $KT$ repre-

sents admission of steam to head end during forward stroke and *SR* to crank end for return stroke. *FH* represents compression n crank end during forward stroke and *GU* compression in head end during return stroke because exhaust lap is negative. $p_1\ q_1$ represents the preadmission to head end before beginning of forward stroke and $pq$ the preadmission to crank end of cylinder for return stroke. As the inside lap at crank end is different from that at head end we must carefully distinguish between the two. Then $m_1 n_1$ is that last part of the forward stroke during which release takes place in the head end and $mn$ that part of the return stroke during which release takes place in the crank end of the cylinder. By dividing these quantities by $2 \times \overline{CO}$, we get the values tabulated in the diagrams. We have omitted the release from these tabulations but it can be easily found from these exact diagrams if desired. In following the steam distribution by means of the diagram, it is well to bear in mind that the valve is to the right of its middle position whenever the valve-circle cuts the crank radius.

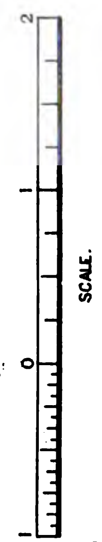
The same letters are used in the other diagrams to represent the same parts of the distribution; the letter *C* always represents the crank position, $E_n$ eccentric position corresponding to *C*, and $D_n$ the diameter of the valve-circle corresponding to the eccentric-setting $COE_n$.

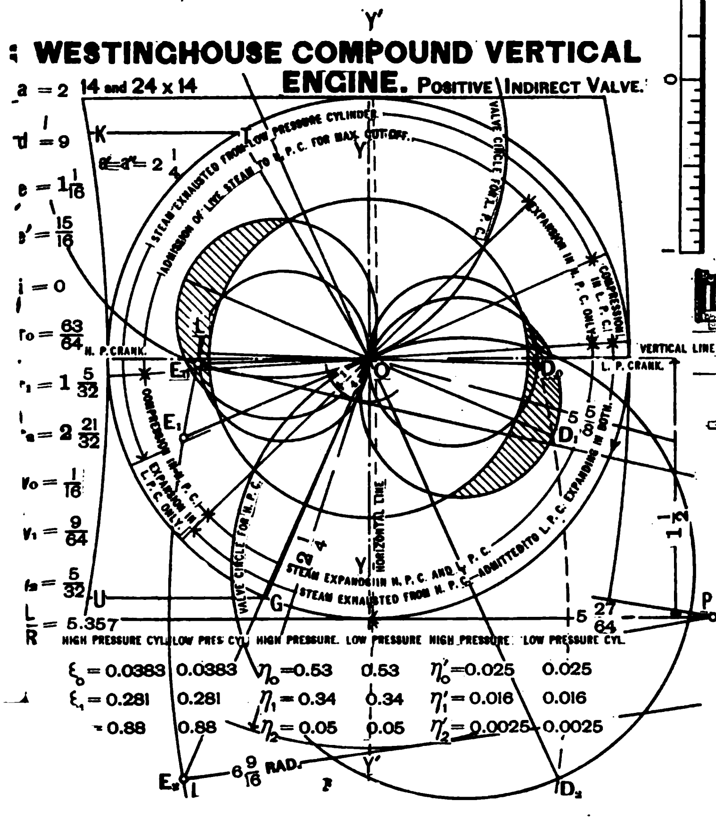
The valve diagrams, for the twelve Single-valve Automatics represented, are here divided into two groups according to the location of the pivot *P* (or center of curvature) relatively to the crank radius. In the first group, Fig. 67-72, the center of curvature of that element of the locus **LL** which crosses the crank lies on the crank's own radius, while in the second group, Figs. 73–78, the pivot *P* lies outside of the crank's radius or its prolongation. As the pivots *P* are often at considerable distance from their loci **LL**, economy of space required that the points be detached from their proper position relatively to their loci and be placed nearer the latter; this has been done in every case and their co-ordinates given so that they can be laid out if desired.

**Figs. 77 and 78.**

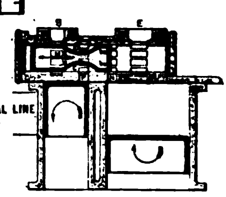



| RIGHT CYLINDER | LEFT CYLINDER | RIGHT CYL. | LEFT CYL. | RIGHT CYLINDER | LEFT CYL. |
|---|---|---|---|---|---|
| $\xi_0 = 0.023$ | 0.023 | $\eta_0 = 0.631$ | 0.631 | $\eta'_0 = 0.035$ | 0.035 |
| $\xi_1 = 0.283$ | 0.283 | $\eta_1 = 0.373$ | 0.373 | $\eta'_1 = 0.0083$ | 0.0083 |
| $\xi_2 = 0.75$ | 0.75 | $\eta_2 = 0.165$ | 0.165 | $\eta'_2 = 0.0025$ | 0.0025 |




| HIGH PRESSURE CYL. | LOW PRES. CYL. | HIGH PRESSURE | LOW PRESSURE | HIGH PRESSURE | LOW PRESSURE CYL. |
|---|---|---|---|---|---|
| $\xi_0 = 0.0383$ | 0.0383 | $\eta_0 = 0.53$ | 0.53 | $\eta'_0 = 0.025$ | 0.025 |
| $\xi_1 = 0.281$ | 0.281 | $\eta_1 = 0.34$ | 0.34 | $\eta'_1 = 0.016$ | 0.016 |
| = 0.88 | 0.88 | $\eta_2 = 0.05$ | 0.05 | $\eta'_2 = 0.0025$ | 0.0025 |



RUSSELL & Co. ENGINE, 14x20.

Fig. 67. In this valve gear the center of the eccentric is moved in a perfectly straight line **LL** across the shaft, at right angles to the crank. The mechanism for accomplishing this (it is not the "swinging eccentric" device) is not given here but can be found in catalogue of the builder. Inspection of the valve-circles shows at once that the steam lead $v$ is constant at all grades of expansion. The lead on the exhaust side is however unequal at the two cylinder ends, for the inside lap $i = 0$ and $-\frac{1}{8}$ at the crank and head end respectively. But this is not a matter of any consequence. The inequality in the inside lap equalizes the beginning of release and of compression for the two cylinder ends as the following table shows:

| | Crank End. | Head End. | | Crank End. | Head End. |
|---|---|---|---|---|---|
| $\varepsilon_0'$ | 0.54 | 0.54 | $\eta_0$ | 0.458 | 0.450 |
| $\varepsilon_1'$ | 0.36 | 0.37 | $\eta_1$ | 0.292 | 0.284 |
| $\varepsilon_2'$ | 0.075 | 0.075 | $\eta_2$ | 0.065 | 0.050 |

The value of $\eta$ includes the preadmission of steam and so does not exactly represent compression proper. But it may be taken as representing the *cushioning*, which is practically equalized for the two ends by this arrangement. The table connected with the diagram gives the variation of cut-off and preadmission for minimum, average and maximum throw; the range of cut-off varies from $\frac{1}{13}$ to $\frac{8}{15}$. This engine uses the Giddings' Balanced Valve, which is divided into two parts, rigidly connected, one part for each end of the cylinder. Each part admits steam through two passages that run through the valve and somewhat resemble the supplementary passage of the well-known Allen valve. Like the latter this valve belongs to the positive direct type and increases the clearance volume during the compression period by the volume of the supplementary passage. The clearance volume is smaller at the beginning than at the end of the

compression. As there are two ports or passages for each end of the cylinder, the admission and cut-off are prompt, and the port opening given by the shaded area of the diagram must be multiplied by 2. The valve is of the flat and balanced variety. The speed of this size of engine is 210 revolutions per minute.

## N. Y. S. S. Power Co., Engine, 12x12.

Fig. 68. The locus **LL** of the center of eccentric is concave to the shaft *O*. The center of this arc is on crank radius, at pivot *P*, and 14″ from center *O* of shaft. The lead diminishes as the cut-off increases. The cut-off ranges from $\frac{1}{17}$ to $\frac{4}{5}$. The beginning of release is given by this table,

| | Crank End. | Head End. |
|---|---|---|
| $\epsilon_0'$ | 0.54 | 0.458 |
| $\epsilon_1'$ | 0.35 | 0.276 |
| $\epsilon_2'$ | 0.065 | 0.046 |

## Payne Engine, 13x12.

Fig. 69. The location of the pivot of the pendulum arm of the swinging eccentric in the smallest engines is different from that here given. In a 5x7 for instance, the pivot *P* and crank-pin *C* are on opposite sides of center *O*; in this case pivot *P* and pin *C* are on the same side of *O*. The pivot is 11½″ from shaft center *O* and lies on the same radial line as crank. The radius of arc **LL** is 10⅝″; this makes the arc convex to the shaft and the lead increases with the cut-off, the latter ranging from 0 to ¾. The fraction of piston stroke occupied by preadmission is very small. The valve takes steam on the inside and is of the flat, balanced variety. This size of engine makes about 250 revolutions per minute.

## Rice Engine, 7x10.

Fig. 70. This engine presents a unique appearance in that no governor is visible. The regulator is within the crank disc and

turns an arbor that passes through the center of the crank-pin $C$. This arbor is the pivot $P$ of the pendulum arm that swings the eccentric-pin across the shaft when the expansion is to be varied. The eccentric-pin and the pendulum arm constitute a sort of return crank. In the figure, both $C$ and $P$ represent the center of pivot and of crank-pin. The locus **LL** is again convex to the shaft and has a nearly constant lead; its radius is $4\frac{1}{2}$. The fraction of the stroke occupied by the preadmission is larger in this engine than in any other of the dozen represented. The cut-off ranges from $\frac{1}{4}$ to $\frac{7}{10}$. The length of the eccentric-rod is only 15″, but the influence of its angularity on the valve-travel is but slight, as the arc $Y'Y'$ shows. This arc was struck from the center of motion corresponding to maximum throw and with a radius of 15″. (The center of motion of valve is half way between the two valve positions corresponding to the crank's two dead points.) The vectors of the valve-circle represent the distance of point $E_n$ from vertical $YY$ and are very nearly equal to the valve-travel from the center of motion. The exact valve-travel at any instant is given by the distance of point $E_n$ from the arc $Y'Y'$. These exact travels could be laid off on the crank positions or their prolongations and thus two exact, polar, diagrams be found (for each eccentric throw) which would cut from the crank the exact distance of the valve from the middle position. But in this case they would differ so little from the intermediate valve-circle that this nicety of construction has been omitted. The valve is of the flat, balanced variety.

## Ball Engine—Older Type, 9 x 12.

Fig. 71. The diagram of this engine is given mainly for comparison with the diagram of a more recent style of valve-gear for the same engine whose diagram is given in Fig. 76. In this earlier form the point $P$ of the swinging eccentric was $13\frac{1}{2}$″ distant from the center of the shaft $O$ and was on the same radial line as the crank-pin $C$. The locus **LL** of the center of the eccentric was convex to shaft $O$ and gave a lead that increased

as the cut-off increased. The range of cut-off was from 0 to $\frac{5}{8}$. The valve was flat and balanced. The length of the eccentric-rod was about $41\frac{1}{2}$ inches and with this as a radius an arc $Y'Y'$ is struck off from the center of motion, in the manner already explained when discussing the Rice Engine; the distance of the center $E_s$ of eccentric from dotted arc $Y'Y'$ gives the exact travel of the valve, while the valve-circle gives the distance of $E_s$ from vertical $YOY$; here again the difference is so slight that we may rest well content with the results furnished by the valve-circles.

## ARMINGTON & SIMS ENGINE, $14\frac{1}{2} \times 15$.

Fig. 72. In this engine a peculiar linkage is employed to vary the position of the center $E$ of the eccentric; for the details of this linkage we must refer the reader to the catalogue of the engine builder. The locus **LL** of this diagram is not a circular arc, though it closely resembles some of the loci that have already been given. It was plotted from an accurately measured linkage belonging to a $14\frac{1}{2} \times 15$ engine. Here also, the lead increases as the cut-off increases, the table on the diagram showing the extent to which this takes place. The range of cut-off is from 0 to $\frac{3}{4}$. As in all the preceding cases the compression or cushioning period increases with the period of expansion. The eccentric-rod which connects the driving eccentric with the $10\frac{1}{4}''$ rocker arm is itself $44''$ long. We may regard the rocker arc as a straight line and repeat the construction of the arc $Y'Y'$ already given with the Rice & Ball engines. Here also the difference between vertical reference line $YY$ and the reference arc $Y'Y'$ is so slight that it may be neglected. The valve is of the hollow piston variety, is $6\frac{1}{4}''$ in diameter, is of the positive indirect type and takes steam simultaneously at two places. One of its steam passages leads through the center of the valve and in principle is like the supplementary passage of the Allen valve that is still used on some locomotives. This size of engine is run at about 280 revolutions per minute.

## SOUTHWARK ENGINE, 9x10.

Fig. 73. It is the first of the second group of Single-valve Automatics. In this group the locus **LL** of the driving-pin or eccentric $E$ of the valve has its center $P$ outside of the crank radius $OC$. In most of these cases the pivot $P$ and crank-pin $C$ are still both on same side of shaft center $O$. In the present case the co-ordinates of $P$ are $5\frac{1}{4}''$ and $-\frac{5}{8}''$ which brings it opposite the middle of the utilized portion of the arc **LL**. The arc is concave to $O$ and its chord is perpendicular to crank radius so that it gives a nearly constant lead, for all grades of expansion. The cut-off ranges from 0 to $\frac{2}{3}$ and the cushioning from $\frac{1}{16}$ to $\frac{1}{4}$. This valve-gear and its governor were designed by Prof. C. B. Richards. The substitution of a pin $E$ in place of the eccentric sheave is an excellent feature as it removes the danger of overheating to which the eccentrics of high-speed engines are so liable.

The rod covering the driving-pin $E$ with the $3\frac{1}{4}''$ rocker is $34''$ long; treating the rocker arc as approximately a straight line, we can repeat the constructions of the arc $Y'Y'$ already given with the three preceding engines. Here too the deviation of the more exact reference arc $Y'Y'$ from the vertical reference line $YY$ is so light that there is no need of substituting exact polar curves for the valve-circles. The valve used in this engine is flat, balanced, and of the positive direct type. The speed of this 9x10 engine is 300 revolutions per minute.

## THE STRAIGHT LINE ENGINE, 11x14.

Fig. 74. The locus **LL** is concave to $O$ and has the point $P$ outside of the crank radius $OC$, but $P$ and $C$ are both on the same side of $O$, the abscissa of $P$ being $12.25''$ and its ordinate $-2\frac{1}{2}''$. The stroke of the valve does not pass through the center $O$ of the shaft, but passes it at a distance of $1\frac{1}{2}$ inches. The dead point line of eccentric will therefore be different for the two ends of

the valve strokes and for the different throws of the valve. An average position has been drawn on the diagram. It is therefore like the case represented in Fig. 66, p. 151. The offset $Ee$ is there shown to be nearly equal to $\frac{c}{l}r$; here $c = 1.5''$, $l = 34''$ and $r$ = eccentricity corresponding to grade of expansion considered. If $r = 2.5''$, then $\frac{1.5}{34} \times 2.5 = 0.11$ is the offset, which laid off from $E_2$ at right-angles to $OE_2$ gives $E_{v\,2}$ which is the center of the virtual eccentric $OE_{v\,2}$. In like manner other offsets may be found for other throws of the valve and thus a curve **L′L′**, passing through the ends of these offsets, can be found which will be the locus of the centers $E_v$ of the virtual eccentrics $OE_v$. The valve-circle belonging to any eccentric can be found as in Fig. 60 and in either of two ways: it can be obtained from the actual eccentric (drawn from $O$ to locus **LL**) by laying off the angle $\sigma$ from the dead point line of eccentric, or it can be obtained from the virtual eccentric $OE_v$ by laying off an angle $\sigma$ from the dead point line of crank. (See p. 137, Fig. 60 and Fig. 66, p. 151). The chord cut from the crank by the valve-circle thus determined equally represents the horizontal distance of $E_n$ from the $YY$ line through $O$ or the horizontal distance of $E_{vn}$ from the vertical line $Y'Y'$. (The chords of the valve-circle do not represent either of these distances exactly on account of an approximation made in the construction, Fig. 66.) In neither of these determinations is the angularity of the eccentric-rod fully taken into account. To do this we proceed as in preceding cases. Thus from $E_2$ as a center, with length of eccentric-rod ($= 34''$) as a radius we strike an arc that cuts the central line or stroke of the valve; with same radius and $E_2'$ as a center a second arc is struck off, again cutting the line of stroke of the valve; the point on the stroke bisecting the distance between these arcs will be the center of motion, from which as a center and $34''$ as a radius the arc of reference $Y''Y''$ can be described. The distance of $E_n$ from this arc gives with great exactness the actual travel of the valve from the center of motion for the assumed eccentric $OE_n$. In

this case, the arc of reference $Y''Y''$ does not hug the line $YY$ so closely that it is evident that the vectors of the valve-circles represent with sufficient accuracy the travel of the valve. We therefore give in the following table the *exact* steam distribution for $r_2 = 2.5$ and $r_1 = 1.48$, the latter value differing a little from the value $r_1 = 1.41$ assumed in diagram. The following values were found by constructing the exact polar curves.

| Distri-bution. | $r_1 = 1.48$. | | Distri-bution. | $r_2 = 2.50$. | |
|---|---|---|---|---|---|
| | Crank End. | Head End. | | Crank End. | Head End. |
| $\varepsilon_1$ | 0.31 | 0.29 | $\varepsilon_2$ | 0.71 | 0.77 |
| $\varepsilon_1'$ | 0.10 | 0.12 | $\varepsilon'_2$ | 0.04 | 0.04 |
| $\eta_1$ | 0.34 | 0.34 | $\eta_2$ | 0.11 | 0.13 |
| $\eta'_1$ | + 0.00 | — 0.005 | $\eta'_2$ | + 0.00 | + 0.00 |

This is almost perfect equalization of the distribution for the two ends. A careful comparison of these results with those given in the diagram show that, with perhaps the exception of cut-offs near $\frac{1}{4}$, the distribution is given with sufficient accuracy by the valve-circles even in this extreme case.

The numerical values inscribed on the diagram show that cut-off ranges from about 0 to $\frac{3}{4}$, that the cushioning is nearly equal for the two ends of the cylinder and that the lead varies from a negative value of 0.11 at minimum throw to positive value of 0.11 at maximum throw. The lead is known to be about equal at the two ends for this throw. But this preadmission is attained in very small fractions of the stroke as the tabulated values $\eta'$ show.

The minimum throw chosen was that one in which the port is opened only for an instant. The almost perfect equalization of the functions of the exhaust illustrates the value for this purpose of making the laps unequal. There seems to be more clearance at head end than at crank end in this engine.

Steam is admitted at two places, the port openings given by shaded area of the diagram must therefore be doubled in order to get the true area available for the passage of the steam. The speed of this size of engine is about 275 revolutions per minute.

The Straight Line Engine was designed by Prof. John E. Sweet and constitutes a most instructive example in steam distribution. I am indebted to Prof. Sweet for the following outline of his practice in securing smooth running engines.

Originally the valve motion was designed to produce a practically constant lead at each end of the cylinder. Later experience convinced Prof. Sweet that a constant lead was exactly what he did not want and he therefore located the eccentric plate pivot *P* so that it would give a variable lead equally at the two ends when a correcting, rocker arm, device was used. Still later he found that the variable lead enabled him to dispense with the correcting arrangement, the lead at the two ends being then somewhat unlike. Prof. Sweet's practice on the testing floor is as follows: the first valve is made too long and with too much inside lap, then with varying loads and the indicator and the exercise of judgment as to smooth running, the valve is altered and tried until the best average is obtained. So much discrepancy has been found between different size engines of the same style and sometimes different engines of the same size that no attention whatever is paid to drawing room figures on the valve or its setting. One reason for this is the possible, disturbing, influence of the governor. Prof. Sweet prefers an increase of compression to an increase of lead at the head end of the cylinder; the compression will show more on the cards, but this is held to be of no practical account as the object is to obtain a smooth running engine rather than good looking cards.

In the smooth running engine for which the diagram was drawn, the distribution was practically equalized for the two ends of the cylinder. So far as this one example goes it would therefore seem that uniformity of distribution is favorable to smoothness of running.

## STURTEVANT ENGINE, 8x12.

Fig. 75. The point $P$ of the pendulum arm lies outside of the crank radius $OC$, but $P$ and $C$ are both on same side of shaft center $O$. The locus **LL**, struck from $P$ as a center, is convex to center $O$ of its shaft, but it is so located that the lead increases with the throw, and the preadmission takes but a small part of the stroke. The cut-off varies from $\frac{1}{7}$ to $\frac{4}{7}$. The piston-valve has a diameter of 3″ and is of the positive indirect type. The steam passages in the cylinder have a cross-section $\frac{7}{8}'' \times 6''$. The speed of this engine is 250 revolutions per minute.

## BALL ENGINE.—RECENT TYPE, 12x12.

Fig. 76. This is the diagram of a more recent type of Ball engine than that discussed in Fig. 71. Here the point $P$ is outside of crank direction $OC$ and $P$ and $C$ are on *opposite* sides of shaft center $O$. The locus **LL** is concave to $O$, but so placed as to permit but little change in the lead as the expansion varies; the preadmission also occupies but little of the piston stroke. The cut-off ranges from 0 to $\frac{2}{3}$. In this engine, as in the Southwark, a pin takes the place of the eccentric sheave, thus affecting a reduction of the work of friction and removing the danger of heating. The valve is of the flat and balanced variety; it exhausts at the outer edges and takes steam at the inner ones.

## WESTINGHOUSE STANDARD VERTICAL ENGINE, 9½x9.

Fig. 77. In this diagram the left-cylinder-crank $C'$ is at the left and the right one $C''$ at the right, both being placed on the vertical line of the engine (not of the diagram). The relative positions of crank and eccentric are as they would appear if viewed from the right-crank end of shaft. The terms right and left cylinder as used here and as used by the manufacturers supposes the engine viewed from the throttle side; then it is the $\left\{\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right\}$ port that admits steam to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ cylinder. The

center line of each cylinder passes the center $O$ of shaft at a distance equal to half the length of crank $OC''$. In the diagram this distance and crank are drawn to a reduced scale.

The pivot $P$ of swinging eccentric is $\frac{7}{16}''$ from the radial line of the left crank and has a negative abscissa of $8\frac{3}{8}''$. The radius of locus **LL** is $PE_2 = 9\frac{7}{16}''$. At maximum cut-off the eccentricity is $1\frac{3}{4}''$ and the radius $PME_2$ is tangent to the arc $M$ described with $OM = 1\frac{5}{16}''$ as a radius; at minimum cut-off the eccentricity is $1\frac{3}{64}'' = OE_0$ and the radial line $PmE_0$ is tangent to the arc $m$, which is struck off with radius $Om = \frac{3}{32}''$. The valve-circles are found as in Fig. 60, p. 137. The lead $v$, for all grades of expansion, was measured on that dead-center line of crank which corresponds to uppermost position of piston and increases with the cut-off. The engine is composed of two single-acting, vertical cylinders which receive the live steam in the top end of the cylinder. As each cylinder has its own crank and connecting-rod this arrangement will cause the angularity of the rod to have precisely the same influence on the steam distribution of each cylinder. This is shown by the values tabulated on the diagram. The cut-offs evidently range from 0 to $\frac{3}{4}$.

The table inscribed on the diagram and valve-circles (in full lines) do not take into account the influence of the length of the eccentric rod. The vectors of the valve-circles drawn from $O$ give the hor. distance of the eccentric center $E_n$ from the line $Y'OY'$ which is perpendicular to the dead center line of eccentric. To take account of this angularity of the eccentric-rod, we make the following construction. Place the crank upon dead point for top of stroke and the eccentric in a corresponding position. The center $E_2$, will then be $4\frac{2}{3}°$ in advance of position shown in the diagram. Taking this new position of $E_2$ as a center and the $20''$ length of the eccentric as a radius, strike an arc that will cut the dead-point line of eccentric; then revolve this eccentric through $180°$ and use the new position of $E_2$ as a center to repeat the former operation. Now bisect the distance between

these two arcs and the point of bisection will be the center of motion to which the valve travel is referred and from which as a center the arc of reference $Y'Y'$ can be described with the 20″ radius. The hor. distance of the eccentric center $E_n$ from this arc of reference will be the exact travel of valve. If this exact travel is laid off on the corresponding crank positions, laying off valve travel to $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of center of motion on $\left\{\begin{matrix}\text{actual}\\ \text{prolonged}\end{matrix}\right\}$ crank repeating this process for second crank also, we will get two pairs of polar curves instead of the two valve-circles, and the intersection of these curves with the proper valve-circles will give the exact steam distribution. One pair of these polar curves, the one for left cylinder crank has been roughly drawn on the diagram for maximum throw. They lie on opposite sides of valve-circle $OD_2$ which may be regarded as their average value. The $\left\{\begin{matrix}\text{outer}\\ \text{inner}\end{matrix}\right\}$ one of the polar curves represents travel to the $\left\{\begin{matrix}\text{left}\\ \text{right}\end{matrix}\right\}$ of the center of motion. The exact steam distribution given by carefully drawn polar curves is contained in the following tabulation:

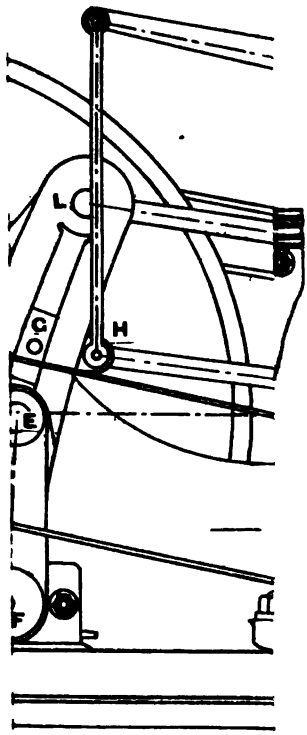
| Half Throw | Right Cylinder. | | | |
|---|---|---|---|---|
| $r_1 = 1\frac{15}{64}$<br>$r_2 = 1\frac{3}{4}$ | $\varepsilon_1 = 0.34$<br>$\varepsilon_2 = 0.68$ | $\varepsilon_1' = 0.21$<br>$\varepsilon_2' = 0.08$ | $\eta_1 = 0.38$<br>$\eta_2 = 0.16$ | $\eta'_1 = 0.01$<br>$\eta'_2 = 0.00$ |
| **Half Throw** | **Left Cylinder.** | | | |
| $r_1 = 1\frac{15}{64}$<br>$r_2 = 1\frac{3}{4}$ | $\varepsilon_1 = 0.28$<br>$\varepsilon_2 = 0.67$ | $\varepsilon_1' = 0.21$<br>$\varepsilon_2' = 0.06$ | $\eta_1 = 0.35$<br>$\eta_2 = 0.17$ | $\eta_1' = 0.00$<br>$\eta_2' = 0.00$ |

The slight inequalities that exist for the two cylinders is wholly due to the influence of the length of the eccentric-rod. The diameter of the piston valve is $4\frac{9}{16}''$ and the speed is 300 revolutions per minute.

### WESTINGHOUSE COMPOUND VERTICAL ENGINE, 14 AND 24X14.

Fig. 78. In this case the stroke of the pistons passes directly through the center of shaft. The single, piston, valve controls two ports $M$ and $N$ (see little figure to right of diagram 78). The passage $P$ leading to high-pressure cylinder is always in communication with the hollow of the valve. The port $M$ therefore admits live steam from pipe $S$ to the high-pressure cylinder, and port $N$, at one edge receives steam from high-pressure cylinder and admits it to the low-pressure cylinder, and at the other edge it delivers steam from low-pressure cylinder to the exhaust pipe $E$. The steam laps $e$ at the inner edges are alike for both ports $M$ and $N$. The exhaust lap is however equal to 0. The value of $e'$ given in table was obtained by multiplying actual lap $e$ by 7/8 ($= \frac{9}{10\frac{3}{8}}$, the ratio of the arms of the bent lever driving the valve) to reduce it and the stroke of the valve to the end of the eccentric-rod. This is a departure from our usual practice of assuming that the actual valve is driven directly by the eccentric, but in this case it was simpler to change only one dimension than to change several.

The pivot $P$ does not lie on either of the two cranks, but it is near to the low-pressure crank and on the same side of center $O$. The abscissa and ordinate of $P$ are 5.42″ and 1.5″ respectively, its total, radial distance from $O$ being 5 5/8″. The locus **LL** struck from this center has a radius of 6 9/16″ and is concave to $O$. The table and data inscribed on diagram state distribution. In this Compound, as in the Standard, the steam distribution is the same for both single-acting cylinders when the eccentric-rod is taken as infinitely long. In this case the rod is finite and 49″ long and its influence on the distribution can be ascertained in the now well-known manner by drawing the arc of reference $Y'Y'$. It is evident from the figure and from what has been said about other diagrams, that the valve-circles give the valve-travel with sufficient accuracy.




O′R – 2¼″

R P – 3″ – RQ′

C R – 52″

OUTSIDE LAP – 0.7

INSIDE LAP – 0.04

LENGTH OF CONN.

RADIUS OF CURVED

## LINK-MOTIONS.

In the second group of single-valve gears (see p. 153), the eccentrics themselves are invariable, the variation in the valve motion being affected by the mechanism between the eccentrics and the valves. All the commom link motions belong to this group; we will here describe and examine but one of them, the Porter-Allen, (or 'Fink'), the only one which in this country has been extensively used on stationary, high-speed engines.*

The Porter-Allen is that special case of the Gooch link motion which is obtained by supposing the centers of the two eccentrics to coincide, thus forming only one eccentric, the two eccentric rods and link then constituting one rigid piece. The best proportion for this linkage are shown in Figs. 79–82, and were taken from blue-prints furnished by the Southwark Foundry & Machine Co., of Philadelphia.

## PORTER-ALLEN VALVE-GEAR, 11½x20.

Figs. 79, 80 and 81 give a side view of this engine, a section through its rocker arm and a skeleton showing the relative position of the three valves when crank is on its forward center. This view and the skeleton of the mechanism given in Fig. 82 are sufficiently full and complete to render any detailed description unnecessary. We will only ask the reader to note that eccentric strap and link constitute one rigid piece, that for a particular grade of expansion we assume that the governor occupies an invariable position vertically, thus making point of suspension *I*

* "Prof. Zeuner's views of the unsuitableness of the 'Fink' link as a *reversing-gear* are correct, and for the reasons he gives. But the use of the half-link in the Porter-Allen engines (it is in connection with these engines that the Fink motion was first brought into extensive use) shows that when compression is judiciously applied, then the cut-off points may be made practically 'symmetrical' up to the half-stroke *at the expense of a variable lead*, the variation of which is in opposite directions for the two strokes. Before cushioning was used in these engines the variability of the lead made it almost impossible to obtain quiet running under varying conditions."

of the hanger $IH$ a fixed stationary point and finally that even then the center $G$ of block does not maintain an invariable distance from the trunnion $E$, but slips a little in its slot.

The view of the relative position of the valves is only a skeleton and does not represent the multiple port-openings; each of the three valves admits steam at four different places thus offering a generous passageway for both the live and exhaust steam. Before taking leave of these general views we desire to call special attention to the rocker arms $O'RPQ'$ which communicate motion to the rods and stems of the two steam valves. From the arrangement we see that each driving arm, $RP$ and $RQ'$, has a period of almost complete rest and two periods of rapid motion. The rapid motion period effects a rapid opening of the port to its full width and then a rapid closing, the period of comparative rest occuring when the port is closed by valve. This arrangement of bent levers does excellent service but modifies greatly the "harmonic motion" of the driving point of the link and requires other diagrams than the valve-circle for the exact representation of the valve-travel. We will return to this matter later on.

We will first show that the travel of a point on the link $EGL$ is approximately like the "harmonic motion" communicated by an eccentric that has an infinitely long eccentric-rod. Let $A$ represent the distance from center of eccentric $B$ to center of trunnion $E$, the latter joining sustaining arm $FE$ and $EGL$ on center line of slot of link; let $\rho$ represent the eccentricity $CB$, $u$ the distance of point $G$ above dead center line of eccentric, $\omega$ the crank or (or eccentric) angle and $\xi$ the distance of eccentric center $B$ from perpendicular through $C$ to eccentric's dead-point line (which is approximately along $CEO'$, the crank and eccentric differing in position by a small angle of $2° 12'$ that is equal to angle included between their respective dead-point lines). Then, for the case in which $\frac{A}{\rho} \geqq 4$, we will show that approximately:

$$\xi = \rho \cos \omega + \frac{u}{A}\rho \sin \omega \qquad (90)$$

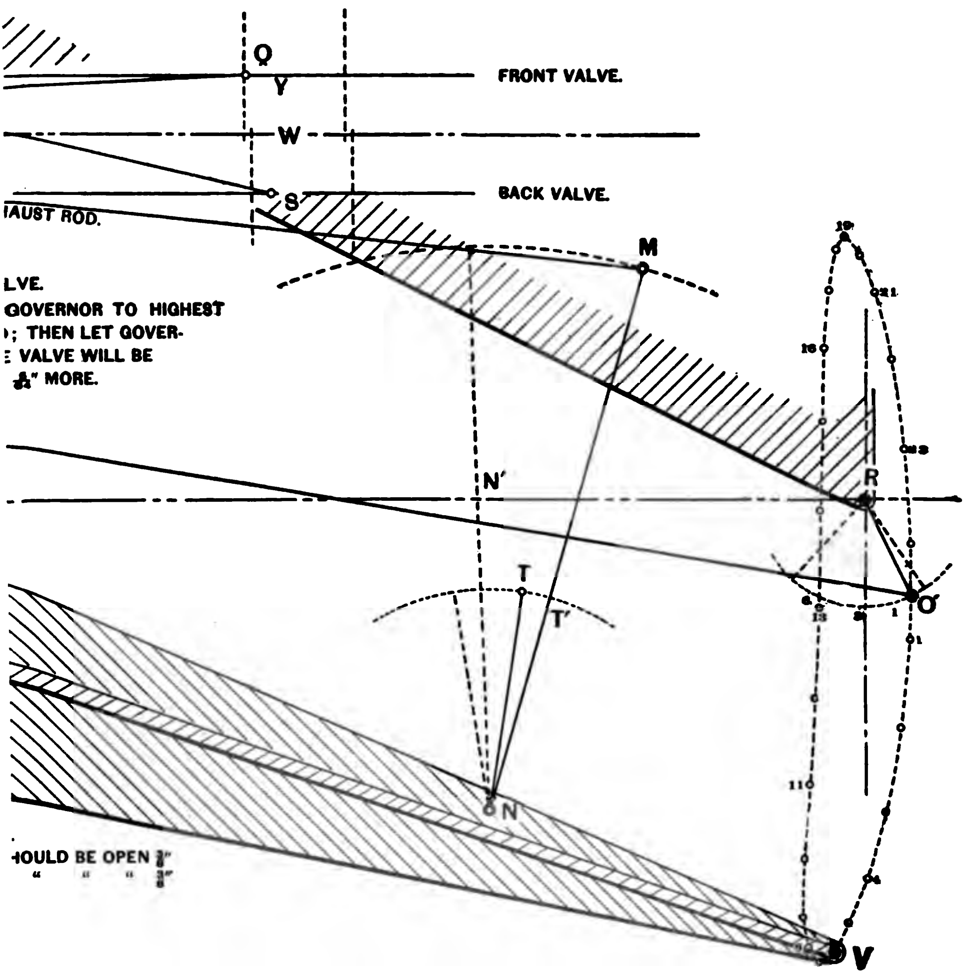
To prove this, we consider the total movement of link to be composed of two parts, of a *horizontal* motion in which the link moves parallel to itself, the character and extent of this motion depending on the *horizontal* throw of the eccentric, and of a *rocking* motion about the trunnion $E$ (or vibrating fulcrum) which is due to the vertical throw of the eccentric. Here the term $\rho \cos \omega$ is the horizontal throw common to all points of the link and the term $\frac{u}{\Lambda}\rho \sin \omega$ is the vertical throw $\rho \sin \omega \times$ the ratio $\frac{u}{\Lambda}$ of the arms of the bent lever constituting the link and rotating about the fulcrum or trunnion $E$. The formula for $\xi$ thus established is the polar equation of the valve-circle and the coordinates $\rho$ and $\frac{u}{\Lambda}\rho$ of the vertex of its diameter show that this vertex lies on a rectilinear locus perpendicular to the dead-point line of the eccentric and at a distance $\rho$ from the center of the shaft. Such a valve-circle is drawn in Fig. 83 for full gear (*i.e.*, for block $G$ about 6″ from trunnion), and the corresponding equivalent, (virtual or resultant) eccentric can be found in the usual way, Fig. 60, from the diameter of this valve-circle.*

To show that the valve-circle thus found represents with considerable accuracy the travel of a point on the link, we

* As the horizontal component motion due to the rocking, is motion relative to trunnion $E$, we can also combine the parallel and rocking components by means of the parallelogram of velocities. We need only regard the rocking component of the motion as directly and independently produced by a separate eccentric, one that would *follow* the actual eccentric by 90° and have a length $\frac{u}{\Lambda}\rho$. As these two eccentrics are both at right angles to, and proportional to, the velocities they impart we can construct the parallelogram of velocities on the eccentrics as sides and the diagonal will give the length and location of a resultant or virtual eccentric capable of imparting directly the whole horizontal motion to the point of the link. The parallelogram and triangle of eccentricities constructed in Fig. 83 and Fig. 84, respectively, are illustrations of this method. The valve-circle can then be found from the virtual eccentric as in Fig. 60.

have in Fig. 83 compared such a valve-circle with a polar curve *OMN* or *OM'N'* representing very exactly the travel of the rocker-pin *O'*. (Figs. 79 and 82.) To simplify matters we have assumed pin *O'* to travel on a straight line that bisected the rise of its arc-path and passed through center *C* (Fig. 82) of shaft. In Fig. 83 we have called this straight line *CO'* (of Fig. 82) the dead center line of eccentric and constructed the valve-circle according to formula 90 found above. The dotted, polar, curves *OMN* and *OM'N'* (Fig. 83) were found by laying off on the crank positions the exact distance of pin *O'* from its center of motion, supposing *O'* to travel on aforesaid dead point line *CO'*. We see that these curves agree very well with the valve-circles and that the latter may unhesitatingly be used to represent the travel of lower rocker-pin *O'* parallel to dead point line *O'C*. Inspection of the setting of the upper rocker-pins *Q'* and *P* shows at once that the horizontal components of their motion could not be represented by valve-circles, and that to represent the valve travel accurately we must construct exact polar curves for both the front and the back valve.

A fairer comparison of the actual travel of a point on the link with that given by the formula or valve-circle is furnished by Fig. 84, which represents by the dotted, polar curves the actual travel, from its center of motion, of the point *T* (Fig. 82) driving the exhaust valve. It is evident that the average direction of the point *M* of the exhaust rocker *NM* is parallel to the average direction of the path of the driving point *L* on link. We may therefore assume that the actual travel of *M* is almost axactly like that of *L*. Now the travel of *T* is 0.39 of that of *M*; it is represented by the dotted, polar curves of Fig. 84, which were obtained by directly measuring the distances of point *T* from its center of motion and then laying off the distances on the corresponding positions of the crank. The valve-circle representing, approximately, the travel of point *L* is drawn on diagram on diameter $OD_v'$ and reducing it 0.39 we get the valve-circles $OD_v''$ representing, approximately, the travel of driving point *T*.



FRONT VALVE.
BACK VALVE.
IAUST ROD.
LVE.
GOVERNOR TO HIGHEST
; THEN LET GOVER-
VALVE WILL BE
" MORE.
HOULD BE OPEN
Q
Y
W
S
M
R
N'
T
T'
O'
N
V

Though the deviation of the two sets of curves is now marked, nevertheless, the deviations occur where they will but slightly influence the beginning of compression or release, which are found as of old at the intersections of polar curves (or circles) with the lap circles.

We have already called attention to the fact that the setting of the driving arms $PR$ and $Q'R$ causes them to communicate to the valves a horizontal motion that is decidedly different from the "harmonic motion" assumed for eccentrics. If we wish to represent the valve's motion accurately by polar curves whose vectors are the crank positions, we must determine accurately the distance of the valve from its center of motion for a complete series of crank positions.

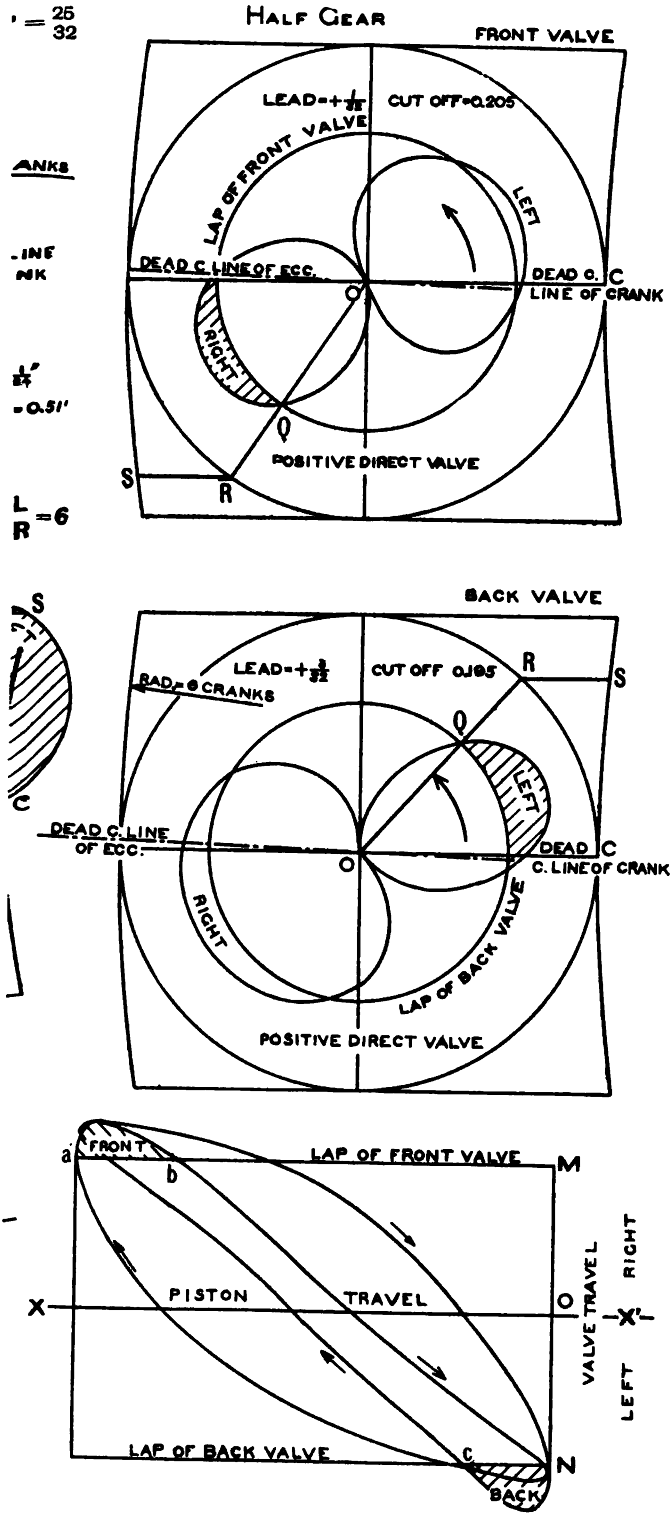
Inspection of the skeleton in Fig. 82 shows that it is composed of two quadric chains $CBEF$ and $HIRO'$ connected by the straight link $GV$ (which in the actual mechanism is represented by the link-block $G$). For the assumed point of suspension $I$ the point $G$ of second quadric chain describes the path $G$, 1, 3, 6, while the point $O'$ of lower rocker arm describes the corresponding path $O'$, 1, 3, 6. In the first quadric chain $CBEF$ the point $L$ occupies points 1, 8, 13, 23, etc., of its path, while eccentric center $B$ occupies points 1, 8, 13, 23, etc., of its circular path. The center $V$ of the slot $GE$ is the point on which rod $GV$ turns and this point is on the line $BE$ of the first quadric chain; the piece $BE$ in the skeleton is supposed to be extended so as to support this point $V$. The path $V$ is given and the numbers correspond to the crank or eccentric positions. We can easily find the position of either valve for any crank position if we know the corresponding position of $O'$. To get $O'$ for any eccentric (or crank) position, we suppose $V$ placed at that point of its path which has the same number as the crank position under consideration; with this point $V$ as a center strike off an arc cutting the path of $G$: then with this intersection of the arc and the path as a center and the steam rod $GO'$ as a radius strike off another arc; it will cut the circular path of $O'$ at the point $O'$ desired.

The position of each valve can be represented by its point $Q$ or $S$ and is estimated from a center of motion that is invariable in position for all grades of expansion, *i.e.*, for all positions of the block $G$ in its slot. To find this center of motion we must first find two peculiar eccentric positions *near* the dead-point line of the eccentric. To get these positions we erect at $C$ (Fig. 82) the perpendicular $CF_1 = EF$ to the dead-point line of eccentric. Then with $F_1$ as a center and $F_1C$ as a radius describe an arc tangent to this dead-point line; the chords $C1$ and $C13$ of this arc will be the peculiar eccentric positions needed. Now find two positions of both $Q$ and $S$ corresponding to these two peculiar eccentric locations and then find a point half way between them; this point will be the desired center of motion from which the travel is to be estimated, and will be found to be the same whatever position is assumed for the point $G$ in its slot (that is, it will be the same whatever fixed position is chosen for the point of suspension $I$ of the hanger $IH$). The center of motion can also be found, and more simply still, in the following way. The steam rod is $\frac{3}{16}''$ shorter than radius of link slot; near the two intersections of path of center $V$ (Fig. 82) with arc of $O'$ lay off $\frac{3}{16}''$ from arc $O'$ towards path of $V$, in the direction of middle radius of the link slot, till a place is found (on each half of $V$) where arc $O'$ and path of $V$ are just $\frac{3}{16}''$ apart. When center of steam rod occupies either of these two positions of $O'$ the block may be moved from one end to the other of link slot without moving rocker pin $O'$ or either of the two steam valves. Hence these two points on arc $O'$ will give exactly the two valve positions that are equidistant from the center of motion; in like manner the two points on path $V$ thus found will give the corresponding eccentric positions and these will be found to be near the peculiar ones described above. The distance of each valve from its center of motion is now laid off on corresponding crank positions and the polar curves shown in Figs. 86 and 87 for full gear ($G$ is then $6''$ from trunnion $E$) obtained, also the polar curves shown in Figs. 89 and 90 for half-gear (block $G$ is $3''$ from trunnion $E$). To ascertain the lap from the data given in

Figs. 89, 90, 91.

MS FOR BOTH STEAM VALVES—

HALF GEAR



Figs. 79–82, we must find positions of point $\left\{\begin{matrix} Q \\ S \end{matrix}\right\}$ when engine is on $\left\{\begin{matrix} \text{forward} \\ \text{back} \end{matrix}\right\}$ center and block $G$ at trunnion $E$; as the edge of the $\left\{\begin{matrix} \text{front} \\ \text{back} \end{matrix}\right\}$ steam port is then $\frac{1}{16}''$ $\left\{\begin{matrix} \text{left} \\ \text{right} \end{matrix}\right\}$ of edge of $\left\{\begin{matrix} \text{front} \\ \text{back} \end{matrix}\right\}$ steam valve, we lay off $\frac{1}{16}''$ to the $\left\{\begin{matrix} \text{left} \\ \text{right} \end{matrix}\right\}$ of $\left\{\begin{matrix} Q \\ S \end{matrix}\right\}$ to get its position when valve edge is exactly at port edge. Now place $\left\{\begin{matrix} Q \\ S \end{matrix}\right\}$ at its own center of motion and measure its distance from its former position when valve and port edges coincided; the distance will be the steam lap desired. Laying off the same steam lap $= \frac{25}{32}$ for both front and back valve, and measuring in each case the distance $RS$ (Figs. 86–90) and dividing it by the stroke $= (2 \times \overline{CO})$, we get the cut-off effected at each end and at each grade of expansion. The shaded areas give in all cases the linear port openings and should be multiplied by 4 for the total openings. The numerical values of the distribution are inscribed on the diagrams and show that it is a practically perfect valve gear in this respect.

The actual valve travel found above can also be laid off as ordinates, on the piston stroke as a base, and then we get another series of valve diagrams of an oval shape, that are known as "motion" curves. The exact polar and oval diagrams give of course identical results. Figs. 85, 88 and 91 respectively represent the travel of the exhaust valve, that of the front and back steam valves for full gear and that of the front and back steam valves for half gear. In Fig. 85 cushioning in crank end is measured by $\overline{aX} \div \overline{OX}$, release in head end by $\overline{bX} \div \overline{OX}$; similarly cushioning to head end is given by $\overline{dn} \div \overline{OX}$ and release in crank end by $\overline{cm} \div \overline{OX}$. In Figs. 88 and 91, $\overline{ab} \div \overline{OX}$ measures the cut-off in crank end and $\overline{Nc} \div \overline{OX}$ the cut-off in

head end. The polar diagrams for the steam valve best show the variation of lead. The speed of this size of Porter-Allen engine is 230 revolutions per minute.

## DOUBLE-VALVE GEARS.*

The characteristic of a double-valve gear is that while the first of its two valves slides on a fixed seat, the second slides on the first, the motion of the second valve relatively to the first being affected by the absolute motion possessed by each of the two valves.

In single-valve gears with variable elements, and in link-motions, we simplify our problem by finding the equivalent or virtual eccentric and then proceed as if the valve were directly moved, to and fro upon its fixed seat, by this virtual eccentric. In double-valve gears a like simplification is introduced, in fact it is the finding of this virtual eccentric which usually constitutes

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* This term, double-valve, has occasionally been applied to a valve consisting of two rigidly connecting halves and also to two separate and independent valves each sliding on a fixed seat. The gear here called double-valve gear has sometimes been defined as gear with plain slide valve and independent cut-off; it has also been described as composed of a plain slide valve and a riding valve. But these are long expressions and are also not free from ambiguity.

We wish to distinguish between gears requiring considering of only absolute motion and those in which relative motion must be considered. Gears possessing but a single valve evidently belong to the former class and the term, single valve, is sufficiently suggestive of absolute motion. As relative motion implies at least two pieces we feel justified in using the term double-valve to suggest this motion. A further justification is supplied by the fact that the principal functions, of opening or closing a steam passage, must be doubly performed (though not simultaneously) when one valve slides on the back of another. We must confess, however, that we are not satisfied with the terms we have chosen and hope that something else as simple but more exact may be found.

the main part of the problem. Such an eccentric drives the expansion valve, over a perfectly stationary main valve, with an absolute motion that is the same as the motion of the expansion valve *relatively* to the main valve when both these valves have their actual motion. In this way the new and complex problem of the relative motion of two valves, is reduced to the old and simple problem of a single-valve sliding on a fixed seat and driven by one eccentric.

Now as to the method of procedure. When the main and expansion eccentrics are both given, the virtual eccentric can easily be found by means of a proposition for which Dr. Zeuner suggests the name of " Parallelogram of Eccentricities."

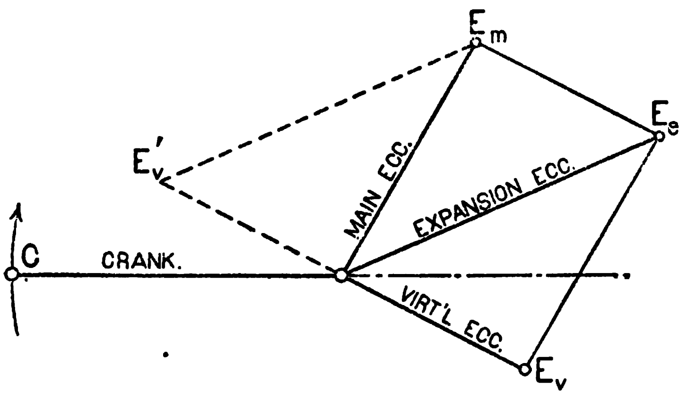



Fig. 92.

It may be stated as follows: *The main, virtual and expansion eccentrics constitute, respectively, the two adjacent sides and diagonal of a parallelogram.*

In Fig. 92, $OE_m$, $OE_v$ and $OE_e$ represent these eccentrics and $OE_mE_eE_v$ the parallelogram which they form. To avoid confusion hereafter it should be specially noted that the *expansion eccentric* is always the *diagonal* of this parallelogram, provided the virtual eccentric $OE_v$ effects the motion of the expansion valve relatively to main valve. But when it is a question of

relative motion of main valve to expansion valve the virtual eccentric must be $OE_v'$, just opposite to $OE_v$, and $OE_m$ must then be the diagonal of parallelogram $OE_e$ $OE_m$ $OE_v'$. In all our work we shall take the *expansion eccentric* as the *diagonal* of the parallelogram.

In *existing* valve gears the main and expansion eccentrics are completely known and the examination of the steam distribution effected by their valves is easily made by means of the main and virtual valve-circles, which are deduced in the usual way from the known main and virtual eccentrics.

In *designing* valve gears, however, the expansion eccentric is the unknown quantity whose variations must satisfy the prescribed conditions as to range of expansion, time of admission, rapidity of cut-off, etc. To find it we must first find the main and virtual eccentrics and with these two as adjacent sides construct a parallelogram whose diagonal will be the desired expansion eccentric.

The main eccentric can be found in the usual way from its valve-circle and this in turn from the given conditions as to lead, port-opening, maximum cut-off, release or compression. The virtual eccentric is found in a similar way, its circle being first obtained from such given conditions as, type of expansion valve, cut-off and time of reopening.

Having thus indicated the principal steps in designing a double-valve gear, we will return to the beginning and show that, when there are two valves and both are in motion, one riding on the other, then a fixed circle can be found which will cut from the crank, or its prolongation, chords whose lengths represent the distances between a point on the expansion valve and a corresponding point on the main valve, the correspondence being simply that the two points are together when the expansion valve occupies its middle position on its seat on the main valve. The distance between two such points at any crank position is evidently equal to the (algebraic) difference of their distances from any common, fixed, reference line. Neglecting the angu-

larity of the eccentric-rods, this difference is equal to the difference in the distances $E_eS_e$ and $E_mS_m$, Fig. 93, of the corresponding eccentric centers $E_e$ and $E_m$ from their common reference line $OP$, which is perpendicular to the dead-point line $NOZ$ of the eccentrics. Our problem is now reduced to finding a

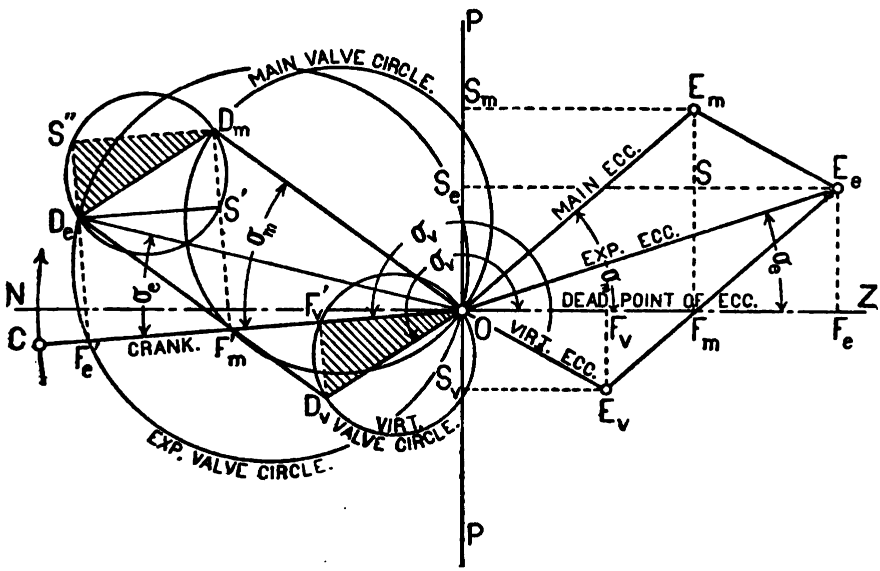



*Fig. 93.

circle that will cut from any crank position $OC$ a chord equal to the corresponding difference $E_eS_e - E_mS_m = E_eS$ of the distances of the centers $E_e$ and $E_m$ from the middle position $OP$ of the eccentrics.

Finding the main valve-circle $OD_m$ from its eccentric $OE_m$ as in Fig. 60, p. 137, we get the chord $OF_m' = OF_m = E_mS_m$ as the distance of $E_m$ from $OP$. The distance $E_eS_e$ of $E_e$ from $OP$ can be found in the same way by constructing the expansion valve-circle $OD_e$. To do this we measure angle $\sigma_e$, from the *right* portion the eccentric's dead-point line $OZ$, in direction

* The inner angle $\sigma_v$ extends from $OZ$ around to $OE_v$. The outer angle $\sigma_v$ extends from $OC$ around to $OD_v$.

opposite to the rotation, and lay off this angle $\sigma_e$ from the crank in the direction of rotation; $OD_e = OE_e$ will be the diameter of the expansion valve-circle and $OF_e' = OF_e = E_eS_e$. Then because $D_mS''$ is parallel to crank position $OC$, we have $D_mS'' = F_e'F_m' = F_eF_m = E_eS$. As $D_mF_m'$ is perpendicular to $OC$, the angle $D_mS''D_e$ is a right angle. But the hypothenuse $D_eD_m$ is constant in position and magnitude for all crank positions; the locus of $S''$ is therefore a circle. If through $D_m$ we draw a parallel to the crank position $OC$ at any instant, this circle will cut from this parallel a distance $D_mS''$ which is equal to the value of $E_eS$ at this instant. By moving this circle parallel to itself so that the point $D_m$ of this circle $D_mS'D_e$ shall shift its position to shaft center $O$, the circle will take up the new position $OD_vF_v'$ and cut from the crank itself (or its prolongation) the distance $OF_v' = D_mS'' = E_eS = E_eS_e - E_mS_m$. In its new position it may be called the *virtual* valve-circle for it possesses all the properties of a valve-circle relatively to its eccentric. For instance, the virtual eccentric $OE_v$ is found from the virtual valve-circle $OD_v$ by laying of the angle $\sigma_v$ in the same way as the angles $\sigma_m$ and $\sigma_e$. Moreover, when the $\left\{\begin{matrix}\text{actual}\\ \text{prolonged}\end{matrix}\right\}$ crank cuts the virtual valve-circle, the virtual eccentric will be to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of its middle position $OP$. As regards the periods during which the riding-valve keeps the ports in the main valve opened or closed, the virtual valve-circle, like the two others, is subject to the conditions of valve-type, etc., laid down in Table XIV. The valve driven by the virtual eccentric is of course of the same type as the expansion valve, the *relative* motion of the latter corresponding exactly to the *absolute* motion of the former.

Before applying these results to the common high-speed engines, there is still another proposition enunciated by Dr. Zeuner which is of the greatest service in the solution of valve-

gear problems. But before stating what it is, we will point out that with double-valves the expansion can be varied:

I. By varying the angle of advance of the expansion eccentric.
II. By varying the throw of the expansion eccentric.
III. By varying both angle and throw of the expansion eccentric.
IV. By varying the lap of the expansion eccentric.

In the last case neither throw or angle of advance of expansion eccentric is varied. The most general case is therefore III;

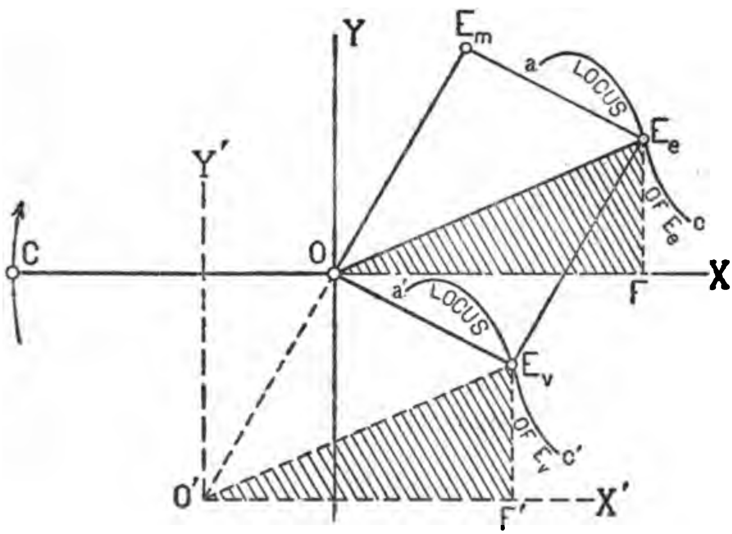



Fig. 94.

it is represented in Fig. 94, where the center $E_e$ traverses any arbitrary locus *ac*. Its reference line is taken to be the crank at left dead point, but any other crank position might have been chosen and the locus *ac* would occupy a correspondingly different position in the plane of the paper. It is simplest to consider the locus *ac* as rigidly attached to the crank, moving with it, and preserving always the same relative position towards it.

Let *OX* and *OY* be coordinate axes rigidly attached to the crank and moving with it, then will *OF* and $FE_e$ represent the

coordinates of any point $E_e$ of the locus *ac*. Prolong main eccentric $E_mO$ to $O'$, making $OO' = OE_m$, and draw a parallel set of coordinate axes $O'X'$, $O'Y'$, also rigidly attached to crank. Then the proposition of Dr. Zeuner, last referred to, consists in the statement that, *The locus* a'c' *of the center* $E_v$ *of the virtual eccentric* $OE_v$ *is a curve exactly equal and parallel to the locus* ac *and the position of* a'c' *relatively to the origin* O' *is exactly like that of* ac *relatively to origin* O.

To prove this it is only necessary to show that the coordinates $O'F'$, $F'E_v$ are respectively equal and parallel to coordinates $OF$ and $FE_e$. By construction $OO'$ is equal and parallel to $E_eE_v$, which makes $OO'E_vE_e$ a parallelogram, hence $OE_e$ is equal and parallel to $O'E_v$ and this makes the two sets of coordinates respectively equal and parallel. Since in designing we start with the valve-circles rather than with the eccentrics it will be convenient to represent this prososition as applied to the diameters of the valve-circles.

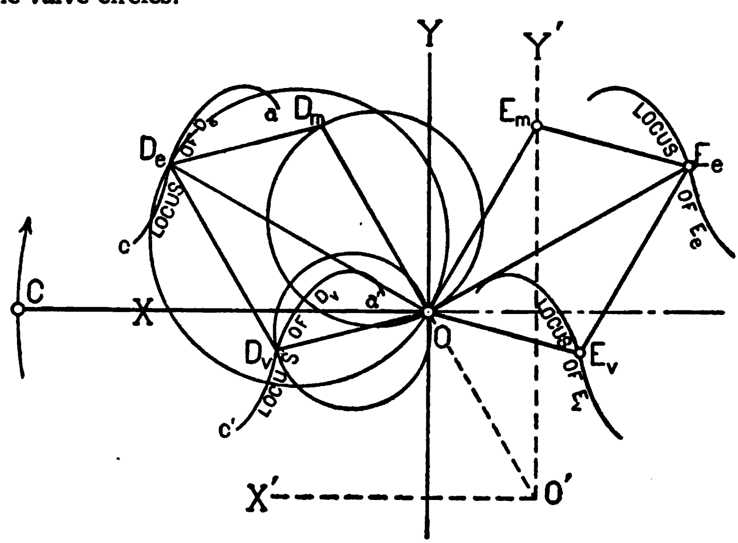



Fig. 95.

In Fig. 95 the curve *ac* is the locus of the vertex $D_e$ of the valve-circle diameter $OD_e$. This curve *ac* is equal to the locus of

$E_e$ and is similarly located. The curve $d'c'$ is the locus of the vertex $D_v$ of the diameter $OD_v$ of the virtual valve-circle, and is equal and similarly situated to the locus of $E_v$. The locus $d'c'$ has the same position relatively to the origin $O'$ that $ac$ has relatively to the origin $O$. These valve-circle-loci, unlike those in Fig. 94, have the great advantage of preserving their places in the plane of the paper during the rotation of the crank.

The subscripts $e$, $m$, $v$ employed in these diagrams relate, respectively, to expansion, main and virtual eccentrics and their valve-circles. The symbols $e_o$, $r_o$ and $\delta_o$ respectively refer to the lap of the expansion valve, the eccentricity and angle of advance of the expansion eccentric. When the subscripts 1 and 2 are annexed to the subscript $o$ the values at minimum and maximum grade of expansion are meant. The other symbols have the meaning already assigned them in the discussion of single-valve gears.

The loci of the center of expansion eccentric vary greatly in form according as the valve-gear elements, angle of advance, throw and lap are varied. In Figs. 100 and 101, representing the expansion valve diagrams of the Cummer engine, the locus of $E_e$ is circle concentric to shaft center $O$ and has $r_o$ as a radius. In Fig. 106, the expansion valve diagrams for a Meyer valve gear, this locus reduces to a point, for the setting and throw of the expansion eccentric does not vary with the grades of expansion but only with the lap. The inscription, "locus of $D_v$," on this diagram is to be understood only as a geometrical construction that is helpful in solving the problem. In Fig. 109, the expansion valve diagram of the Rigg engine, the locus $E_e$ is $OE_{e1}E_{e2}$, a straight line passing through $O$. In Fig. 112, the locus of $E_e$ is a circle that is not concentric to $O$. In Figs. 115, 116 the locus $E_e$ is a straight line that does not pass through $O$ and in Figs. 118, 119 locus of $E_e$ is approximately a circle whose circumference passes through shaft-center $O$.

In designing a double-valve gear, the steps are: ($a$) to find the main eccentric suitable for release, compression, and beginning

of admission, (*b*) to find the virtual valve-circle that will give the desired cut-off, reopening, etc., for the assumed type of expansion valve, (*c*) to find the corresponding virtual eccentric, and (*d*) to find the desired expansion eccentric by combining the main and virtual eccentrics.

When very great accuracy in the determination of valve travel is desired, we must substitute for the virtual valve circle a pair of exactly drawn curves whose vectors represent at once the crank's position and the valve's travel from its center of motion (see definition of this center, p. 151). To construct these polar curves we must therefore find this valve travel. Accordingly we should, in Fig. 93, replace the middle position or reference line $POP$ of the main eccentric by a reference *arc* struck from the center of motion with eccentric rod as radius. The distance of $OE_m$ from this arc will be $E_mS_m'$. In like manner we must replace the reference line $POP$ of the expansion eccentric by the reference arc (of the valve) struck from the corresponding center of motion and the eccentric rod as radius. The distance of $E_e$ from this arc will be $E_eS_e'$ and the true relative position or travel of the expansion valve will be $E_eS_e' - E_mS_m'$ and this difference laid off on the corresponding crank position will give one point on the desired polar curve.

The polar curve obtained by laying off on the crank the distance $E_vS_v$ (of center $E_v$ of the virtual eccentric from the common arc of reference) is in general not a closer approximation than the valve-circle itself.

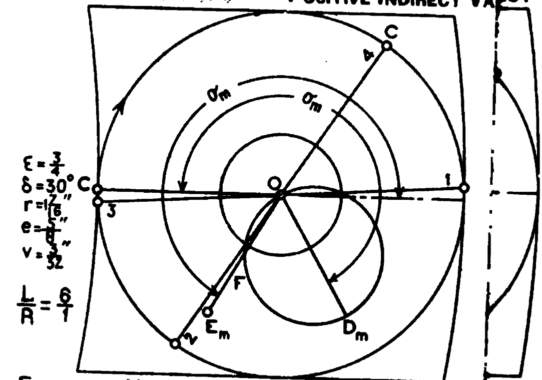
When there is a reversing lever between one valve and its eccentric, the actual distance of each from the arc of reference must still be taken, remembering however to take the $\left\{\begin{matrix}\text{difference}\\ \text{sum}\end{matrix}\right\}$ of $E_eS_e'$ and $E_mS_m'$ when the *valves* are on the $\left\{\begin{matrix}\text{same side}\\ \text{opposite sides}\end{matrix}\right\}$ of their center of motion.

Figs. 96, 97, 98.

# BUCKEYE ENGINE.

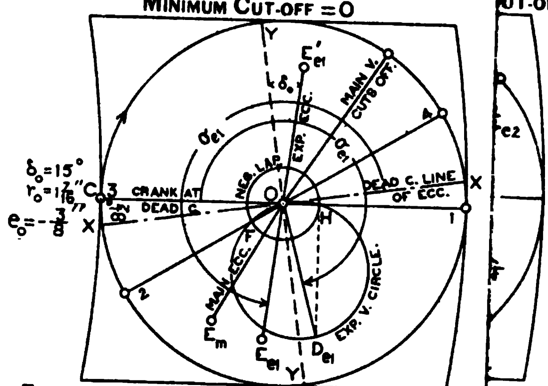
EXPANSION VARIED BY ANGLE OF ADVANCE. ANSION

MAIN VALVE DIAGRAM. POSITIVE INDIRECT VA ECT VALVE. M



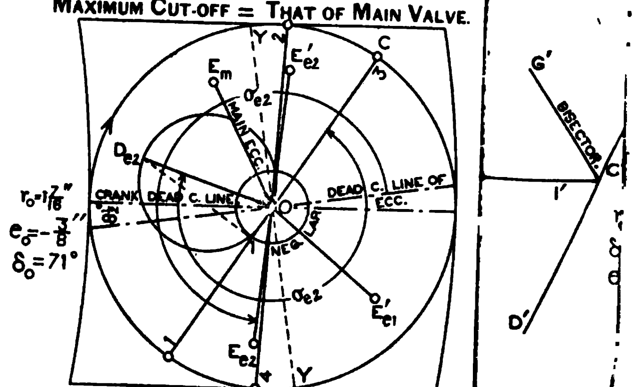

EXPANSION VALVE DIAGRAM. NEGATIVE DIRECT V RECT VALVE

MINIMUM CUT-OFF = 0 UT-OFF = 0.5




EXPANSION VALVE DIAGRAM. NEGATIVE DIRECT V RECT VALVE.

MAXIMUM CUT-OFF = THAT OF MAIN VALVE.




REVERSING ARMS BETWEEN ECCENTRIC AND VAL

In the great majority of cases the valve-circles will be sufficiently accurate; in the few remaining cases it will rarely be necessary to construct the whole polar curve; a few points of the curve (for the crank positions of the most practical interest) will usually suffice.

## EXAMPLES OF DOUBLE-VALVE DIAGRAMS, FIGS. 99-119.

In these mechanisms the expansion valve moves on the main valve and controls the ports or steam passages in the latter. Since both valves move their relative motion depends upon the separate, absolute, motions and is one of some complexity, but it can be simplified by considering the main valve stationary and supposing the expansion valve moved by a new special eccentric that imparts to it an absolute motion equal to the relative motion which it possesses when both valves move under the influence of their own, actual, eccentrics. This eccentric has been called the virtual eccentric and for a particular grade of expansion has all the simplicity and properties of the ordinary fixed eccentric. Its valve-circle is found from the virtual eccentric in the usual way and gives the same sort of information concerning the crank positions at which the port in the main valve opens and closes.

In this set of diagrams the port openings have not been indicated by sectional areas. With negative valves the port opening is measured *inward*; from negative lap-circle to valve-circle, this intercept lying wholly or partly *outside* of the valve-circle.

When there is a reversing arm between an eccentric and its valve, we assume in diagramming the same type of valve but an eccentric capable of giving the valve motion directly, without such an intermediate lever. At the end of the procedure this is allowed for by placing the resultant expansion eccentric 180° from that given by diagram.

As this series of figures (99–119) is intended to illustrate the double-valve system of varying the expansion, we have neglected the exhaust side of the distribution in our main valve diagrams. But this may easily be supplied from what has gone before.

## BUCKEYE ENGINE, 13x24, FIGS. 96, 97, 98.

We have already touched upon the kinematic character of these valves in the foot-note, p. 154.

Generally when there are two valves there are two eccentrics and these usually give a resultant or virtual eccentric, but not always. There is one exception in the Buckeye Engine. In this case the mechanism between expansion eccentric and expansion valve is such that the rod of this expansion valve receives a motion nearly equal to that of the main valve thus making the relative motion of the expansion valve on the main valve dependent simply on the motion of the expansion eccentric. The latter therefore is at once expansion and virtual eccentric. In this case of double valves and double eccentrics we need speak only of the *expansion* eccentric and valve-circle.

Fig. 96. The main valve in this case covers the port in its middle position and cuts off with its inner edges which brings it into the Positive Indirect class of valves. The location of the corresponding valve-circle $OFD_m$, Fig. 96, can easily be chosen by the help of Fig. 62 and the remarks made on pp. 142, 143. The eccentric can now be found as in Fig. 60. It is evident from Table XIV that left cylinder port is open for steam admission between crank position 3 and 4 and closed during crank's motion through positions 4 — 1 — 2 — 3, closing at 4 and reopening at 3.

Fig. 97. For the sake of economy of construction we will take $r_e = r$ and describe a circle with $O$ as center and $r$ as a radius. To find the valve-circle for minimum cut-off, when negative lap $e_0$ is given, we erect at $H$ a perpendicular $HD_{e_1}$ to dead

center line $OH$ of crank. Then will $OD_{e_1}$ be the diameter of the expansion valve-circle for minimum cut-off. Inspection of Table XIV and Figure 97 shows that this negative direct valve keeps its left port in the main valve closed from crank position 3 to 4 and opening during 4 — 1 — 2 — 3, the reopening taking place at 4 after main valve has closed the left cylinder port. In getting the position of the expansion eccentric it should be noticed that in this particular problem the dead center lines of eccentric and crank do not coincide differing by about 8½°. After the manner of Fig. 60, we find $OE_{e_1}$, to be the position of expansion eccentric at minimum cut-off, when crank is at left dead point. As there is a reversing lever between eccentric and valve, we must reverse $OE_{e_1}$ to get position $OE'_{e_1}$ of expansion eccentric in the actual case.

Fig. 98. In order that the expansion may have the widest possible range the expansion valve at maximum cut-off should close its port at the same instant that the main valve closes the corresponding cylinder port. To realize this place the crank at position belonging to main-valve's cut-off and then find, according to Fig. 62, the location of the proper valve-circle. As the lap $e_e$ does not change the perpendicular $ID_{e_2}$ to the crank position $CO1$ will give $OD_{e_2}$ as the diameter of the valve-circle desired. Laying off the proper angles as in Fig. 60, we get $OE_{e_2}$ for the position of expansion eccentric at maximum cut-off, when crank is at cut-off position and no reversing lever exists between eccentric and valve. As there is such a reversing lever, we must change by 180° the position of $OE_{e_2}$ and thus get $OE'_{e_2}$, the position of the actual expansion eccentric when crank is at position of maximum cut-off. For this same position $CO$ of the crank the main eccentric is at $OE_m$ and the expansion eccentric's position corresponding to minimum cut-off is at $OE'_{e_1}$. The angles of advance $\delta_0$ for the extreme cut-offs are easily obtained provided the definition of this angle, given on p. 150, is borne in mind. The angle $E'_{e_1} OE'_{e_2}$ measures the variation of angular position of

expansion eccentric for the whole range of cut-off. Table XIV again shows that the left port in the main valve is open during the crank's motion 4 — 1 — 2 — 3, and there will therefore be a large lead by the expansion valve at the beginning of the forward stroke. The cut-off given on this set of diagrams are approximate only. They can be obtained more accurately from the accompanying piston travel diagram in which $\frac{L}{R} = 6$. In this connection see method of finding cut-off in Fig. 63.

## Cummer Engine, 11x20, Figs. 99, 100 and 101.

The functions of the steam distribution are divided among three pairs of valves, three valves for each end of the cylinder. There is a so-called main valve which however only regulates he beginning of the admission of steam to the cylinder, though it also has some influence on the relative velocity with which the cut-off valve opens and closes its ports in the main valve. Next there is the cut-off or expansion valve which as its name implies regulates only the grade of expansion. Finally there is the exhaust valve which regulates the beginning of release and compression for each end of cylinder making them both constant for all grades of expansion. In each of these three pairs of valves the two halves are rigidly connected. There are at each end *three* steam ports and *four* exhaust ports, all valves being of the gridiron variety. None of these flat valves are balanced, because the valve stroke is small and therefore the work of friction so small that there would be little gained by balancing. The port openings are large for both admission and exhaust.

In this engine the expansion is varied by changing the angular position of the eccentric, as in case of the Buckeye Engine. In the case of the Cummer Engine however, the expansion and virtual eccentric are not identical, the latter being a side of the parallelogram of eccentricities constructed on main and expan-

sion eccentrics, as in the full lines of Fig. 92. Here the complex motion of two valves is reduced to one valve driven by an equivalent eccentric called the virtual eccentric. When this is known the expansion eccentric is known and the problem is solved. The principal steps of the solution are: (*a*) To find the locus of the vertex of the diameter of the virtual valve-circle. (*b*) To construct these valve-circles for the extreme cut-offs. (*c*) To find the virtual eccentrics from the diameters of the valve-circles.

Fig. 99. The construction of the main valve diagram presents nothing new. We may therefore omit a description of it. We simply call attention to the fact that with this positive, direct, main, valve the left cylinder port is open from crank position 1 to position 2 and closed more than half the revolution, namely 2 — 3 — 4 — 1, (see Table XIV). This and the expansion diagrams are drawn to twice the usual scale because of the smallness of the elements of the problem. The lead $v$ is only $\frac{1}{32}''$ instead of $\frac{1}{16}''$ as given in diagram. The width of port is $\frac{3}{8}''$.

The exhaust valve's diagram has not been given, though it deserves attention because there is a reversing lever between it and the eccentric of the main valve. This eccentric drives both the exhaust and the main valve, but the latter is driven directly. These two valves are both of the positive direct type, but move always in exactly opposite directions. If the exhaust valve kept its present motion but were driven directly its eccentric would have to be 180° from its present position and its valve-circle would be drawn 180° from the equal one shown in Fig. 99. But this figure, as it stands, may be used for the exhaust valve provided we (draw the inside lap and) remember that then the $\left\{ \begin{matrix} \text{actual} \\ \text{prolonged} \end{matrix} \right\}$ crank cuts the present valve-circle when exhaust valve is to the $\left\{ \begin{matrix} \text{left} \\ \text{right} \end{matrix} \right\}$ of its middle position. The lap on the exhaust side may easily be ascertained by assuming that cushioning takes place during the last fifth of the stroke; the beginning of the release can be determined from the inside lap thus found.

Fig. 100. Given $e_0$ and $r_0$. As the eccentricity $r_0$ of the expansion eccentric does not change, the locus of the center of the latter will be a circle *FLM* described about *O* with radius $r_0$. The vertex of the diameter of the expansion valve-circle has the self-same locus *FLM* about *O* as a center. Consequently according to principle enunciated in connection with Fig. 95 the locus of the vertex $D_v$ of the diameter of the virtual valve-circle must be a circumference *NQP* struck with $r_0$ as a radius and $O'$ as a center. As the negative lap is given, we have only to erect the perpendicular $HD_{v1}$ to the crank position $C'OH$ at minimum cut-off to get a second locus of $D_{v1}$ and thus get the diameter $OD_{v1}$ of the desired valve-circle. We see that the reopening of the expansion valve takes place just a little before the crank reaches the dead point. By laying off the angles as in Fig. 60, we find $OE_{v1}$ as position of virtual eccentric for $\frac{1}{10}$ cut-off and then, by means of the parallelogram, $OE_{e1}$ as the position of the corresponding expansion eccentric when crank is at dead point.

Fig. 101. Given $e_0$, $r_0$ and the extremes of cut-off. In like manner we find $OE_{e2}$ is the position of expansion eccentric at the maximum cut-off. If we transfer the eccentric position $OE_{e1}$ in Fig. 100, for minimum cut-off and crank at dead point, to the present figure, the angle $E_{e1}\ OE_{e2}$ will measure the total change of angular position of the expansion eccentric for this range of cut-off.

Two limits may be placed upon the time of reopening of the expansion valve. The first limit is set by the condition that the reopening of its port at either end by the expansion valve must surely occur *after* the main valve has closed the steam port at that end; otherwise there will be two admissions of steam during the same stroke, and the benefit of expansion will be destroyed. The second limit is prescribed by the condition that, for the same cylinder end, the expansion valve must open port as soon as, or sooner than, the main valve. The two crank positions at which the positive main valve cuts off and admits steam at the

same port, are always more than 180° apart. Hence, with a negative direct expansion valve, we have only to guard against a premature opening and therefore need to consider only the first of the two limits mentioned above.

Let us assume new data, namely, that $r_0$ is given, also the extremes of cut-off $\epsilon_{o_1}$ and $\epsilon_{o_2}$ and the crank position at which main valve cuts off; required a suitable lap $e_0$.

Now the diameter of a valve-circle lies half way between the two vectors that contain its intersections with the lap circle. If for minimum cut-off, we bisect the angle between the crank at this cut-off and the crank position when main valve closes, and prolong this bisector till it cuts the given circular locus, this bisector will constitute the limit to $OD_{v_1}$. A perpendicular dropped from the intersection $D_{v_1}$ of this bisector with the circular locus will give the maximum lap $e_0$ that can be used in this gear with a negative direct valve. Such a lap will give the largest port openings, but if these are already ample a smaller lap $e_0$ may be chosen which will favor the average cut-off in the matter of rapidity of closing. In Figs. 100 and 101 a smaller lap than the maximum given above is used.

Let us again assume a new set of data in which the lap $e_0$, the crank positions at the extreme cut-offs of the expansion valve and at the closing of the main valve are given, while the radius $r_0$ of the expansion eccentric is required.

We know from the nature of the gear that $r_0$ is constant and that $O'$ is the center of the, as yet unkown, circular locus of $D_v$. Where the lap circle cuts the prolongations of the crank positions corresponding to minimum cut-off and closing of main valve, erect perpendiculars to the crank and find their intersection $D_{v_1}$. Then $O'D_{v_1}$ will be the radius of expansion eccentric which can be employed with the given lap $e_0$.

The small laps of the expansion valve and the small throws of its eccentric in this case are due to the gridiron type of valve used. It is one of the good points of this engine that it thus

cuts down the work of valve friction without decreasing the excellence of the admission.

The cut-offs can be found more exactly from the accompanying diagram of piston travel, for which $\frac{L}{R}$ is again 6. (See method used in Fig. 63.)

## MEYER VALVE GEAR.

In this arrangement the expansion eccentric preserves an invariable throw and an invariable position relatively to the crank, the expansion being varied by changing the amount of the lap of the valve. The mechanism for doing this is shown in all the elementary text-books of the steam engine and concerning it we will simply say, that the expansion valve consists of two halves that may be moved apart to any desired extent by means of right and left-hand screws located on the rod connecting these two halves. The separation may be carried so far as to change the valve from one having negative lap to one having positive lap. As the expansion eccentric does not change at all with the grade of expansion, there is in this case no locus of the center $E_e$ of the expansion eccentric and then, strictly speaking, (in an existing gear), there is no locus of the center $D_v$ of the virtual eccentric. The locus of $D_v$ inscribed on Fig. 106 is simply to be regarded as a geometrical help in finding the angle of advance $\delta_0$ when the eccentricity $r_0$ is known. As the expansion, like the main, eccentric is fixed as regards its setting and throw, there will be only one parallelogram of eccentricities (or of diameters), only one virtual eccentric and only one virtual valve-circle.

Before solving any problem in connection with this gear it will simplify matters to first demonstrate a couple of auxiliary propositions. The first of these relates to the finding of the locus $D_v$ for a special case and the second to the equalization of the cut-off in the Meyer gear.

It can be shown that, if the difference $e_2 - e_1 = E$ is prescribed or given by the proportions of the expansion valve, the vertex $D_v$ of diameter $OD_v$ of virtual valve-circle must lie somewhere on a straight line $SV$, Fig. 102 and Fig. 106, parallel to the bisector $OQ$ of the angle $\psi$ ($= NOL$) included between the crank positions for maximum and minimum cut-offs, the distance of the locus from the bisector being

$$\overline{SR} = \frac{E}{2 \sin \frac{\psi}{2}}. \qquad (91)$$

Let $OL$ and $ON$ represent the prolongations of the crank positions at the extreme cut-offs, $OQ$ the bisector of the angle

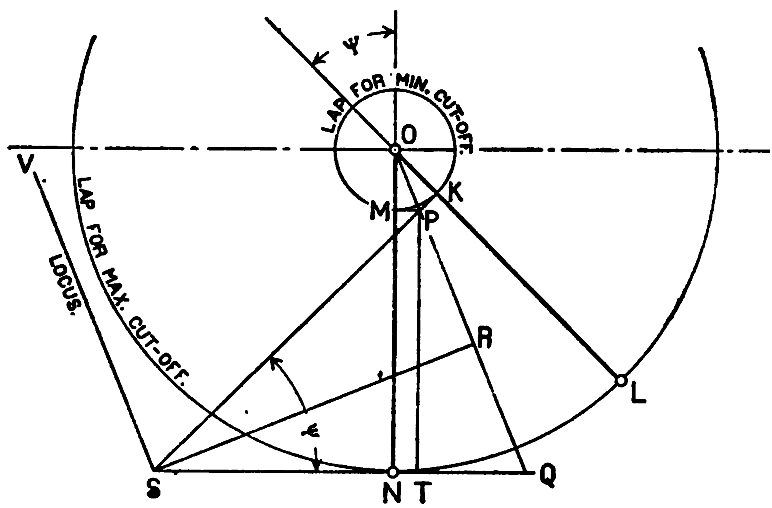



Fig. 102.

$\psi = LON$ and $KL = MN = PT = E = e_2 - e_1 =$ given difference of outside laps. Then

$$\overline{SP} = \frac{E}{\sin \psi} \text{ and } \overline{SR} = \overline{SP} \cos \frac{\psi}{2} = \frac{E}{2 \sin \frac{\psi}{2}}.$$

The remarks and constructions given on pp. 194, 195 in connection with the Cummer engine, are also applicable here.

As equality of cut-offs is conducive to smooth running, Dr. Burmester has given a solution of the problem of equalization for the Meyer valve, which consists in making the laps unequal for the two ends of the valve, the difference between the laps being a constant quantity.

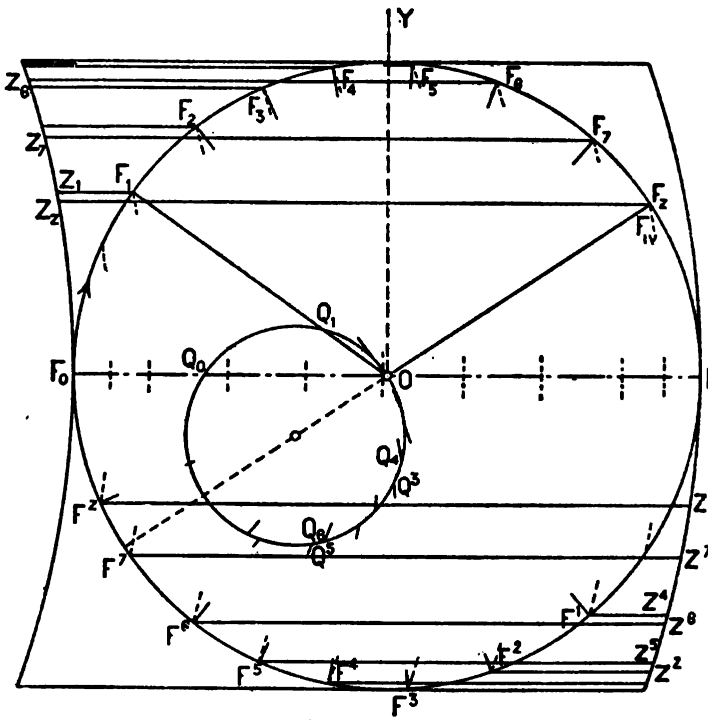



Fig. 103.

The procedure is as follows: In Fig. 103 the stroke $F_0F^0$ is divided into eight equal parts and through these divisions, with the connecting rod as radius, arcs are struck intersecting the crank circle at $F_1$, $F_2$, $F_3$, etc., then $Z_1F_1$, $Z_2F_2$, $Z_3F_3$, etc., represent 1/8, 1/4, 3/8, etc., of stroke, or cut-offs, and $OF_1$. $OF_2$, $OF_3$, etc., the corresponding crank positions. These cut-offs 1/8, 1/4, etc.,

are laid off as abscissas in Fig. 104. Now let the valve-circle for this case be represented by $OQ_0\,Q_5$; it will cut from the crank the chords $OQ_0$, $OQ_1$, etc.; these represent the travel of any point of the valve and are laid off as ordinates $T_0m_0$, $T_1m_1$, in Fig. 104.

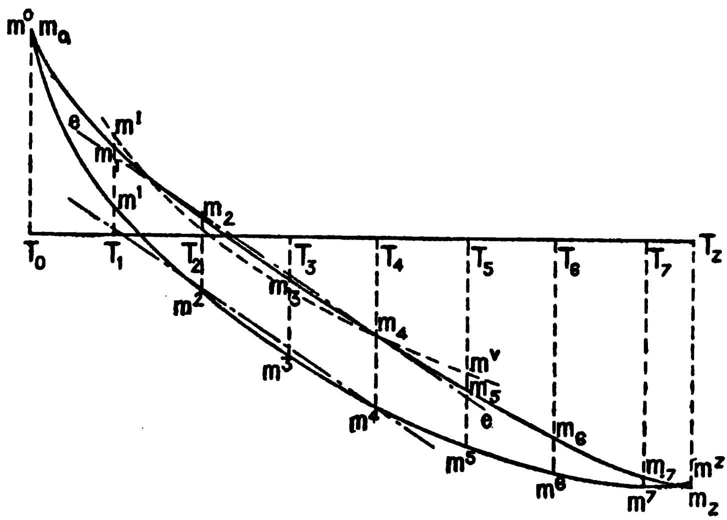



Fig. 104.

Each of these chords also represents the lap of the valve effecting the cut-off at the crank position on which the chord lies. Hence the ordinates of curve $m_0m_1m_2$ . . . $m_z$ also represent the laps effecting the cut-offs represented by their corresponding abscissas, the ordinates above the base representing positive, and those below negative, laps. In like manner the ordinates of curve $m^0m^1m^2$ . . . . $m^z$ represent the laps effecting the cut-offs during the return stroke, It is evident from the figure that for the same cut-off the laps for the return stroke are greater than those for the forward stroke. In this diagram these differences can be made to (nearly) disappear by moving the lower curve upward through a distance $m^4m_4$; it will then occupy the position shown by the dotted *curve* $m^1m^v$. In the

mechanism itself, the corresponding step is to start with a symmetrical valve having the same lap on its two parts, then set the right-hand half of the expansion valve, nearer to its left-hand half by just this amount $m^4m_4$. Approximately equal cut-offs will then be effected on each stroke.

It is evident from the diagram, Fig. 104, that a straight line *ee* will approximately represent the two-lap curves between the cut-offs ⅛ and ⅝. To equalize the cut-off it is therefore unnecessary to move the two halves by two screws of unlike pitch. Both screws may be of the same pitch provided the two halves of the expansion valve are placed on their seat on the main valve with the unequal laps given above.

The method of finding the unknown elements in this (Meyer's) valve gear of course varies with the character of the data. We shall assume in all the special cases discussed, that the range of cut-off of the expansion valve is given and that the laps, setting and throw of the main valve are known. We therefore know the crank position at which main valve cuts off and care must be taken that the expansion valve does not permit a second admission of steam during the same stroke. Although the locus of $E_e$ (and locus of $D_v$) is reduced to a point for this gear, it still remains true that the point $D_v$ must lie at the distance $r_o$ from the point $O'$; this point, we know from Fig. 95, is found by prolonging $OD_m$ and making $OO' = OD_m$. The problems and their solutions are arranged in the following order:

(1.) Given $r_o$ and $\delta_o$; required $e_{o2}$ and $e_{o1}$.
(2.) Given $r_o$ and $e_{o2}$; required $\delta_o$ and $e_{o1}$.
(3.) Given $r_o$ and $e_{o2} - e_{o1}$; required $\delta_o$, $e_{o2}$ and $e_{o1}$.
(4.) Given $\delta_o$; required $r_o$, $e_{o2}$ and $e_{o1}$.
(5.) Given $\delta_o$ and $e_{o2} - e_{o1}$; required $r_o$, $e_{o2}$ and $e_{o1}$.

(1.) Fig. 106. If the radius $r_o$ and the angle of advance $\delta_o$ of the expansion eccentric are both given, the problem is readily solved, for then a parallelogram constructed on the main and expansion eccentrics (Fig. 92) will determine the virtual eccentric

and this (Fig. 60) the virtual valve-circle. The intersections of this circle with the crank positions corresponding to the limits of cut-off will determine the laps desired, provided the expansion valve does not open prematurely. In this gear, it is only at maximum cut-off that there is any danger of an opening of this valve before the main valve has closed the port at the cylinder end in question.

(2.) Fig. 106. Given $r_0$ and $e_{02}$. With $O'$ as a center and $r_0$ as a radius, describe a circumference; the vertex $D_v$ will lie somewhere on it. At the cut-off point $1'$ erect a perpendicular $1'D_v$ to $O1'$, where it cuts the circular locus will be point $D_v$ desired, provided the second intersection $2'$, of valve-circle on $OD_v$ with lap circle $e_{02}$, is such that the expansion valve's opening does not occur till after the closure of the port by the main valve, otherwise the data must be regarded as incompatible with good running. The lap $e_{01}$ can be found from the intersection of this virtual valve-circle with the crank position at minimum cut-off.

By means of the parallelogram, the position of $OD_e$ may be found from $OD_m$ and $OD_v$ and then by method of Fig. 60 the position of $OE_e$. This determines at once the setting of the eccentric and by applying the rule given on p. 149 the angle of advance $\delta_0$ can be found if desired.

(3) Fig. 106. Given $r_0$ and the difference $e_{02} - e_{01} = E$. We know from the remarks on p. 200 that $D_v$ must lie somewhere on the circumference described with $O'$ as a center and $r_0$ as a radius, and from the proposition given with Fig. 102 we know that $D_v$ must also lie on a line $SV$, parallel to bisector $OQ$ (Fig. 102) of angle $\psi$ = angle $303'$ = angle $101'$ and at a distance from it equal to

$$\overline{SR} = \frac{E}{2 \sin \frac{\psi}{2}}.$$

In general there will be two intersections of this circumference and line $SV$, which will give two solutions of the problem. That one should be chosen which prevents a premature opening of the

expansion valve. In the present case $E = 1\frac{3}{8}''$, $r_0 = 1\frac{3}{8}''$ and $\psi = 54°$, then $\overline{SR} = 1.514''$. This gives two values of $OD_v$, $1.82''$ and $2.29''$; of these the former guards thoroughly against a premature opening for the whole range of expansion; the second value of $OD_v$ permits premature opening for nearly the whole range of expansion. Of these two valves, $OD_v = 1.82''$ is decidedly the preferable one and is the one chosen for illustration in Fig. 106. By the aid of the parallelogram and the method of Fig. 60 we can now get the position of the expansion eccentric $OE_e$.

(4.) Given $\delta_0$. This determines the location of the radial line $OE_e$ of the expansion eccentric relatively to the crank $OC$, Fig. 106. Consequently (according to Fig. 95) the vertex $D_v$ of the virtual eccentric must lie on the line $O'F$, the position of this line relatively to a vertical and horizontal axis through $O'$ being the same as the position of $OE_e$ relatively to a vertical and horizontal axis through $O$.

The bisector of the angle $1'02'$, between crank position when main valve cuts off and the crank when expansion valve effects its maximum cut-off, will intersect line $OF'$ in a point which is a limit of the possible vertices $D_v$ on $O'F$. The intersections, of the valve-circle described on $OD_v$ as a diameter with the cut-off positions, will give the maximum and minimum laps desired. The distance $O'D_v$ will be equal to the eccentricity $r_0$ of the expansion eccentric.

(5.) Given $\delta_0$ and the difference $e_{o_1} - e_{o_2} = E$. As in case (4), we find $O'F$ to be one locus of $D_v$. The construction of Fig. 102 gives a second locus $SV$ of $D_v$. The intersection of these two loci will give $O'D_v = r_0$ provided $D_v$ is located on, or to the left of, the bisector of the angle $1'02'$. Perpendiculars dropped from $D_v$ on to the crank at the extremes of cut-off will then give $e_{o_1}$ and $e_{o_2}$.

Fig. 107. The Rider Valve gear is only a particular case ot the Meyer. In the Rider the expansion valve plate instead ot

being flat, as in the Meyer, is bent into a cylindrical shape. This cylinder is rotated by a governor when the grade of expansion is to be changed, the rotation effecting the necessary change of lap. The principle of this construction is shown in Fig. 107, where the cylinder's surface and its seat are both developed into a plane. Here the corresponding edges of the two ports are inclined to each other but are parallel to the edges of the valve. When the latter is in the position shown by the full lines the cut-off = 0.5, the lap is negative and equal to 1.67″; when the valve has been shifted through *fe* into the position indicated by the dotted lines the cut-off is 0.1 and the negative lap = 0.30, the difference of the two laps being *fg* = 1⅜″. The amount of shifting *ef* in the flat plate is equal in the cylindrical valve (with helical edges) to the circular arc through which any point on its surface is moved. In order that the valve may take up some of the wear between its face and seat, this circular seat should be considerably less than 180°; a value of about 100° will be found to be suitable. The proportions and distribution given in Figs. 105–107 are suitable for a Rider valve gear. A Meyer gear can easily be arranged so as to cut off from 0 to ⅞.

## Rigg Engine, Figs. 108, 109, 110.

(See Rigg's Treatise on the Steam Engine, Pls. 17, 18, pp. 95–97.)

In this engine the expansion is varied by a changeable eccentricity, the setting of the eccentric remaining the same at all grades of expansion. The centers $E_e$ of the expansion eccentric must therefore lie on the same radial line $OE_{e_1}E_{e_2}$, Fig. 109, and the the center $D_v$ of the virtual eccentric must likewise lie on one straight line $O'D_{v_1}D_{v_2}$ whose location with respect to the axes $X'O'Y'$ is like the location of $OE_{e_1}E_{e_2}$ or $OD_{e_1}D_{e_2}$ to the axes $XOY$ (see also Fig. 95).

We again assume the extremes of cut-off as given and the limits within which the expansion valve may open without per-

mitting a second admission during the same stroke. The laps, setting and throw of the main valve are also known. As these data are like those for the Cummer gear the remarks on p. 194 concerning the limitations to which the locatien $OD_v$ is subjected apply here also.

We shall discuss the problems connected with this gear as follows:

(1.) Given $e_o$, required $\delta_o$, $r_{o_1}$ and $r_{o_2}$.
(2.) Given $\delta_o$, required $e_o$, $r_{o_1}$ and $r_{o_2}$.
(3.) Given $r_{o_2} - r_{o_1}$, required $\delta_o$, $e_o$, $r_{o_1}$ and $r_{o_2}$.

(1.) Given $e_o$. This case is solved in Fig. 109. $JO$ and $KO$ are the crank positions corresponding to minimum and maximum cut-off, respectively. $OH = OI = e_o$ is the given, negative, lap of the expansion valve. Here the diameter $OD_{v_1}$ of the virtual valve-circle for minimum cut-off is taken as located at one of its limits, namely that one which corresponds to a reopening of the expansion valve at the very instant that the main valve closes. It would have been better to have assumed some margin. Position $OD_{v_1}$ is therefore a bisector of the angle included between the prolongation of crank position $JO$ and the prolongation of crank position corresponding to the closing of the cylinder port by main valve. A second locus of $D_{v_1}$ is the perpendicular at $H$ to $JOH$; the vertex $D_{v_1}$ lies at the intersection of this bisector and this perpendicular. For this kind of gear all the points $D_v$ lie on a straight line through $O'$, therefore by joining $D_{v_1}$ and $O'$ we get a locus of the unknown vertex $D_{v_2}$. A second locus of $D_{v_2}$ is obtained in the perpendicular erected at $I$ to the crank position $KOI$ for maximum cut-off. We have $O'D_{v_1} = r_{o_1}$ and $O'D_{v_2} = r_{o_2}$; the remaining unknown element $\delta_o$ can be found by means of the parallelogram and Fig. 60, as in the preceding valve gears, or more directly as indicated in the figure. The crank is supposed to be at $OX$ when the expansion eccentric is on radius $OE_{v_1}E_{v_2}$.

(2.) Given $\delta_o$. This is the preceding case worked backward.

We therefore use the same figure, 109. As the locus $OE_{e_1}E_{e_2}$ of the expansion eccentric is given we find at once $OD_{e_1}D_{e_2}$ and its parallel $O'D_{v_1}D_{v_2}$. The latter is the locus we need and its intersection $D_{v_1}$ with the limiting position of diameter $OD_{v_1}$ must be projected on crank position $JOH$ to get the lap $OH = e_0$. With this lap we can now easily find $OD_{v_2}$. Then will $O'D_{v_1} = r_{o1}$ and $O'D_{v_2} = r_{o2}$.

(3.) Fig. 110. Given $r_{o2} - r_{o1}$. In this case we do not know either of the eccentricities of the expansion eccentric at the cut-off limits, we simply know the *change* of eccentricity which our mechanism will permit. We again assume, for the location of the diameter of virtual valve-circle corresponding to minimum cut-off, the limiting line $Ol$, Fig. 110.

We can easily find a locus for the vertex $D_{v_2}$ belonging to maximum cut-off. For this purpose draw through $O'$ a series of lines $O'L$, $O'M$, $O'N$ . . . . . , which cut line $Ol$ in points $l$, $m$, $n$, . . . . . . ; then lay off distances $lL$, $mM$, $nN$, . . . . . each equal to the given variation of eccentricity $r_{o2} - r_{o1}$. The curve $LMND_{v_2}$ thus obtained is part of a conchoid and is the locus mentioned. Each point on this curve represents a different maximum cut-off, for the constant difference $r_{o2} - r_{o1}$, and our task now is to find the point corresponding to the given maximum cut-off. This is most easily accomplished by a trial process, which we will now give. Inspection of Fig. 109 for the preceding cases, shows that the intersection $C$ of the two perpendicular $CH$ and $CI$ dropped from the vertices $D_{v_1}$ and $D_{v_2}$, falls on the line $GC$ bisecting the angle $JOK$ included by the crank positions at the cut-off limits. This suggests the following tentative method: Reproduce, on a separate piece of tracing cloth, (see right half of Fig. 110), the bisector $G'C'$ and the perpendiculars $C'I'D''$ and $H'C'D'$ so that angles $G'C'D'' = GCD_{v_2}$ and $G'C'D' = GCD_{v_1}$. Then place this tracing on the drawing so that $G'C'$ on tracing will coincide with $GC$ on drawing and slide bisector $G'C'$ on bisector $GC$ till the intersection $D_{v_1}$ of perpen-

dicular $C'D'$ with line $OD_{v_1}$ and the intersection $D_{v_2}$ of perpendicular $CD''$ with conchoid $LMN$ both fall on the same straight line $O'D_{v_1}D_{v_2}$ drawn through $O'$. When this occurs all the conditions of this particular case are fulfilled and the size and setting of expansion eccentrics can be found in the now well-known manner.

Before leaving this problem it should be noticed that the locus of the vertex $D_{v_2}$ (of the diameter $OD_{v_2}$ of the valve-circle corresponding to a given maximum cut-off), is an hyperbola $ROD_{v_2}Q$, whose asymptotes $PL$ and $PT$ are parallel respectively to the perpendicular $CID_{v_2}$ and the diameter $OD_{v_1}$. The origin $O$ is on the one branch of this curve and $O'$ on the other branch. Points on the hyperbola are easily obtained as follows: Take any point $C$ on the bisector $GC$ and drop the perpendiculars $CH$ and $CI$ on $JH$ and $KI$. Then prolong $HC$ till it cuts given line $Ol$ in some point, say $D^v_1$. Join this point with $O'$ and prolong the connecting line till it cuts the second perpendicular $CI$ in $D_{v_2}$. This will be a point on the hyperbola. It is evident that the intersection of conchoid and hyperbola give at once exactly the desired point $D_{v_2}$. But the construction of both these curves, or even partial arcs of both of them, will seldom be advisable, as the problem can be much more easily solved by the trial method detailed above.

In the preceding double-valve gears only one of the elements, (lap, setting and throw) has been varied at a time. In the remaining gears, two elements, or their equivalents, are varied simultaneously.

## STURTEVANT ENGINE, 12 X 24, FIGS. 111, 112, 113.

In this engine the expansion is varied by a simultaneous variation of the throw and angle of advance. This variation is effected by a suitable mechanism, of the swinging eccentric type, the center of the expansion eccentric being compelled to travel on the circular locus $E_e$ (Fig. 112) when the expansion is to be

varied. This locus passes $O$ at a distance of $\frac{3}{4}''$. Its center and radius are inscribed on the figure. According to Figure 95 the locus of $D_v$ has, relatively to $O'$, the same position as locus $E_e$ to $O$.

Here also we assume laps, setting, throw and distribution of main valve as completely determined. The range of cut-off for expansion valve, and the limits of the opening of this valve are likewise supposed to be known. On the diagram of Fig. 112 the minimum cut-off is stated to be 0. This is approximately true but not exactly. A tangent touching both lap circles $AO$ and locus of $D_v$ will determine, at its point of contact $A$, the crank position $AOC$ for least cut-off.

To find the position of expansion eccentric at 0 cut-off, we erect at $A$ (Fig. 112) the perpendicular $AD_{v_1}$; it will touch the locus of $D_v$ in $D_{v_1}$ and $OD_{v_1}$ will be the diameter of the virtual valve-circle. The dotted parallelogram $OD_{v1}D_{e_1}D_m$ gives the relative position and magnitude of the diameters of main, expansion and virtual valve-circle; the equal parallelogram $OE_{v1}E_{e_1}E_m$ gives the relative position and magnitude of the corresponding eccentrics. $OE_{e_1} = r_0$, is the position of the expansion eccentric relatively to crank $OC$ when cut-off $= 0$.

Fig. 113. In like manner we find that $OD_{v_2}$ is the diameter of the virtual valve-circle for the maximum cut-off $= 0.7$. The reopening of the port by expansion valve occurs just after main valve closes. For this cut-off the position and size of expansion eccentric is given by $OE_{e_2}$ when crank is at dead point $OC$. The expansion eccentric therefore travels on locus from $OE_{e_1}$ to $OE_{e_2}$ while varying the expansion.

## Double-Valve Gears with Invariable Driving Eccentrics.

There are two kinds of double-valve gears in which expansion valves are driven by non-adjustable or invariable eccentrics. In the first kind the expansion valve is driven by one or more invariable eccentrics, and its absolute motion is entirely indepen-

dent of the main eccentric. In the second kind the absolute motion of the expansion valve does depend, more or less, upon the main valve's eccentric. In both kinds there is between the expansion valve and the invariable eccentric a mechanism whose action is equivalent to driving this value by a variable eccentric.

A valve gear by G. Schuhmann, of Reading, Pa., belongs to the first kind. Between the expansion eccentric and its valve there is a rocker with two arms, a variable and invariable one, the invariable one driving the expansion valve and the variable one receiving its motion from the expansion eccentric through a block whose position on the arm is regulated by the governor. The locus of the center $E_e$ of the expansion eccentric is a straight line which, when crank is on back center, passes below and to the right of the shaft center $O$ making an angle of 30° with the line of dead centers. The main valve in this engine belongs to the positive indirect, and the expansion valve to the negative indirect, type.

The valve gears of Polonceau (or Borsig), of Guinotte and of Bileram are examples of the second kind. Polonceau's is described and discussed on p. 203, second English edition, of Zeuner's valve gears. Here both main and expansion valves are driven by Gooch's link motion and the locus of the center $E_v$ of the virtual eccentric is a vertical line through center of shaft. Of these gears we will discuss only those of Guinotte and Bilgram.

## Guinotte's Valve Gear, Figs. 114, 115, 116.

The complex mechanism and theory of this gear are both given by Dr. Zeuner (see p. 214, second English edition of Valve Gears). It permits the reversal of engine without touching the expansion gear. The character of the locus of the vertex of the diameter of the virtual eccentric is such as seems to deserve special treatment here. We therefore reproduce it although it is

fully given by Dr. Zeuner. In this gear the vertex $D_v$ of the diameter of the virtual valve-circle lies in a rectilinear and inclined locus *NN*, Fig. 115. This is similar to the locus *MM* of the corresponding vertex $D_e$ of the expansion valve-circle and this in turn corresponds to locus *LL* of the center of the *equivalent* expansion eccentric. We say *equivalent* because the actual expansion eccentric has a constant eccentricity $OD_0 = r_0$ and a constant angle of advance, $\delta_0 = 90°$. But the mechanism between it and the expansion valve has the effect of moving the latter as if it were driven by an expansion eccentric whose center had a locus *LL* for the different grades of expansion. According to the principle underlying Fig. 95, the position of *NN* relatively to $O'$ as an origin is the same as that of *MM* relatively to $O$ as an origin and the same as *LL* relatively to $O$.

At position 3 (Fig. 115), the main valve cuts-off at the left cylinder port and in order that the expansion valve shall shut its left (main valve) port at the same time we must give the diameter of the virtual valve-circle a position $OD_{v_2}$, which is found by erecting, at the lap-circle *I*, the perpendicular $ID_{v_2}$ to the cut-off position $3O1$. The distance $OD_2$ will be the corresponding diameter of expansion circle $OE_{e_2}$ the corresponding position of the *equivalent* expansion eccentric. In the mechanism itself this corresponds to the position of the block at the upper end of the link. It is evident that with negative lap the virtual valve-circle $OID_{v_2}$ will cut-off at the same instant that the main valve closes.

Fig. 116. For a minimum cut-off we will choose 0 or cut-off at dead point. To find the corresponding virtual valve-circle we erect to the dead center line, at the lap circle *H*, the perpendicular $HD_{v_1}$ and prolong it to the locus *NN* of the vertex $D_{v_1}$. The distance $OD_{v_1}$ will be the diameter of the virtual valve-circle $OHD_{v_1}$. It is evident that this gives the desired cut-off and that the reopening of the left port in the main valve will not take place till crank reaches position $O4$, that is after the main valve

has closed the left cylinder port. The position of the equivalent expansion eccentric will be near $OD_{e_1}$ in this case and in the mechanism it will be found that this requires that the block shall be at the bottom and not at the central portion of the curved slot in the link.

## Bilgram Valve Gear, Figs. 117, 118, 119.

The mechanism of this gear has been given on p. 119 of Bilgram's work on Slide Valve Gears, also in Halsey's Valve Gears, p. 123. There is only one eccentric which drives the main valve directly and the expansion value indirectly.

The inventor gives the favorable porportions, for most of which the reader must look to the references just given. The mechanism between the main eccentric and the expansion valve, in its action on the latter, is nearly equivalent to the action of an expansion eccentric whose center travels on the circular locus $OLE_e$, Fig. 118, while passing from one extreme of expansion to the other. A line $\overline{OE_e} = 2 \times \overline{OE_m} = 2r$ placed 50° in advance of the main eccentric radius $\overline{OE_m}$ is the diameter of this locus. The corresponding circular locus $OMD_v$ of the vertex $D_v$ of the diameter of the virtual valve-circle is found in accordance with the principle illustrated in Fig. 95.

Fig. 118. The inventor recommends a negative lap for expansion valve that is $\leqq$ than the outside lap of the main valve. We have here taken ½″ for this negative lap because this is favorable to quick cut-off. To get a cut-off at the dead point erect to $OH$ the perpendicular $HD_{v_1}$ and prolong it till it cuts the locus $OMD_{v_2}D_{v_1}$ in $D_{v_1}$. Then will $OHD_{v_1}$ be the required virtual valve-circle for minimum cut-off. It is evident that the reopening of the left, main valve, port, occurs at position $4OE$ after the main valve has closed the corresponding cylinder port.

Figs. 118, 119. The cut-offs at one-quarter and one-half stroke are treated in the same way; the corresponding virtual

valve-circles are $OID_{v_2}$ and $OJD_{v_3}$ (Fig. 119). Inspection of these circles at the points $I$ and $J$ shows that the cut-off is very quick, as is claimed.

The range of cut-off is great, extending from nothing up to that effected by the main valve. The virtual valve-circle corresponding to this maximum cut-off is $OKD_4$, Fig. 119.

## Practical Data and Directions.

The rapidity of cut-off for any gear depends upon the speed of the valve at the instant and the number of ports through which the steam is admitted to one end of cylinder. It can easily be estimated by finding the position of the virtual eccentric at this instant and dropping a perpendicular from its center upon the "dead point" line of the eccentric. The product of the number of ports and the ratio of this perpendicular to the perpendicular dropped from crank-pin on piston stroke is, other things being equal, perhaps as fair and convenient a measure as any for comparing the rapidity of cut-off of the different gears.*

The distance from the port of a valve edge not engaged in steam distribution should be more than sufficient to prevent its opening the port at maximum travel.

To avoid two admissions of live steam during the same stroke the expansion should not reopen its port in the main valve till after the main valve has closed the corresponding cylinder port.

Arrange the valve gear so that it will realize the steam distribution required by the desired indicator diagram. The release

* A more exact measure would be the product of the number of ports by the ratio of valve speed to piston speed at instant of cut-off. The valve speed is measured by the intercept included between the shaft's center and the eccentric-rod, the intercept being taken on the perpendicular through this center to the stroke of the valve end of the rod. The piston speed, to the same scale, is measured by the intercept on the perpendicular to its stroke included between shaft center and connecting-rod. The ratio of valve speed to piston speed is the ratio of these two intercepts.

may begin at $\frac{11}{12}$ of the stroke, unless it has already been fixed by the compression assumed in the earlier part of the work. As American practice now is to use balanced valves, large throws of eccentrics are permissible. The cut-off should be made as rapid as possible.

The width of bridge (metal between steam and exhaust ports) should be greater than $r - (b + e)$ to avoid fresh steam passing directly into exhaust. It is often made equal to the thickness of the cylinder wall. An empirical formula given by Zeuner is $0.4 + 0.5b$.

To prevent steam leaking between valve face and seat they should have a contact-width of at least 0.4 of an inch.

The exhaust port should have such a width $b_0$, that when valve is at end of travel, it will still have the width $b$, of the steam port, that is, $b_0 = r + b + i - (0.4 + 0.5b)$

$$= r + 0.5b + i - 0.4.$$

The valve seat should have such a length that the valve will pass beyond the outer edges of the seat, thus avoiding the formation of a shoulder in the seat. These should always remain, however, a 0.4″ contact to prevent leakage.

When cut-off $\varepsilon$, lead $v$ and maximum port-opening $b \pm k$ ($+ k =$ over-travel, *i.e.*, beyond edge of port) are given, the half-throw $r$ can be found by trial, or at once and exactly, by Zeuner's formula:

$$r = \frac{2(b \pm k) - v + 2\sqrt{(b \pm k)(b \pm k - v)(1 - \varepsilon)}}{2\varepsilon} \qquad (92)$$

The following tables will be useful in the solution of problems relating to the main valve. When the outside lead is known as a fraction of the eccentricity $r$, for example $v = mr$, we may transform the formula into

$$\frac{b + k}{r} = 1 + \frac{m}{2} - \sqrt{1 + \frac{m^2}{4} - \frac{m^2}{4\varepsilon} - \varepsilon}. \qquad (93)$$

## VALUES OF $\frac{r}{b+k}$.

| Values of $\frac{v}{r} = m$ | Values of $\varepsilon$ = apparent cut-off. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 |
| 0.05 | 3.140 | 2.821 | 2.545 | 2.307 | 2.094 | 1.905 | 1.730 | 1.567 | 1.410 |
| 0.075 | 3.018 | 2.721 | 2.464 | 2.240 | 2.039 | 1.859 | 1.693 | 1.537 | 1.386 |
| 0.10 | 2.906 | 2.626 | 2.387 | 2.176 | 1.987 | 1.815 | 1.657 | 1.508 | 1.362 |
| 0.125 | 2.792 | 2.536 | 2.314 | 2.116 | 1.937 | 1.774 | 1.622 | 1.479 | 1.339 |

## VALUES OF $\frac{e}{r}$.

| Values of $m = \frac{v}{r}$ | Value of $\varepsilon$ = apparent cut-off. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 |
| 0.05 | .681 | .645 | .607 | .566 | .522 | .475 | .422 | .362 | .291 |
| 0.075 | .669 | .632 | .594 | .554 | .510 | .462 | .409 | .349 | .278 |
| 0.10 | .656 | .619 | .581 | .540 | .497 | .449 | .396 | .337 | .266 |
| 0.125 | .642 | .606 | .568 | .527 | .484 | .436 | .383 | .324 | .253 |

For any assumed value of $\frac{r}{b+k}$ we can immediately get the angular advance from

$$\sin \delta = \frac{e+v}{r} = 1 - \frac{b+k}{r} + \frac{v}{r} = 1 + m - \frac{b+k}{r}. \tag{94}$$

$k$ should be taken as small as is consistent with sufficient port opening. The shorter cut-offs given in table are more favorable than the longer ones in preventing the reopening of the expansion valve before main valve closes, on the other hand the shorter cut-offs involve a greater throw of main valve.

In slow going engines the outside lead is often taken $= \frac{1}{16} r$, while in fast running engines $\frac{1}{8} r$ is often taken.

## VALVE GEARS WHICH DO NOT POSSESS "HARMONIC" MOTION.

On p. 134 we divided all valve gears into two classes: those in which the valves move approximately like the "slotted cross-head" and those which did not, that is, into valves with and without "harmonic" motion. It is the first class that is most extensively used in practice and the one which has been discussed in the preceding pages. Even in these there is some deviation from "harmonic" motion which is not due to the angularity or length of the eccentric rod. As the valve stem is often at a considerable distance from the plane of the eccentric, sometimes at the side, and at other times on top, of the cylinder, rocker arms are interposed between eccentric and valve to transmit the motion. Usually they produce less deviation from "harmonic" motion than the angularity of the eccentric-rod. But they may be arranged so as to produce a rapid opening of the port followed by a period of comparative rest for the valve. The Corliss valves driven by a wrist-plate and the Porter-Allen valves driven by the bent lever *O'R*, *PQ* in Figs. 80–82, are examples. Mr. F. A. Halsey has shown, in his Slide Valve Gears, pp. 54–63 and 89–98, that the rocker arms may be arranged so that both cut-off and lead are equalized. In the Ide engine they are used to keep the lead constant at each end but greatest at the cylinder end farthest from shaft. All such devices produce some deviation from "harmonic" motion.

The most characteristic members of this group are the so-called "Radial" valve gears, of which the Joy is an excellent representative and perhaps the one which is best known in this country. The link driving the valve rod in these gears has two of its points guided in curves, one of which is closed and the other open or closed, while a third point drives the valve rod or stem. The motion is usually of a complex character and the valve travel is got at by plotting the paths of the three points of the driving link, in much the same fashion as the travel of the Porter-Allen valves was obtained. The relation between valve

and piston travel may also be shown by polar or oval diagrams like those given Figs. 85-91 for the Porter-Allen valve gear.

This group of non-"harmonic" valve motions (see p. 134), has these two following sub-divisions:

(*a*.) Gears in which the valve slides on a stationary seat and only its absolute motion is considered.

(*b*.) Gears with two valves, of which one slides on the other, and in which the relative as well as the absolute motions of the valves must be considered.

The Porter-Allen motion, already discussed, belongs to the first sub-division and may be regarded as composed of three "single-valve" gears, each valve sliding on a fixed seat.

The Payne-Corliss engine belongs to the second sub-division. It has two sets of double-valves, one set for each end of the cylinder. Between the eccentric and the valves there are Corliss wrist-plates, but not the Corliss releasing-gear.

# APPENDIX A.

## Crank Angle Corresponding to Maximum Velocity of Slide.

Dr. C. L. Schadwill in a thesis presented in 1876, and entitled "Das Gliedervierseit," proved that the configuration of the slider-crank giving zero acceleration (or maximum velocity) of the slide, see Fig. 120, existed when line of instantaneous centers $\overline{13\text{–}24}$

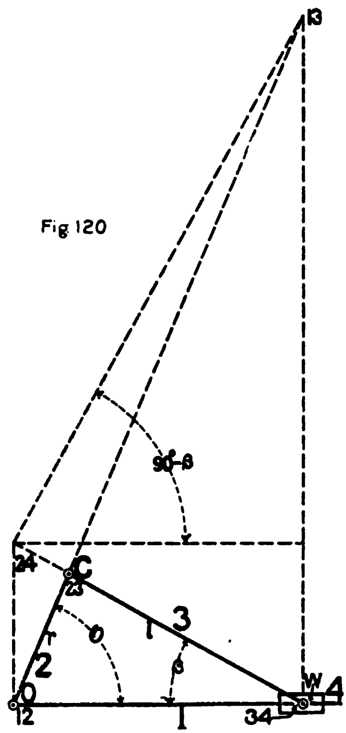



Fig 120

is perpendicular to rod's direction. (It has occasionally been erroneously assumed that this configuration existed when line $\overline{13\text{–}23}$ is perpendicular to rod.)

Starting with this proposition we will now find a cubic equation for the relation between this particular crank-angle $\theta$ and the ratio $\frac{r}{l}$ of crank to rod.

Figure 120 readily furnishes the relation.

$$\overline{OW} \tan \theta = \overline{OW} (\tan \beta + \cot \beta) = \frac{\overline{OW}}{\sin \beta \cos \beta}$$

or $$\cot \theta = \sin \beta \cos \beta$$

and since $$\sin \beta : \sin \theta = r : l$$

we get, after reduction,

$$\sin^2 \theta + \frac{r^2}{l^2} \sin^4 \theta - \frac{r^4}{l^4} \sin^6 \theta = 1.$$

In terms of the rod angle $\beta$ the cubic equation takes an even simpler form,

$$\sin^2 \beta + \sin^4 \beta - \sin^6 \beta = \frac{r^2}{l^2}.$$

In either of these equations we have an exact solution. Grashof in his Machienenlehre, Bd. II, p. 133, found approximately,

$$\cos \theta = \frac{r}{l} - \tfrac{3}{2} \left(\frac{r}{l}\right)^3,$$

which is correct to within $\left(\frac{r}{l}\right)^5$.

This result is of course sufficiently exact for engineering purposes.

$$\text{For } \frac{r}{l} = \tfrac{1}{6},\ \cos \theta = 0.1597 \quad \text{and} \quad \theta = 80^\circ\ 49'.$$

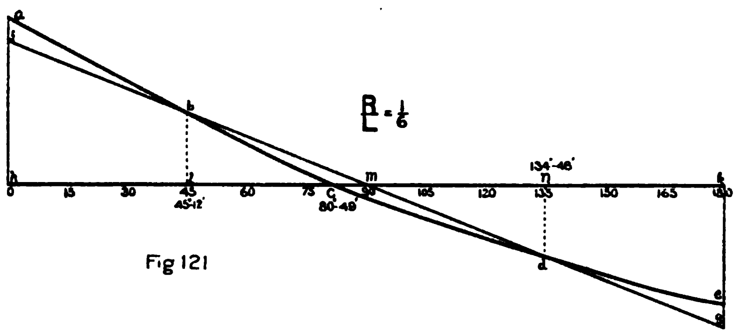



Fig 121

On pages 36 and 37 it is shown that the slide acceleration is approximately, $\theta = \frac{v^2}{R}\left[\cos\theta \pm \frac{r}{l}\cos 2\theta\right]$. This means that ordinary finite rods have same velocity as an infinite rod when crank angle is about 45°. Thus, for $\frac{r}{l} = \frac{1}{6}$, this angle is $\theta = 45^\circ\ 12'$ or $134^\circ\ 48'$.

This gives another convenient method of constructing approximately the curve of slide accelerations *abcde*, Fig. 121. For we have for the infinite rod the straight line *fbmdg*. For the finite rod we have exactly $af = \frac{1}{6}\,hf$ and $eg = \frac{1}{6}\,ek$, and approximately, there is equality in the acceleration of the two rods at *bl* and *nd*. Point *c* we can find closely from Grashof's formula given above. This enables us to get easily five points *abcde* on the curve of slide accelerations and leads to a ready and sufficiently accurate construction of this important curve in Steam Engine Dynamics.

# APPENDIX B.

## Diagram for Determining Inertia-Resistance of the Rod, the Horizontal and Vertical Components of this Resistance and the total Force Exerted by Rod.

The method of finding the resultant of the weight of the rod and of its resistance to change of velocity has been given on page 103, Figure 47, and the method of finding the pin pressures is illustrated on page 108, Figure 49. In both the figures, the main train of engine (crank, rod and slide) is supposed to be drawn to some small scale to bring it within the limits of the paper. Even then, many of the construction lines are long, and separate, enlarged, diagrams of the polygon of forces $WVbMs$, etc., are found necessary. These disadvantages can in large measure be removed by omitting the connecting-rod from the drawing, then using in its place the image $Cw$ (see figures 43 to 46) of the rod, and making the constructions employed with the image, similar to those used with the rod. The following figure, 122, illustrates how this may be done.

$Cgwz$ is the image of the portion $C\mathfrak{G}WZ$ of the rod. Triangle $jgfz$ is the image of triangle $J\mathfrak{G}ZF$ and is the triangle $j'g'f'z'$; these three triangles are all similar to each other by construction.

The point $J$ through which passes the resultant of rod's inertia resistance is obtained by finding that point $Z$ of rod which has acceleration along the axis of rod and using it as one point of mass concentration, the other point $J$ of mass concentration being taken so as to satisfy Eqs. 68 to 70 on p. 95, that is, so that

$$K^2 = J\mathfrak{G} \times \mathfrak{G}Z.$$

This will necessitate the rod's inertia-resistance passing through point $J$.

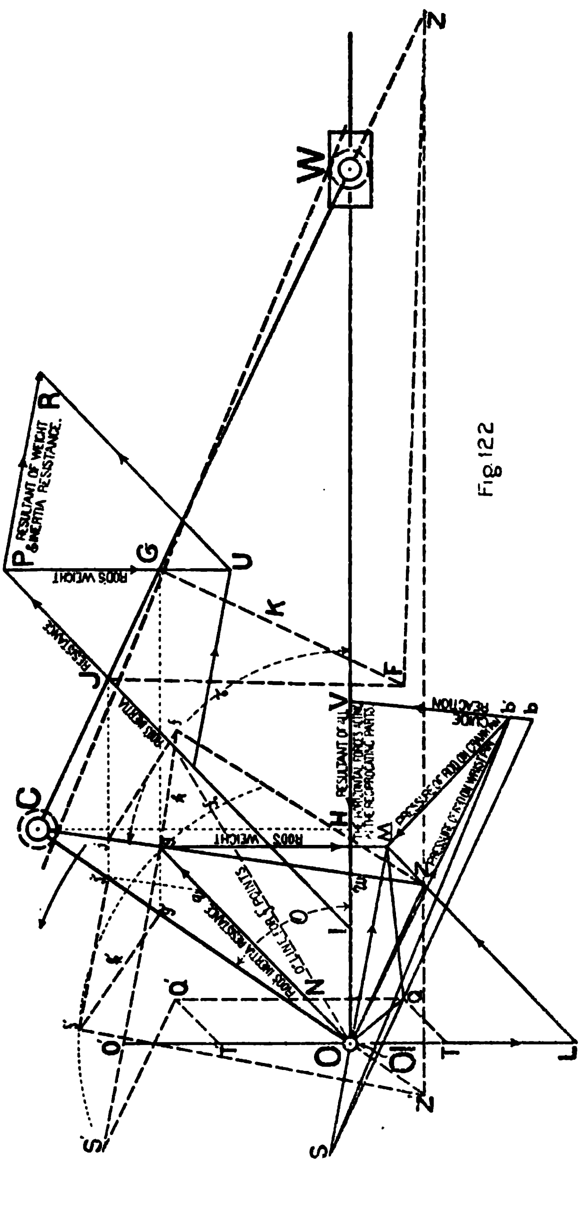


Fig. 122

Point $Z$ is horizontally projected on rod from point $z$ of image which point in turn was found by drawing $Oz$ parallel to the center line $CW$ of the rod. When rod is omitted from the figure, point $z$ is projected to $z'$ on crank $OC$ and the right-angle triangle $z'f'j'$ is used to determine $j'$ (the horizontal projection of point $J$.) Here the perpendicular

$$k' = \frac{OC}{WC} \times K = \frac{R}{L} K.$$

The point $f'$ can be laid off on a circle of its own, its zero line for angles being along line $Of_o$. All points $j'$ are easily found from the $z'$ and $f'$ points of the figure. We will assume, without again giving the construction, that the $j'$ points of Fig. 122 have been correctly found and then determine the $q$ points of division on $Og$. Then will $qg$ and $gO$ respectively represent the components at wrist-pin and crank-pin that make up, and are parallel to, the whole force of acceleration of rod.

To find the resultant of this force and the weight we may proceed as in Fig. 47, page 103, but this involves drawing the crank train. Or on $Og$ and $OL$ (here the weight $OL$ is, for convenience, taken very large relatively to the inertia $Og$) we may construct the parallelogram $OgML$ and $OM$ is the resultant desired. By laying off the weight $OL$ in an opposite direction, $OO'$, and completing parallelogram $OO'gM$ we can get the desired resultant $O'g$ as the upper side of the parallelogram. It is the last construction which is most convenient and is the one used on plate.

We next resolve this force $O'g = OM$ into its actual, component, pressure at wrist-pin and crank-pin. This is done as in Figs. 48 and 49, the distance $O'g$ (or $OM$) Fig. 120, taking the place of $WZ$, Fig. 48 or of $WM$, Fig. 49; and the point $O'$ (or $O$) then taking the place of point $W$ in Figs. 48 and 49. The first step is to find the point $S$ on resultant which will divide it into the two parallel components $SO$ and $SM$ ($SW$ and $SZ$, Fig. 48) acting at wrist-pin and crank-pin respectively. It is evident from equation 72, page 107, that the point $S$ must lie on

a line $QS$ ($SVt$ Fig. 48 or $SZ$ Fig. 49) which is parallel to the line of centers $WC$ and passes through the intersection $Q$ of any pair of components of $OM$, for instance $OQ$ and $QM$. It is evident that we shall know $Q$ if either component $OQ$ or $QM$ is completely known. In the figure we have resolved the weight $OL$ into components $OT$ and $TL$ parallel to itself and acting at wrist- and crank-pin respectively, the components satisfying the condition $OT : TL = C\mathfrak{G} : \mathfrak{G}W$. In like manner the resistance to inertia is resolved into the two components $Eg$ and $EO$, satisfying the proportion $Eg : EO = CJ : JW$.

Transferring $qg$ to $ON$ and combining latter with $OT$ we get $OQ$, one component of $OM$. We now draw through $Q$ the line $QS$ parallel to line of centers $CW$ of rod, and then through $S$ the line $Sb$ (see also Fig. 49) parallel to line of internal stress, *i. e.*, parallel to the proper tangent to the friction circles. In the figure extravagantly large friction circles have been placed at $C$ and $W$ in order to show the construction more clearly.

In the figure on plate the forces are transferred from center of shaft $O$ to point $O'$ above, the latter at distance $OO' = OL =$ weight per □″ of piston. Then (Figs. 122 and 124) $O'g = OM =$ resultant of weight and inertia-resistance of rod. Furthermore it is evident that this resultant $gO'$ can be divided at $S'$ into the same segments as $OM$ at $S$, by laying off $NQ' = TL$ and drawing $Q'S'$ parallel to center line $CW$ of rod.

Here it will be well to tabulate the value of component $O'S'$ in some table of forces; it can then be conveniently used in the diagram for finding crank- and wrist-pin pressures. In the present example, the values of $O'S'$ are:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 20° | 40° | 60° | 80° | 100° | 120° | 140° | 160° | 180° |
| 1.59 | 1.68 | 1.10 | 0.60 | 0.39 | 0.00 | −0.90 | −4.41 | 5.21 |
| 200° | 220° | 240° | 260° | 280° | 300° | 320° | 340° | 360° |
| 2.21 | 1.54 | 1.39 | 1.28 | 1.40 | 1.91 | 2.61 | 3.59 | 6.00 |

The minus sign indicates that point $S'$ has fallen outside of space $O'g$.

In order to construct curve *aaaa* in the diagram of shaking forces, Fig. 50, we must know the horizontal and vertical components of the inertia-resistance of the rod. By projecting the point $g$ (Figs. 122 and 124) to $g'$, on the vertical through $O'$ we get in $gg'$ the horizontal, and in $Og'$ the vertical, component of the inertia-resistance $Og$ of rod. The values of these components should be tabulated in the table of forces shown with Fig. 50. They can then be easily transferred to the diagram for finding wrist- and crank-pin pressures; this is particularly desirable if scale of the pin-pressure diagrams differs from that of the rod's inertia-diagram.

## CONSTRUCTION OF DIAGRAM OF INERTIA–FORCES OF ROD.

For ease of construction and accuracy it is well to make the crank of diagram *about* 12 inches long. The exact length of crank is determined by scale of forces, which should be a convenient one. In this case 1 pound per □″ of piston area to the linear inch will be a convenient scale, making the radius

$$\overline{OC} = \left(\frac{Wv^2}{gR}\right) \div A = 13.02;$$

for in this 14x14 engine represented in the diagram, $A = 153.94$, $W = 136$ pounds and revolutions per minute 273, while weight of reciprocating parts is 321 pounds. Take the crank positions 20° apart and then find the foot $w$ of the image by computing the distance $Ow$ by means of table on page 39. We have

$$Ow = 13.02 \times \text{tabular quantity } c.$$

The rod is supposed to be six cranks long.

When the segments $O'S'$ and $Sg$ of the total force $O'g$ of the rod are to be found for each one of a series of crank positions, the work of locating on the image the point $g$ corresponding to the center of gravity of the rod can be greatly simplified by dividing crank-radius $OC$ of diagram, Fig. 124, at point $g'$ into the segment $Og'$ so that $\frac{Og'}{OC}$ equal ratio $\frac{W\oplus}{WC}$ in rod, (see Fig. 122), and

then describing about $O$ a circle with the radius $Og'$. In Fig. 124 this radius is 0.573 of the diagram crank. Where this circle cuts the crank positions draw horizontal lines to point $g$ of images corresponding to these positions. The lines $Og$ represent the total inertia-resistances offered by rod at these crank angles.

When the crank is upon its dead points the method followed above fails because then the image coincides in direction with the radius. In this case the point $g$ on image may be found by making, for the crank-angle 0°, the distance

$$Og = R\left(1 \pm \psi\frac{R}{L}\right), \text{ when } \psi = \frac{C\mathfrak{G}}{L}, \text{ (Fig. 122);}$$

here $C\mathfrak{G}$ is the distance of the center of gravity from crank-pin center.

The reason for this construction is that wrist-pin $W$'s acceleration at $\left\{\begin{matrix} 0° \\ 180° \end{matrix}\right\}$ is, in our diagram, measured by $R\left(1 \pm \frac{R}{L}\right)$ respectively. Here $\frac{R^2}{L}$ represents the length of the image to our scale. As the $g$ point always divides the image into segments having the same ratio as the segments on each side of center of gravity of rod, that is, in the ratio $\psi$, we get, $Og =$ radius $\pm\ \psi \times$ image as above.

To obtain inclination $\alpha$ of rod readily we can compute it from $\sin \alpha = \frac{R}{L} \sin \omega$, or, we draw in lower half of diagram a semicircle having a radius ½ that of diagram crank and horizontally project this circle's intersections with the crank on to the crank-pin circle. The radii joining these projections with the center $O$ will evidently have the inclination of the rod corresponding to the crank positions.

We have already described, with the help of Fig. 122, the steps that must be taken to divide the total force $O'g$ of rod into two parallel components $S'O'$ and $S'g$ acting at wrist- and crank-pin respectively. It is therefore unnecessary to repeat it with present figure as these two figures are lettered alike. In Fig. 124 the

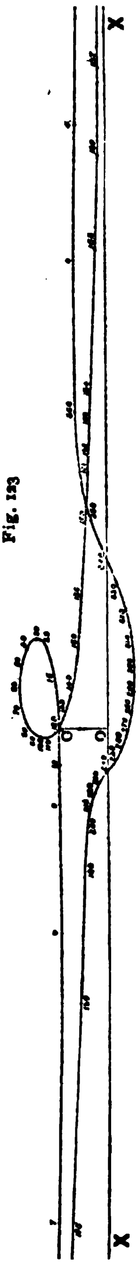


Fig. 123

wrist-pin pressure $S'O'$ is not drawn, for the sake of clearness; its value can however be taken directly from diagram by measuring from fixed point $O'$ to end $S'$ of the segment $gS'$ that is drawn. At the dead points (0° and 180°) the point $E$ on line $Og$ representing inertia-resistance becomes indeterminate, but the resultant $O'g$ of inertia and weight of rod can nevertheless be resolved into its two parallel components at crank and wrist-pin. First it is evident that this resultant cuts the line of rod at the center of gravity itself and its components at the pins will be inversely as the distance of the pins from the center of gravity. The curve connecting the points $S'$ is very irregular, has a loop, and has four branches running out to infinity. For the present data its course is as in Fig. 123.

The following enumeration may be of assistance in expediting the work of constructing the diagram.

## Steps, Stated in Their Order.

1. Draw crank positions for every 20° of rotation. (Additional positions may subsequently be added to fill out parts of other diagrams of special interest.)

2. Draw three concentric circles, for point of concentration, center of gravity and angle of rod.

3. Draw *images* of rod, using table on page 39.

4. Project horizontally the intersections, of crank with circles (mentioned above in 2), on to image and on to crank-pin circle. It is evident that the projecting lines will be the same for symmetrical crank positions in first and second quadrants.

5. Draw parallels to rod, as in diagram of rod angles, Fig. 124.

6. Lay off weight $OO'$ upwards from center $O$ and divide it into the two components that act at crank and wrist-pin.

7. Draw direction of acceleration of center of gravity by joining center of shaft with point $g$.

8. Draw the line $Oz$ parallel to direction $CW$ of rod till it cuts in $z$ the prolongation $Cwz$ of the rod's image. Through this intersection draw a parallel $zz'$ to stroke of $W$ till parallel cuts in $z'$ the crank $CO$ or its prolongation.

9. On the hypothenuse $Cz'$ construct the right-angled triangle $c'g'j'f'$ similar to $Z\mathfrak{G}JF$. A simple way will be to make perpendicular

$$g'f' = k' = \frac{CO}{CW} K = \frac{R}{L} K;$$

this need only be done once. With $O$ as a center and $Of'$ as a radius describe a circle; this will be the locus of all the $f'$ points of the series of crank positions.

10. Having found $j'$ as directed by 9, draw the parallel $j'q$ to image, cutting acceleration of center of gravity at $q$ and dividing the inertia-resistance $Og$ into two segments.

11. Lay off $ON$, from center $O$, equal to $qg$ the smaller of these segments, on this same line of acceleration of center of gravity; then lay off upward the greater of the gravity components. These two components can be laid off at same time on, and from, the symmetrically placed $Og$ line in the lower semi-circle of diagram, Fig. 124.

12. Through the upper end of the greater, gravity, components draw lines of internal stress parallel to inclination $a$ of connecting rod, (see inclination lines in lower half).

## Diagram of Pressures at Crank- and Wrist-Pin.

### Figure 125.

The preceding diagram gives the total force $O'g$ exerted by the rod, its segments $S'O'$ and $S'g$ representing those components of the pin-pressures that are parallel to total force of rod. If we combine the internal stress (see Fig. 49 and page 110, line 23) with each of these components we get the actual pin-pressures.

The wrist-pin pressure thus formed must be equal to the resultant of the guide reaction and of the pressure acting along axis of cylinder. This pressure is the algebraic difference between the effective pressure of steam and the inertia-resistance of reciprocating parts.

The principle underlying the construction of these pin pressures has already been fully explained in connection with Figs. 48 and 49. We shall therefore confine ourselves to stating how the procedure may be simplified and drawing-paper economized.

In the first place we must, in this diagram, represent our forces to a smaller scale than in the preceding diagram of rod forces; for the pressures along axis of cylinder are much larger than the rod forces. Generally one-half or one-third of scale used in first diagram will answer. We will choose the latter reduction for Fig. 125, that is, we shall assume that in this figure the scale of forces is 3 pounds per □″ of piston area to the linear inch. To lay off exactly, in direction and intensity the total force $WM$ of rod in the new diagram, we divide by 3, the tabulated value of its rectangular components, and lay them off on the proper one of the four pairs of axes we shall use. Each of these four pairs of axes corresponds to a quadrant of the crank's motion. As the cylinder is supposed to be to the right, the axial pressure will be to the $\left\{\begin{matrix}\text{left}\\ \text{right}\end{matrix}\right\}$ for the $\left\{\begin{matrix}\text{first}\\ \text{last}\end{matrix}\right\}$ two quadrants and the axis of $X$ is made to extend to the $\left\{\begin{matrix}\text{right}\\ \text{left}\end{matrix}\right\}$ of origin $W$.

The following tables give the values of the rectangular components $O'g'$ and $gg'$ for the example illustrated in the present diagrams.

VERTICAL COMPONENTS OF TOTAL FORCE OF ROD PER □″.
+ AND — RESPECTIVELY REPRESENT UP AND
DOWN COMPONENTS OF ROD.

| Crank Angles—Forward Stroke. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0<br>180 | 10<br>170 | 20<br>160 | 30<br>150 | 40<br>140 | 50<br>130 | 60<br>120 | 70<br>110 | 80<br>100 | 90<br>90 |
| —0.88 | 0.40 | 1.67 | 2.86 | 3.93 | 4.87 | 5.62 | 6.18 | 6.52 | 6.72 |
| Crank Angles—Return Stroke. | | | | | | | | | |
| 180<br>360 | 190<br>350 | 200<br>340 | 210<br>330 | 220<br>320 | 230<br>310 | 240<br>300 | 250<br>290 | 260<br>280 | 270<br>270 |
| —0.88 | —2.16 | —3.43 | —4.62 | —5.69 | —6.63 | —7.38 | —7.94 | —8.28 | —8.48 |

HORIZONTAL COMPONENTS OF TOTAL FORCE OF ROD PER □″.
+ AND — REPRESENT FORCE ACTING TO THE RIGHT
AND LEFT, RESPECTIVELY.

| Crank Angles—First and Fourth Quadrants. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0<br>360 | 10<br>350 | 20<br>340 | 30<br>330 | 40<br>320 | 50<br>310 | 60<br>300 | 70<br>290 | 80<br>280 | 90<br>270 |
| 14.02 | 13.77 | 13.00 | 11.80 | 10.21 | 8.25 | 6.11 | 3.79 | 1.40 | —0.93 |
| Crank Angles—Second and Third Quadrants. | | | | | | | | | |
| 100<br>260 | 110<br>250 | 120<br>240 | 130<br>230 | 140<br>220 | 150<br>210 | 160<br>200 | 170<br>190 | 180<br>180 | |
| -3.14 | -5.18 | -7.02 | -8.58 | -9.89 | -10.87 | -11.61 | -12.04 | -12.17 | |

Having laid off the total force of rod to the assumed scale we next lay off from the origin *W*, the smaller *O′S′*, Fig. 124 (or *WS* of Fig. 49), of the two segments of the total force and through the point *S* thus found draw the line *Sb* (Fig. 49 and Fig. 122) parallel to the line of internal stress, which is very nearly a tangent to the friction circles at crank and wrist-pin. These tangents are drawn, in the middle of the diagram, Fig. 125, for the inclinations of connecting-rod corresponding to the different crank angles. The proper tangents for each position

are decided upon according to the method indicated in Figs. 38 to 41, pages 89 and 90.

Having drawn, Fig. 125, the line *Sb* parallel to the proper tangent, we next lay off on each axis of $X$ that force $WV$, along axis of cylinder, which equals the difference of effective steam pressure and inertia of reciprocating parts. From its end $V$ lay off the guide's reaction $Vb$, inclined, from the normal, against the slide's motion, an amount equal to the angle of friction. This reaction will determine point $b$ and thus determine the actual wrist-pin pressure $bW$ and the crank-pin pressure $bM$.

Of course the total labor of taking these various steps is considerably diminished by taking the same step for each of the crank positions before proceeding to take the next step in the series. In laying off the total force of rod it may be of some help to notice that the vertical components of this force are alike in quadrants I and II for symetrically placed crank positions and are also alike in III and IV for such crank positions.

It is evident from the figures corresponding to the different quadrants that friction has very little influence on the intensities of the pin-pressures. It does influence the vertical component of the wrist-pin pressures to a notable degree because these are small any way. As it is just these components that determine whether slide shall press upward or downward and as a very slight upward component will suffice to effect this reversal of pressure and cause a knock, we cannot say that the friction's influence is everywhere practically unimportant. Moreover we must not conclude, because pin pressures are not appreciably affected by friction, that the latter's influence on engine friction is likewise inappreciable. The crank-pin's pressure for instance makes its influence felt to a notable degree by acting tangent to friction circle in such a way as to diminish its crank leverage by the radius of the friction circle.

When it is only a question of finding when reversal of pressure on slides occurs, the determination of pin-pressures will not be necessary for every crank position of Fig. 124. Inspection of

Fig. 124 will show where this reversal is liable to take place and the pressure determinations of Fig. 125 may then be confined to the crank positions of that vicinity.

In each of the upper and lower semi-circles, the images are exactly parallel for crank positions symmetrically situated with respect to the $\overline{90°–270°}$ line.

This parallelism can be shown as follows: Let $\tau$ represent the angle $CwW$ made by image with stroke of slide. Then will

$$\cot \tau = \frac{wH}{CH} = \frac{R \cos \sigma - \theta}{R \sin \sigma}$$

$$\cot \tau = \pm \frac{1}{\sin \sigma}\left[\frac{\sin^2 \sigma}{\sqrt{\frac{L^2}{R^2} - \sin^2 \sigma}} \mp \frac{L^2}{R^2} \frac{\cos^2 \sigma}{\left(\frac{L^2}{R^2} - \sin^2 \sigma\right)^{\frac{3}{2}}}\right]$$

Here $\theta$ has the meaning given it on page 37 and its factor $\frac{v^2}{R}$ has the value $R$ in diagram, Fig. 124. From this it is evident that $\cot \tau$ will be the same for any two supplementary angles $\sigma$ and angles $180° - \sigma$.

When $\cot \tau = 0$, the image becomes vertical; this is always so for infinite rods and for common ratios of $\frac{L}{R}$ when crank-angle $\sigma$ is nearly 45°.

With the help of this expression for $\cot \tau$, we can show

$$\overline{OI} = \chi L\ (\cos \sigma + \sin \sigma \cot \tau) \text{ exactly,}$$

and that the distance $OI$ is practically a constant quantity for ordinary rods. This would enable us to find still another easy method of getting the location of the inertia-resistance of the rod; but we omit it as it is not any simpler than the method we now have. See note on page 100. $\chi = \frac{CH}{L}$, Fig. 45.

Fig. 126

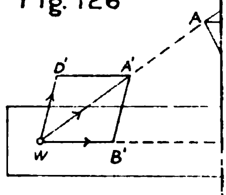



Fig. 127

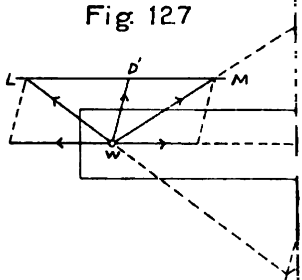
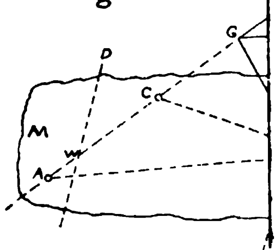



Fig. 128



# APPENDIX C.

## EXACT CONSTRUCTION FOR THE FORCES ACTING IN AND UPON THE CONNECTING ROD.

BY R. C. H. HECK, M.E.,
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1. When the forces developed in the connecting-rod, due to its own mass and motion, are not considered, the rod is taken as simply a connecting link, serving to transmit force from one pin to the other. The pin-pressures, equal in intensity and opposite in direction, act along the same straight line. This force line, or line of internal stress in the rod, passes through the pin centers, friction being neglected. Taking account of friction, the line is tangent to the friction circles. But in any actual engine, there are certain forces developed in the connecting-rod, due to gravity and inertia, generally acting transverse to the center line of the rod. It is therefore necessary to discuss what takes place under these conditions.

2. In Fig. 126, let $WC$ represent any rigid body, upon which acts the force $Q$ along the line $XZ$. Impose the condition that the body shall be held in equilibrium by two forces passing through the points $W$ and $C$ respectively. These two must intersect in the line $XZ$; but there is evidently an infinite choice in the location of this point of intersection.

Let $WX$, $CX$, be the lines of action of one pair of possible forces, $P_w$ acting at $W$ and $P_c$ at $C$. Lay off $XY = Q$, and find its components, $AX = P_w$ and $BX = P_c$. Now, through $A$ draw $AD$ parallel to $WC$, making two force triangles $AXD$, $YAD$. In $AXD$, $P_w$ is resolved into the component $AD$ parallel to $WC$ and $DX$ along $Q$. $P_c$ is likewise resolved into $DA$ and $YD$.

A similar resolution of $P_w$, at $W$, by the force parallelogram $WD'A'B'$, and of $P_c$ at $C$ by $CD''A''B''$, gives the same components as the figure $XADY$; and, what is more, it gives the components on their lines of action, or locates them. For, since the forces $P_w$, $P_c$ act upon the rigid bar $WC$ at the points $W$ and $C$, their components, by whatever resolutions, must also act at these points. Considering now the components of $P_w$, $P_c$ we have $WD'$ ($= DX$) and $CD''$ ($= YD$), forming with $Q$ a system of three parallel forces in equilibrium; we will call these the *equilibrium components*, and designate them $P_{we}$, $P_{ce}$. Besides these there are $WB' = AD = P_{ws}$ and $CB'' = DA = P_{cs}$, which we will call the *internal stress* components, and which simply balance each other as they act along the line $WC$.

3. Since the equilibrium components are taken parallel to $Q$, they must be invariable as long as they act at $W$ and $C$, no matter what two coincident $P$ forces are applied at these points. From the laws of parallel forces we would have

$$P_{we} + P_{ce} = Q \quad \text{and} \quad P_{we} \cdot WZ = P_{ce} \cdot CZ.$$

This latter relation may also be demonstrated from Fig. 126:

| From triangles $AXD$, $WXZ$, | and from triangles $AYD$, $CXZ$, |
|---|---|
| $\dfrac{XD}{DA} = \dfrac{XZ}{WZ}$; | $\dfrac{AD}{DY} = \dfrac{CZ}{XZ}$. |

Whence $\dfrac{XD}{DY} = \dfrac{CZ}{WZ}$, or $P_{we} : P_{ce} :: CZ : WZ$.

4. Since every possible $P_w$ must have $WD'$ as one of its components—the other component being along $WC$—it is evident that a line drawn through $D'$ parallel to $WC$, as $LM$, Fig. 127, is a locus of the end of $P_w$ when this is laid off from $W$ as an origin. Similarly, $L'M'$ is the locus of the end of $P_c$. Fig. 127 illustrates the properties of these loci, and shows clearly how the internal stress components vary as $P_w$ and $P_c$ change.

5. The validity of the preceding discussion depends upon the right to select some particular point on the line of action of a orce as its point of application, through which point all the com-

ponents of the force must pass. Now forces acting upon a rigid body, as *MN* Fig. 128, can act and react upon each other only through the medium of stresses developed in the body-stresses which are distributed over its cross-section, but whose effect in any direction must be represented by a single resultant force. To select a particular point of application is simply to make that the point where the external force meets the resultant of the internal stresses; and obviously all components of the external force must pass through this point. But in the general case of Fig. 128, no reason can be assigned for the selection of any particular point on, for instance, the line *AE*; not even that the point must fall within the body. The total effect, along the line *AE*, of the internal stresses will not vary as the point moves; but their component effect, in any assigned direction, will vary with the same component of the external force, equilibrium being always preserved.

It will be well to realize that, when we make the force resolution of Fig. 126, as at *EGKF*, Fig. 128, and call *AB* the line of internal stress, that only a part of the internal stress effect acts along *AB*; there are also developed stresses, closely analogous to those in a beam, under the action of the parallel forces $Q$, $P_{we}$, $P_{ce}$.

Fig. 128 is intended to illustrate choice of line of internal stress (for the force resolution of Section 2), which exists in the general case of any rigid body in equilibrium under the action of three forces, showing two out of an infinite number of possible cases. It is evident that we may impose some condition, as for instance $P_{we}$ shall act along the line *WD*, which will at once determine the point of application of $P_w$.

## Construction for the Pin Pressures, when Friction is Neglected.

6. In Fig. 129, the resultant of weight and inertia of rod is supposed to be known, and is represented by $XY = Q$. The pin-pressures must pass through the centers of the pins; the force

diagram for determining them will be constructed at $W$; for $P_w$, besides balancing its share of $Q$, must also be the resultant of the known effective horizontal pressure on the wrist-pin and of the guide reaction whose direction is known; these two conditions determine it fully.

Lay off $WN$, parallel, equal, and opposite in direction to $Q$. Let it be divided at $L$, so that $WL : LN :: CZ : ZW$; then $WL$ is the equilibrium component of $P_w$. The line $LM$, parallel to $WC$, is, in reference to the point $W$, a locus expressing graphically the first condition, as is shown in Section 4. Now measure off $WV =$ effective horizontal pressure, and draw $VM$ parallel to the guide reaction; then $VM$ embodies the second condition; and the intersection of these two loci at $M$ fixes $WM$ as the actual pressure of wrist-pin on connecting-rod. The line $MN$, completing the triangle $WMN$, is the crank-pin pressure, whose line of action is $CX$.

## Pin Pressures when Friction is Considered.

7. In Figs. 130 and 136, the circles around $W$ and $C$ are the friction circles of the respective pins, greatly exaggerated. The mechanism is simply outlined. The pin pressures act tangent to these circles; they must, of course, hold $Q$ in equilibrium. But the lack of a point common to all the forces possible of action at each pin, and which may be taken as their point of application, makes the graphical expression of this condition much more difficult than in the preceding case. Fig. 130 illustrates the construction of an exact polar locus of the force $P_{wf}$, the same in its conditions as $LM$, Fig. 127. It is obtained by drawing pairs of pin-pressure lines to various points on the line of $Q$ as $X_1$, $X_2$, $X_3$, and resolving $Q$ along these lines; then laying off from $W$ as a pole the various $P_{wf}$'s thus found and drawing a curve $DD$ through their ends.

This curve has some interesting properties. If we draw $WD$ parallel to $Q$, it corresponds to the case of pin-pressures parallel to $Q$. Note how $WL$ differs from $WL$ in Fig. 129 (the figures

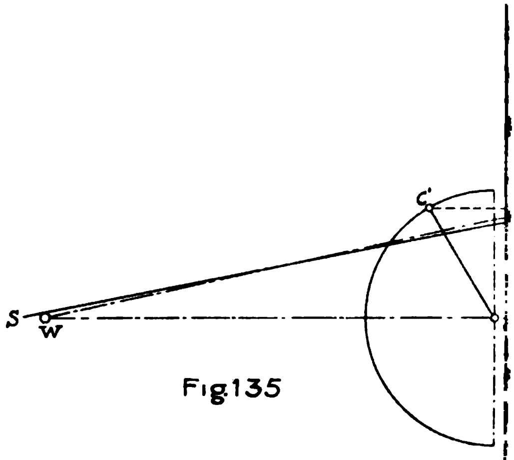


Fig. 135

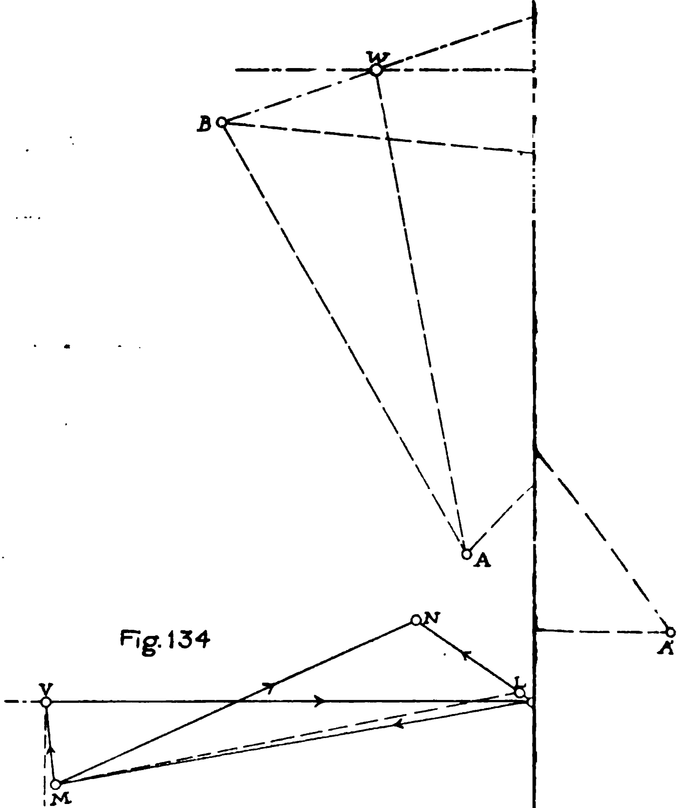


Fig. 134

being alike, aside from friction). The branch $DD_2$, to the right of $D$ corresponds to the case of rod in compression. It reaches $\infty$ when the pin pressures are conceived as acting along the line $EF$, tangent to the friction circles. For the left-hand branch $DD_3$, rod in tension, the similar limiting case is the force direction $KH$. If the direction of motion of the mechanism were reversed, the entire locus would be changed.

8. To establish an approximate construction of practical utility, we impose the condition that the equilibrium components of the pin-pressures, parallel to $Q$, shall pass through the pin centers; in Fig. 131 their lines of action are $WL$, $CU$. This makes them the same as in Fig. 129. Now draw from any point $X$ on the line of $Q$ the pin-pressure lines $XR$, $XU$; the intersections $R$, $U$, become the points of application of pin pressures, through which all their components must pass, and $RU$ is the line of the internal stress components (see end of section 5). If we were to draw through $W$ a line parallel to $RX$ and measure off the length of $P_{wf}$: then a parallel to $RU$, through $L$, would pass through the end of $P_{wf}$ (compare $WM$, $LM$, Fig. 129), and at that point it would agree with the locus $DD$ of Fig. 130; so that we have here stated, in a slightly different form, the method of obtaining a point on that locus.

When the force $Q$ of the rod is not taken into account, the internal stress line is $ST$, tangent to the friction circles, as stated in Section 1. The approximation which we now make is to consider that this line $ST$ practically agrees in direction with the variable line $RU$, which in the actual rod is true within the limits of graphical accuracy. Then $LM$, parallel to $ST$, becomes a locus exactly similar to $LM$ of Fig. 129 or of Fig. 127. The pin-pressure construction is now the same as in Fig. 129, differing only in that the guide reaction is inclined from the normal through the angle of friction. If the rod be in tension the locus is $LM'$ parallel to a tangent drawn like $KH$ of Fig. 130. Note that in Fig. 131 the actual lines of action of $P_{wf}$, $P_{cf}$ are not shown.

Drawing this locus $M'LM$ in Fig. 130, we can get a good idea of the error of the approximate method. It is evidently greater when $Q$ is large relatively to the force transmitted by the rod. The lines $LM$, $LM'$ are asymptotes to the true locus; for at $\infty$ $RU$ agrees with $ST$ and the construction is exact. In Fig. 131 the condition was stated that the pin-pressures must be tangent to the friction circles. To substitute $ST$ for $RU$ as the line of internal stress is to make $S$ and $T$ the points of application of all pin-pressures found by the approximate loci, thus making these pressures differ slightly from those found by the exact construction. The approximation to tangency may be easily estimated by inspection.

## The Rod Force and its Components.

9. Referring to Sections 6 and 8, it appears that, in order to construct the pin-pressures it is necessary to know the total rod force and its parallel components at the pin centers, the latter being a function of its location on the rod. In Fig. 132 is developed a method for determining these quantities.

Refer to pp. 93-100, and foot note p. 44.

In Fig. 132, $WCO$ represents the mechanism. $CO$ is taken to represent the acceleration $\frac{v^2}{R}$ of $C$; and $WO$, equal acceleration of $W$, is found as in Fig. 16, page 41 or by Eq. 36. Now by producing $Cw$ to $W'$, making $CW'$ equal $CW$, and completing triangle $CW'A$ similar to triangle $CwO$; then revolving it into the position $CWA$, we find the center of acceleration in $A$.

$$\left(\text{For } \frac{wO}{AW} = \frac{CO}{AC}, \text{ and angle } AWO = \text{angle } ACO.\right)$$

$Cw$, being the acceleration image of the rod, we have only to find center of gravity, $g$, of the image from $G$, and draw $gO$, to get the intensity and direction of the acceleration of the center of gravity $G$ of the rod. Again, letting $OC$ represent to a suitable scale of forces, the inertia (centrifugal) force which the mass of

the rod would have if concentrated at $C$; then to the same scale, $Og$ represents the actual inertia force of the rod.

10. To locate this force we proceed as follows:

The actual distributed mass of the rod may be replaced, for dynamic purposes, by two simple concentrated masses $m_1$, $m_2$, located on any straight line through the center of gravity, at the distances $h_1$, $h$, from the latter; the conditions being—

1. $m_1 + m_2 = M$, (same mass).
2. $m_1 h_1 = m_2 h_2$, (same center of gravity).
3. $m_1 h_1^2 + m_2 h_2^2 = Mk^2$, (same moment of inertia).

$k$ = principal polar radius of gyration, and $k^2 = h_1 h_2$.

The points of mass concentration chosen, and their accelerations found from the image, then the inertia forces of $m_1$, $m_2$ will have the direction thus determined; and through the intersection of these inertia forces, components of the total inertia force of the rod, the latter force must pass.

If we choose one mass point so that its inertia acts along the line $WC$; then the direction of the inertia of the other mass point is of no importance, for it will intersect the first force in its own mass point, and the latter will serve to locate the resultant, or total inertia of rod. Construction: make angle $WAB$ = angle $CWO = \alpha$, so that angle $ABW$ = angle $AWO$; whence acceleration of $B$ is along $BC$; at $G$ erect $GK$ perpendicular to rod, and equal to $K$; complete the right-angled triangle $BKJ$, making $BG \times GJ = \overline{GK}^2$, then $J$ is the second mass point, through which must pass the inertia force $QE = Og$.

The weight and inertia being combined at $Q$, their resultant $QF$ cuts the rod at $P$; and $QF$, divided in the ratio $CP$: $PW$ is what we wish to obtain, but by a more compact and convenient construction.

11. Lay off $O'O$ = weight of rod, then $Og = QF$ = total rod force $Q$. Construct on the image $Cw$ a figure similar to $CBKJ$; determine $b$ by $Ob$ parallel to $CW$; lay off $kg$ in proper ratio $Cw$, and find $j$; draw $jJ$ parallel to $wO$, and $J'e$ parallel to

16

*Cb*, then $ge : eO = Cj : jw = CJ : JW$. That is, the inertia force is divided into its parallel components at the pins.

Now divide the weight $OO'$ at $T$ so that $O'T : TO :: CG : GW$, separating the weight also into its parallel components at the pins. Starting from $O'$, combine $O'T$ and $TR$ ($= eg$), the wrist-pin components of weight and of inertia. Their resultant $O'R$ is a possible wrist-pin pressure $P_w$ (acting in the direction "rod on pin"); its end $R$ must therefore lie on the $P_w$ locus of which $O'$ is the pole. This locus is $RS$ parallel to $WC$, and it cuts from $O'g$ the wrist-pin component $O'S$. (Compare Figs. 126 and 127.) This completes the solution of the problem.

$Cw$ changes with the crank position, and with it $kg$. To avoid making repeated computations for the latter, we can make part of this image construction on the crank $OC$ instead of on the acceleration image $Cw$. Project $b$ to $B'$, lay off $G'K'$ in proper proportion to $OC$ and obtain $J'$. The entire construction is now independent of the rod, except for the direction of the lines $Ob$, $RS$.

12. Fig. 133 shows the whole construction for one crank position, and will serve to illustrate the following instructions. Having necessary data, including weight, center of gravity, and radius of gyration of rod:

1. Draw circle $ACD$, with radius $= \dfrac{F_o}{A}$ for rod, to a convenient scale of forces, and space off $C$ points.

2. From p. 37 compute and lay out $Ow$; draw images $Cw$.

3. Strike circle $HG$, $G$ being center of gravity of crank image of rod; obtain $g$ by projection; lay off $OTO'$, draw $Og$, $O'g$.

4. Draw $GK$ *once*; then strike circle $LK$, and from each $G$ locate the corresponding $K$ on this circle.

5. Rod angle construction: $\dfrac{Om}{OA} = \dfrac{R}{L}$; $OC$ cuts circle $mpn$ at $p$; project $p$ to $u$; $uO$ is parallel to rod, for crank position $C$ (and for $180° - C$); keep this construction below the center line $AO$.

6. Having $b$ project to $B$, draw $BKJ$, $Je$.

7. Lay off $OE = eg$, $ER = OT$; draw $RS$ parallel to $uO$.

To avoid confusion:

(*a*) Complete each step in the above for all the crank positions before beginning the next.

(*b*) Mark the points $C$, $w$, $G$, $g$, $O'$, $T'$, $K$, $b$, $B$, $J$, $e$, $E$, $R$, $S$, as found, by *inking* a small circle about each, the first six and last three in black, the others in red. Letter one construction fully, and number each set of points with the degrees of crank angle, from 0° to 360°; likewise, designate the rod line $uO$ with the angles to which they belong.

(*c*) Draw as few lines as possible; thus, on upper half of figure, $CO$ need not come inside of circle $HG$; below, of circle $mu$; $ub$ need not come to $O$; projection lines, as $Gg$, $bB$, $BK$–$KJ$, $Je$, need not be drawn in.

13. Fig. 133 completed, the pin-pressures are found by Figs. 134 and 135. Fig. 135 is drawn first. To as large a scale as paper will allow, strike arc $AB$, with rod-length $L$ as radius; draw circle $OC$ with radius equal $R$; put in crank position $OC'$, project $C'$ to $C$; $WC$ is rod angle, and $ST$ is line of internal stress, with friction. The play of forces, as affecting tangency of $ST$ to friction circles, must be found from the diagram of effective horizontal pressures on wrist-pin; modified at critical points by the inertia of the rod itself, as affecting direction of crank-pin pressure. It is possible to have one end of the rod in tension, the other in compression. Mark each $ST$ with its crank angle. Fig. 134, is the same as the pin-pressure diagram of Fig. 131, except that all the forces are reversed, so as to give pressure of rod on pins rather than of pins on rod. Bunch the constructions in four groups on Plate 7, one for each quadrant of crank circle. $NLW$ is $O'Sg$ to reduced scale.

# APPENDIX D.

## The Link Motions.

In this set of valve gears, the eccentrics, the throw and location of each eccentric, are invariable. These Link Motions occur principally in the locomotive, which, to be sure, is a species of high-speed engine, but has so many other special problems connected with it that it cannot be fully treated within the limits of this work. Nevertheless, in order to give a certain degree of completeness to the general subject of valve-gear design we will here give an outline of the Kinematics of Link Motions. The Fink Motion, as used in the Porter-Allen Fngine, has already been discussed in the body of the text.

In the valve gears known as Link Motions the same sort of steam distribution occurs as in the gears having a "Swinging Eccentric," that is, the locus of the eccentric centers is of the same general character. But the mechanism employed is very different, the adjustment of the Link Motions by hand causing the invariable eccentrics to produce the desired variations of valve motion. About the only case in which the adjustments are automatically made by the governor occurs in the Fink or Porter-Allen Motion.

Dr. Zeuner found, in his well-known treatise on Valve-Gears, this locus of eccentric centers by analytical means, but it may be found much more easily graphically and, so far as practical applications are concerned, with the same degree of accuracy.

The basis of the graphical treatment is Fig. 66, which contains the method of finding the virtual eccentric for a valve motion whose stroke does not pass through the center of the shaft.

We will here use the figure and demonstration given by Pro-

fessor A. Fliegner in his work " Die Umsteuerungen der Locomotiven.*"

Fig. 136 represents what may be regarded as the general case of all the link motions, extensively used in practice, which impart

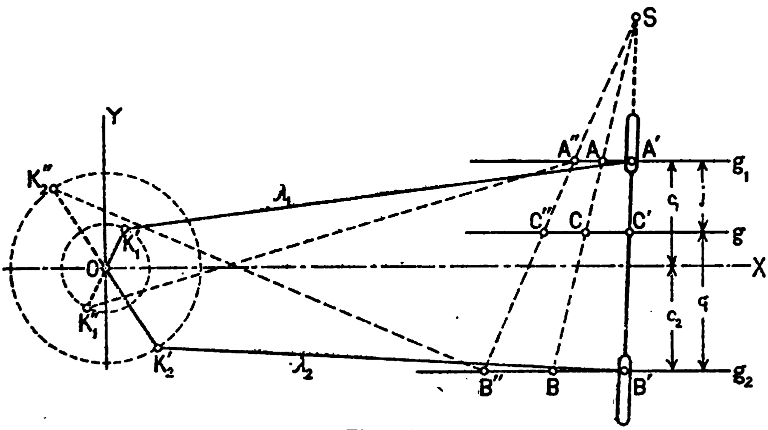


Fig. 136.

to the valve a motion like that which it experiences when directly driven by a single, simple, eccentric. This figure and Fig. 137 show how two eccentric motions may be combined so as to be equivalent to the motion of a single eccentric, called the *virtual* eccentric. This, indeed, is the gist of almost all the link-motion problems.

$A'B'$ is regarded as the link provided with slots at the ends to permit the eccentric ends $A'$ and $B'$ to move in the perfectly straight lines, $g_1$ and $g_2$. The point $C'$ on this link directly drives the valve and is itself assumed to move in a straight line $g$, which is parallel to the lines $g_1$ and $g_2$. The rods are of unequal lengths, $\lambda_1$ and $\lambda_2$; the eccentricities $\rho_1$ and $\rho_2$ are also unequal and have the unequal angles of advance $\delta_1$ and $\delta_2$. The position shown in full lines corresponds to crank position at left dead point; the

* Dr. Burmester has also given an elegant kinematic presentation in his "Lehrbuch der Kinematik," pp. 696–702, from which some of the special cases can be more rigorously deduced.

broken lines of the figure correspond to the position of the crank at the right-hand dead point.

Here it will be well to note the difference between link-motions with *open* and with *crossed* rods. When crank is at dead point and eccentrics are placed between engine shaft-center $O$ and link $A'B'$, then the mechanism is said to have *crossed* rods if the eccentric rods cross in this position and *open rods* if the eccentric rods do not then cross. Fig 136 is an example of the open-rods variety.

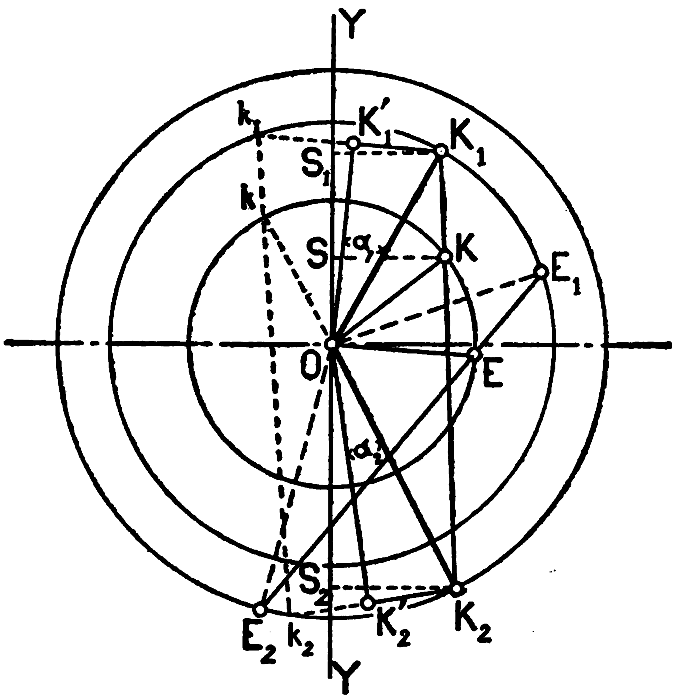


Fig. 137.

Take $A$ and $B$ as the centers of travel of points $A'$ and $B'$ and join $A$ and $B$; then will $AB$ pass through the intersection $S$ and be the reference line from which the travel $CC'$ of any point $C'$

on link $A'B'$ can be estimated for any crank position. It is evident from the figure that the travel $CC'$ at any instant is

$$CC' = AA' + \frac{j}{j+q}(BB' - AA').$$

Let us take these eccentrics $OK'_1$, $OK'_2$ separately and on a larger scale construct in Fig. 137, after the method of Fig. 66, the virtual eccentrics $OK_1$ and $OK_2$, whose distances $\left\{ \begin{matrix} K_1S_1 \\ K_2S_2 \end{matrix} \right\}$ from the vertical line $OY$ are respectively equal to the travels $\left\{ \begin{matrix} AA' \text{ of point } A' \\ BB' \text{ of point } B' \end{matrix} \right\}$, see Fig. 136. To do this we make

$$K'_1 K_2 = \frac{c_1}{l_1} \times \overline{OK'}_1 \text{ and } K'_2 K_2 = -\frac{c_2}{l_2} \times OK'_2$$

for the case of open rods,

$$\text{and } K'_1 k_1 = K'_1 K_1 = -\frac{c_1}{l_1} \times \overline{OK'}_1$$

$$\text{and } K'_2 k_2 = K'_2 K_2 = +\frac{c_2}{l_2} \times \overline{OK'}_2 \text{ for crossed rods.}$$

Now join $K_1$ and $K_2$ and divide at $K$ the distance $K_1K_2$ so that the ratio

$$\frac{K_1K}{K_2K} = \frac{j}{q},$$

in other words, in the same ratio that the valve-stroke-line $g$, (Fig. 136) divides the total distance between $g_1$ and $g_2$, the lines of travel of the eccentric pins.

This will make the distance $KS$ of point $K$ from the vertical $OY$ equal to

$$KS = K_1S_1 + (K_2S_2 - K_1S_1)\frac{j}{j+q} = AA' + (BB' - AA')\frac{j}{j+q}.$$

When the crank turns into some other position, the distance between pins $A'$ and $B'$ in Fig. 136 is still divided into the same ratio $\frac{j}{j+q}$ by the line of stroke $g$ of the valve, and in Fig. 137 the triangle $OK_1\ KK_2$ goes into some other position $OE_1\ EE_2$,

where the distances of $E_1$ and $E_2$ from $OY$ equal the corresponding distances of pins $A'$ and $B'$ from their centers of motion $A$ and $B$, and the distance $E$ from $OY$ is equal to the distance of the driving point $C'$ of the valve from its center of motion $C$. The complicated motion with which we started has therefore been reduced to the simple motion produced by a virtual eccentric $OK = OE$.

We will now show the applicability of this method to Stephenson's, Gooch's and Allan's link motions and will assume the reader to be acquainted with the general kinematic features of these mechanisms. In each of these motions the distance between the lines $g_1$ and $g_2$ (Fig. 136) remains unchanged.

In Stephenson's motion the block or driving point $C'$ keeps its line of stroke $g$, but the pins $A'$ and $B'$ change their lines of stroke $g_1$ and $g_2$ to some other parallel position in such a way that the distance $c_1 + c_2$, between them remains the same, while the ratio $\frac{c_1}{c_2}$ is changed. As $c_1$ and $c_2$ change the virtual eccentrics $OK_1$ and $OK_2$ will change; the size and position of $OK$ depend on this change and also on the ratio of $j$ to $q$. The locus of $K$ for different positions of the link we will construct later on.

In Gooch's motion the link is stationary, the pins $A'$ and $B'$ keeping their lines of stroke $g_1$ and $g_2$. The locus of the center $K$ of the virtual eccentric $OK$ will therefore for this mechanism always be the line $K_1K_2$ because $c_1$ and $c_2$ do not change. The position of $K$ on this line will depend on the ratio $\frac{j}{q}$.

In Allan's motion both link and block are shifted and in opposite directions. Each of the three lines, $g_1$, $g$ and $g_2$ changes its place, the values of $c_1$, $c_2$, $j$ and $q$ change but so that $c_1 + c_2 = j + q =$ constant. The locus of the center $K$ of the virtual eccentric depends on $c_1$, $c_2$, $j$ and $q$ and is found as before.

The Fink or Porter-Allen link motion may be regarded as that particular case of the Gooch in which the two eccentrics

are equal and have an angle of advance of 90° causing them to coincide and thus reducing it to a link motion with only one eccentric. (The link itself then becomes part of the eccentric strap and may be regarded as a bent lever rocking on a vibrating fulcrum). In the actual link motion this would have the effect of causing the two eccentric rods and the link to form one rigid piece. This would prevent any angular motion between either rod and the link and, in the Fink, the link would therefore receive the whole of the rocking motion due to the throw of the single eccentric. The result is to make the *position* of the locus of the centers $K$ of the virtual eccentric in Fink just what it should be if it were strictly a special case of the Gooch, but the location of $K$ on this locus cannot be found without adding still another approximation to the series, namely, to substitute for length $\lambda$ of the eccentric rod the average distance $\Lambda$ of the trunnion of link from the center of either eccentric. This substitute $\Lambda$ will generally be shorter than $\lambda$ and gives a greater throw by making ordinate $\frac{u}{\Lambda}\rho > \frac{u}{\lambda}\rho$. The result will then agree with that found by analytical investigation.*

In constructing the locus of the centers of the virtual eccentric and the diagrams of the four grades of expansion for forward motion we will use the examples of the Gooch, Stephenson and Allan gears taken by Dr. Zeuner for the same purpose in his Valve Gears.

Gooch's Link Motion. Dimensions $r = 2\frac{3}{8}''$, $\delta = 20°$, $2c = c_1 + c_2 = 12''$, $l = 48''$, $e = 0.91''$ and $i = 0.23''$. Let the

* This is a doubtful derivation of the Fink throw from that of the Gooch, for which only the writer is responsible Dr. Burmester preferred to obtain it independently. Below we have given a simple analytical proof of the travel of the valve in the Porter-Allen link motion.

four grades of expansion correspond to the four cases in which block (*i.e.*, line $g$) is at $\frac{1}{4}c$, $\frac{1}{2}c$, $\frac{3}{4}c$ and $c$ from the line of centers

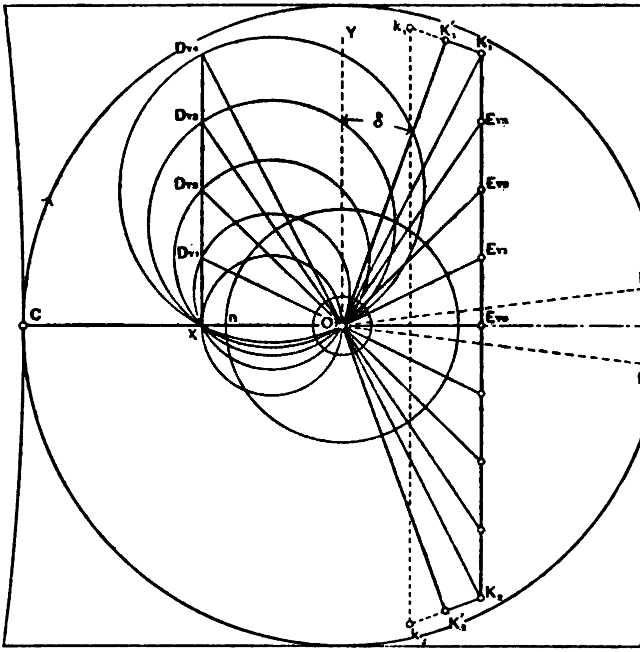


Fig. 138.

$OX$ (Fig. 136). In Fig. 138 $c_1 = c_2 = c = 6''$, $\frac{c}{l} = \frac{1}{8}$ and $K_1K_1' = 0.297 = K_2K_2'$. The value of the ratio $\frac{j}{j+q} = \frac{3}{8}, \frac{1}{4}, \frac{1}{8}$ and 0. Make the offsets $K_1K_1' = K_2K_2' = 0.3''$, join $K_1$ and $K_2$ then divide $K_1E_{v_0}$ into four equal parts and draw $OE_{v_1}$, $OE_{v_2}$, $OE_{v_3}$, and $OK_1$. The link $K_1K_2$ is the locus of the centers of the virtual eccentric for forward and backward motion and $OE_{v_1}$, $OE_{v_1}$, $OE_{v_3}$ and $OK_1$ represent in length and position the virtual eccentric themselves for the forward motion when the crank is at the dead point. The diagram thus found agrees exactly with that obtained from Dr. Zeuner's approximate formula. The diagram shows that lead $nX$ constant. The locus for crossed rods is shown at $k_1k_2$.

PORTER-ALLEN LINK MOTION. The relation between the Fink and the Gooch motion was pointed out above, p. 245. If the angle of advance of each eccentric is made equal to 90°, then the two eccentrics $OK_1'$ and $OK_2'$ reduce to one $OK$ and the locus $K_1K_2$ of the virtual eccentrics takes the position $K'KK''$. For the reason already given, p. 246, the offset $KK' = KK''$ for full grade should be greater than $K_1K'_1 = K_2K'_2 = \frac{c}{\lambda}\rho$. According to Zeuner it should be $\frac{c}{\Lambda}\rho$ for the same grade, where $\Lambda$ is the distance from center of eccentric to center of trunnion; (this trunnion in Porter-Allen gear joins sustaining arm and link on the center line of the slot). For any other grade of expansion Zeuner gives offset $= \frac{u}{\Lambda}\rho$, where $u$ is the perpendicular distance of block above line of centers $OX$.

When $\frac{\Lambda}{\rho} < 4$ we should plot positions of block to ascertain the valve travel but when $\frac{\Lambda}{\rho} \geqq 4$ we may for the Fink gear, as realized in the Porter-Allen link motion, determine the travel $\xi$ as follows: Consider the total movement of link to be composed of two parts, a horizontal motion parallel to itself which is due the horizontal throw of the eccentric and a rocking motion about the trunnion (or vibrating fulcrum) which is due to the vertical throw of the eccentric. These two motions together cause the block to move a distance $\xi$ equal to the sum

$$\rho \cos \omega + \frac{u}{\Lambda}\rho \sin \omega = \xi.$$

Here the first term is the horizontal throw common to all points of the link and the second term is the vertical throw $\rho \sin \omega$ of the eccentric $\times$ the ratio $\frac{u}{\Lambda}$ of the arms of the bent lever constituting the link and rotating about the trunnion.

The formula just given is the polar equation of the valve-circle and the coordinates $\rho$ and $\frac{u}{A}\rho$ of the vertex of its diameter show that this vertex lies on a rectilinear locus perpendicular to the line of centers and at a distance $\rho$ from the center of the shaft, which agrees with the result so irregularly deduced above.

ALLAN LINK MOTION. Dr. Zeuner gives the following dimensions of an existing valve gear with *crossed* rods.

| | | |
|---|---|---|
| Eccentricity, | $\rho = 2.75''$. | |
| Angle of advance, | $\delta = 30°$. | |
| Length of each eccentric rod | $\lambda = 49.2''$. | |
| Half length of link, | $c = 6''$. | |
| Length of the radius rod, | $\lambda_1 = 60''$. | |
| Distance of trunnion of hanger of radius rod from valve stem | $\lambda_0 = 49''$. | |
| Short lifting arm of the lever, | $a^1 = 2.88''$ | ratio $= 2.37$. |
| Long lifting arm of the lever, | $b^1 = 6.81''$ | |
| Outside lap, | $e = \frac{15}{16}''$. | |
| Inside lap, | $i = \frac{3}{16}''$. | |

Ratio $\gamma$ of block to link movement, $\frac{u_2}{u_1} = \frac{b'}{a'} \times \frac{\lambda_1}{\lambda_0} = 2.9$.

Ratio of distance $u$ of block from dead point of link to distance $u_1$ that link is shifted from line of centers is $\gamma = 3.9$.

| For the four grades of expansion, | $u = c$ | $u = \frac{3}{4}c$ | $u = \frac{1}{2}c$ | $u = \frac{1}{4}c$ | $u = 0$ |
|---|---|---|---|---|---|
| Corresp'd'g amount of link shifting $\frac{u}{\gamma}$ | $0.256c$ | $0.192c$ | $0.128c$ | $0.064c$ | $0.0$ |

Since $c_1 = c - u_1$ and $c_2 = c + u_1$ approximately,

| | | | | | | |
|---|---|---|---|---|---|---|
| | $\frac{c_1}{\lambda}\rho$ are | 0.249 | 0.270 | 0.292 | 0.314 | 0.335 |
| the corresponding values of | $\frac{c_2}{\lambda}\rho$ are | 0.420 | 0.399 | 0.378 | 0.356 | 0.335 |
| | $\frac{j}{q}$ are | 0 | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ |

Lay off $K_1'k_1 = 0.25$ on a line perpendicular to $OK_1'$, also $K_2'k_2 = 0.25$ on a line perpendicular to $OK_2'$; the points $k_1$ and $k_2$ will be points on the locus corresponding to $u = c$, or full gear forward and backward respectively. On each of these two perpendiculars lay off from $K_1'$ and $K_2'$, in the direction $K_1'k_1$, and $K_2'k_2$ the distance 0.335, and join the ends of these distances; where this connecting line cuts $OC$ will be the points $p$ of the locus corresponding to mid-gear. This gives us three points $k_1$,

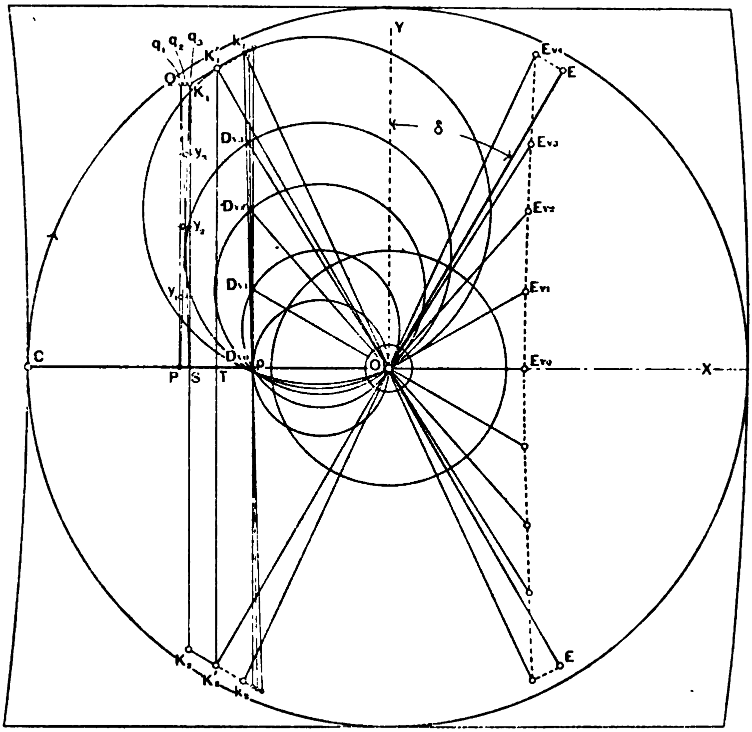


Fig. 139.

$p$ and $k_2$ on the desired locus; a circle through these three would closely represent the curve but the arc is so very flat that its center would fall off the drawing. We will, therefore, construct the arc. The lines are so close together that only the construction for half gear forward is shown. We make the offset at $K_1'$ equal to 0.292 and that at $K_2'$ equal to 0.378 and join the ends of these

offsets. This connecting line is divided at $D_{v_2}$ in the ratio of 1 to 3, and therefore $D_{v_2}$ is another point on the locus. The other points are found in like manner. This locus can be shown to be a parabola and for the present case of *crossed* rods is convex to the center $O$.

For *open* rods the locus is $K_1P$ for forward gear and is again a parabola but is not exactly like locus $k_1p$ for forward gear and is again a parabola but is not exactly like locus $k_1p$ for crossed rods. We have however $TP = Tp$ and $K_1'K_1 = K_1'k_1$. The locus for open arms is concave to center $O$. To find it we will use, without proof, a simpler demonstration given by Burmester. On the diagonal $K_1P$, joining the known points $K_1$ and $P$, construct the rectangle $K_1QPS$; draw the second diagonal $QS$. To find points of the locus on the parallels through $y_1$, $y_2$ and $y_3$ we draw through these points parallel to $QS$ to their intersection with $K_1Q$ at $q_1$, $q_2$ and $q_3$ respectively. The intersections of the lines $Pq_1$, $Pq_2$ and $Pq_3$ with the corresponding parallels will be points on the desired locus.

The valve-circles have only been drawn for the case of crossed rods, forward gear. They show but little variation in the lead. The corresponding eccentric positions, when crank is at dead point, are shown on the other side of the figure.

STEPHENSON'S LINK MOTION. The dimensions taken by Dr. Zeuner for his diagram: $\rho = 2.36''$, $\delta = 30°$, $\lambda = 55''$, $c = 5.9''$ $e = 0.94''$ and $i = 0.27''$. We will consider the grades of expansion represented by

$$u = c, \quad u = \tfrac{3}{4}c, \quad u = \tfrac{1}{2}c, \quad u = \tfrac{1}{4}c, \quad u = 0$$

for *open* rods. Since $c_1 = c - u$ and $c_2 = c + u$ we have the corresponding values of

$$\frac{c_1}{\lambda}\rho \quad = \quad 0 \quad 0.064 \quad 0.127 \quad 0.191 \quad 0.254$$

$$\frac{c_2}{\lambda}\rho \quad = \quad 0.508 \quad 0.444 \quad 0.381 \quad 0.317 \quad 0.254$$

$K_1'$ and $K_2'$ are points of the locus of the centers ot the virtual eccentrics which correspond to $u = +c$ for full gear foward and to $u = -c$ for full gear backward. To find the other points on the locus, make the offsets $K'_1t_3$, $K'_1t_2$, $K'_1t_1$, $K'_1\ K_1$ respectively equal to, 0.06, 0.13, 0.19, 0.25; also on the perpendicular $k_2K_2$ make

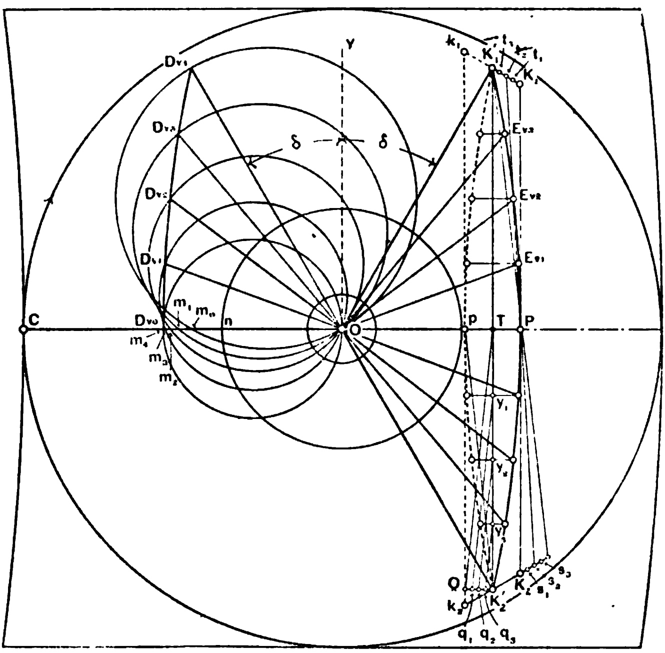


Fig. 140.

the offsets $K'\ s_3$, $K'_2s_2$, $K'_2s_1$, $K'_2K^2$ equal to the corresponding values 0.44, 0.38, 0.32, 0.25. Join $s_1$ and $t_1$, $s_2$ and $t_2$, $s_3$ and $t_3$, $K_1$ and $K_2$. On $s_3t_3$ make $t_3E_{v_3} = \frac{s_3t_3}{8}$, on $s_2t_2$ make $t_2E_{v_2} = \frac{s_2t_2}{4}$ and on $s_1t_1$ make $t_1\ E_{v_1} = 3/8\ s_1t_1$. The points $E_{v_3}$, $E_{v_2}$, $E_{v_1}$ and the mid-point $P$ of $K_1K_2$ are points on the desired locus. $OK'_1$ is at once actual and virtual eccentric, while $OE_{v_3}$, $OE_{v_2}$, $OE_{v_1}$ and

$OP$ are all virtual eccentrics corresponding to the given grades of expansion for forward motion.

The locus $K'_1PK'_2$ for *open* rods is concave towards center $O$. The locus $K'_1pK'_2$ for *crossed* rods is convex towards center $O$. The two loci are exactly equal and symmetrical to the line $K'_1K'_2$. That for crossed rods can be found in the same manner as the one for open rods, but we have preferred to find it by a much simpler method given by Burmester,* omitting the proof.

In the lower part of Fig. 140 on the diagonal connecting the known points $K'_2$ and $p$ we construct the rectangle $pQK'_2T$ and then draw the second diagonal $Tk_2$. To find points of the locus on any parallels $y_1, y_2$ and $y_3$ we draw through points $y_1, y_2$ and $y_3$ parallels to $Tk_2$ letting them cut $QK'_2$ in $q_1$, $q_2$, $q_3$. Then the intersections $pq_1$ with parallel $y_1$, of $pq_2$ with $y_2$, of $pq_3$ with $y_3$ will give three points on the desired locus. Of course the locus for open rods might have been found in the same manner.

Drawing the valve-circles on the corresponding diameters $OD_{v_4}$, $OD_{v_3}$, $OD_{v_2}$, $OD_{v_1}$, $OD_{v_0}$ and the lap circles we get, in the usual manner, the steam distribution for the different grades of expansion. The varying lead, $nm_0$, $nm_1$, $nm_2$, $nm_3$ and $nm_4$ is particularly noticeable. We will show later how this variation may be reduced for this link motion. It is however not so serious a defect as has been commonly believed.

PORTER-ALLEN LINK MOTION. Dimensions are taken from the 11½ x 20 engine shown in text, Figs. 79–82. Here $\rho = 1''$, $A = 6''$, $\delta = 90°$, $c = 6''$, $e = 1\frac{1}{16}$, $i = \frac{1}{16}$. The engine "runs over" and the expansion and exhaust valves are both of the positive direct type. Between the link and expansion valves there are reversing levers whose arms multiply the motion of the link four-thirds. The diagram should be constructed as if the valve were driven by a correspondingly enlarged link-motion. There is also a lever between link and exhaust valve, but this does not reverse

* This method and its demonstration are given in the "Lehrbuch der Kinematik" p. 722. For another easy method see Fliegner's "Umsteuerungen der Locomotiven," p. 81.

the motion, it reduces the travel in the ratio of 4$\frac{5}{8}$ to 11$\frac{1}{16}$. The exhaust valve does approximately follow the law of eccentric motion but the steam valve does not do it as well, for the reversing is effected by a *bent* lever of such an angle as to effect a differential motion like that of the wrist-plate of the Corliss engine. This valve motion has been satisfactorily discussed in the text, see Figs. 83–91 inclusive, and the accompanying notes. For the purpose of comparison however the graphical methods of the preceding link motion may be followed. Then for the grades of expansion:

$$u = c \qquad u = \tfrac{3}{4}c \qquad u = \tfrac{1}{2}c \qquad u = \tfrac{1}{4}c \qquad u = c$$

we have $c_1 = c_2 = c$ and the corresponding values of $\dfrac{j}{j+q}$:

$$0 \qquad \tfrac{1}{8} \qquad \tfrac{1}{4} \qquad \tfrac{3}{8} \qquad \tfrac{1}{2}$$

Then on a scale of $\frac{2}{1}$ make the offset $\frac{4}{3}\,\frac{c}{\Lambda}\rho = \frac{4}{3}''$.

## ADJUSTMENTS OF LINK MOTIONS SO AS TO EQUALIZE MOTION.

"The requirements of a perfectly equalized link motion are: perfect quality of cut-off, of exhaust closure, lead opening and maximum port opening, together with absence of block-slip, between the forward and return stroke of the piston for every suspension of the link from full gear forward to full gear back. Such theoretical excellence is absolutely impossible with ordinary link motion. But good practical qualities may be obtained by sacrificing the *non-essential* to the *essential* points of the motion."

Auchincloss, from whose well-known work these remarks are quoted, then proceeds to lay out the motion by making the leads equal at mid-gear and the cut-offs equal at half and full gear forward and back, thus indicating that these elements constituted the *essential* points of the motion. At $\frac{1}{2}$ cut-off however the condition favorable to minimum slip is introduced. The lead at full gear and the slip of the block then existing in the designed

link-motion are examined and the directions in which modifications should be made are suggested.*

Zeuner, Burmester and Fliegner, on the other hand, seem to lay greater stress on equality of lead at the at ends for each grade of expansion and on the reduction of the slip. As these link motions are usually accompanied by long connecting rods the inequalities of cut-off due to the angular motion of the rods is not very great and we believe them to be comparatively unimportant, particularly at high speeds. We shall therefore follow the presentation given by Drs. Zeuner and Burmester. There are two leading principles or conditions:—

1. A regular and correct distribution of the steam requires that the valve should move symmetrically on each side of a point (the center of motion) that remains fixed for all grades of expansion.

2. A minimum slip of block requires that the trunnions of both the link and the radius rod should have as little vertical motion as possible, in other words, the chords of the arcs described by the lower ends of the hangers should be parallel to the central line of motion and the hangers themselves should be as long as possible.

The first condition is satisfied in Gooch's motion by making radius of link-slot equal to length of the radius-rod; in Stephenson's motion by making the arc of this slot equal to the length of the eccentric rod; and in Allan's motion by making the arms

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* To reduce the slip of the block if it should be excessive, Auchincloss recommends alterations in the following elements, (enumerating them in the order of relative efficiency), (1) Increasing the angle of advance, (2) Reducing the travel of the valve, (3) Increasing the length of the link, (4) Shortening the eccentric rods. When the proportions are unusual, as in link-motions for small launch-engines, considerable deviation from the standard arrangement is profitable, the irregularity introduced curing greater irregularities in the more important functions. In such cases it will always be well to check and correct work by a model.

of the lever carrying the hangers (which move link and radius-rod) bear to each other the ratio $\frac{b}{a} = \frac{\lambda_0}{\lambda}\left(1 + \sqrt{1 + \frac{\lambda}{\lambda_1}}\right)$. In all three motions the effect is to give *equal* leads at the two ends for each grade of expansion, but the lead is not *constant* for all grades of expansion, except in the case of Gooch's motion. In Allan's motion the lead is slightly variable $\left\{\begin{matrix}\text{increasing}\\\text{decreasing}\end{matrix}\right\}$ from full gear to mid-gear in case of $\left\{\begin{matrix}\text{open}\\\text{crossed}\end{matrix}\right\}$ rods. In Stephenson's motion there is still greater variation and in the same direction as in Allan's gear.

Dr. Zeuner has shown however that giving the eccentrics unequal angles of advance would tend to correct this. A simple way of doing this is to change the angle between crank and eccentrics by moving *back* the crank when link drives the valve directly and by moving the crank *forward* (*i. e.* in direction of rotation) when there is a reversing lever between link and valve. The amount of this change in the setting is about 5°, or more exactly, it is the angle whose tangent is equal to $\frac{c_1}{\lambda}$. When this modification is introduced the Stephenson becomes the simpler and more compact gear of the three and equally efficient.

The second condition is not satisfactorily fulfilled in the case of the Allan motion as ordinarily constructed. The upper ends of the hangers should move in parabolic curves that are convex to each other. Instead of that they are guided in circular arcs that are concave to each other. In this case make the hangers as long as possible, make total length of lever $a + b = \lambda_1 - \lambda_0$ and give fulcrum of lever an abscissa (measured from center of shaft) equal to $\lambda + a - \frac{c^2}{2\lambda}$ and an ordinate equal to length of hanger.

In Gooch's motion the second condition is met by placing trunnions of link at center of chord of the link-arc are utilized* and upper end of the hanger is placed so as to have an abscissa $\lambda - \frac{c^2}{2\lambda}$ (estimated from center of shaft) and an ordinate $\lambda_2$ = length of hanger. The abscissa is obtained by placing link at mid-gear with center of chord at points on central line of motion that correspond to the dead points of crank and then bisecting the distance between them. The condition is also fulfilled for the hanger of the radius-rod by finding the abscissa of point of suspension of the hanger in a similar way, namely, by placing crank at both dead points (for each grade of expansion) and the radius rod in corresponding positions (which will be nearly parallel) and then bisecting the horizontal distance between the two trunnion positions of the rod. The ordinate will be the length of the hanger and, erected perpendicular to the central line of motion at the point of bisection belonging to each grade of expansion, will give the locus of suspension for the uppper end of the hanger of the radius-rod. This locus is a circle with radius $\lambda_0$ struck from a center which is at the distance $\lambda_3$ above the mid-position of end of valve stem.

In Stephenson's motion the second condition is only partially satisfied by the method of suspension usually employed in American practice. Concerning this matter of suspension the authorities differ according to their view of what constitutes an excellent gear. Dr. Zeuner's rule, of guiding the upper end of the hanger in the arc of a parabola, (having a parameter $2\lambda$ and a vertex whose coordinates are $\lambda - \frac{c^2}{2\lambda}$ and $\lambda_2$) or in the arc of a

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* Dr. Burmester chooses a different point than middle of chord for the trunnion. He finds that each point of suspension for hanger, on the vertical having the abscissa $\lambda - \frac{c^2}{2\lambda}$, has a particular trunnion point on the link-chord which will make the center of that chord travel most closely to the central line of motion. We do not reproduce the construction here, referring the reader to p. 711, Fig. 704, of Burmester's "Lehrbuch der Kinematik."